CSE 250

Data Structures

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Hash Functions

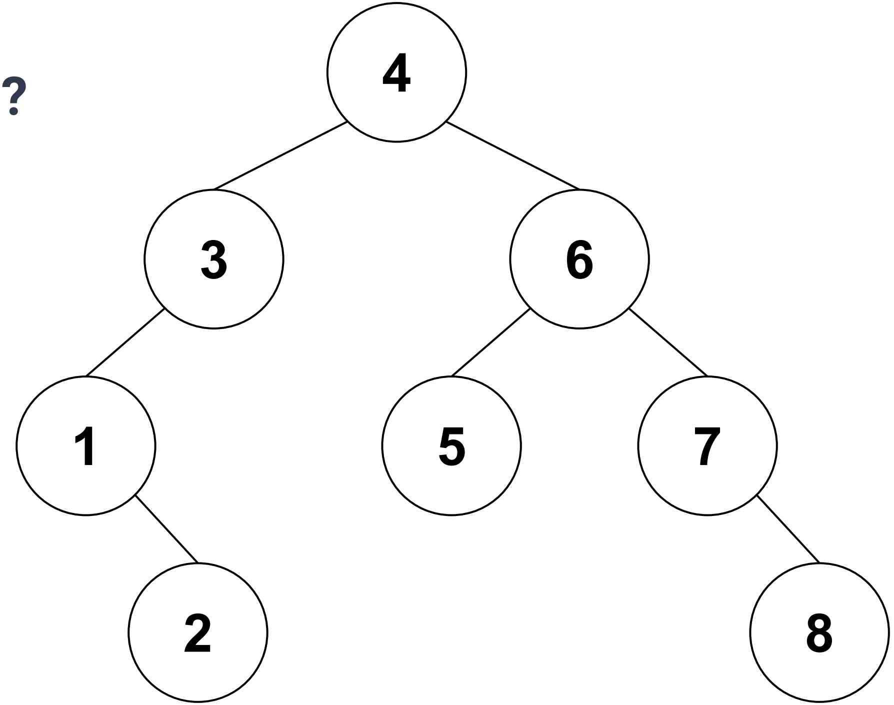
Announcements

- WA3 released, due 4/23/23 @ 11:59PM
    - Make sure you have the correct version

Clarification on Red-Black Trees

Is the following a valid Red-Black Tree?

What depth are the EmptyTree nodes?



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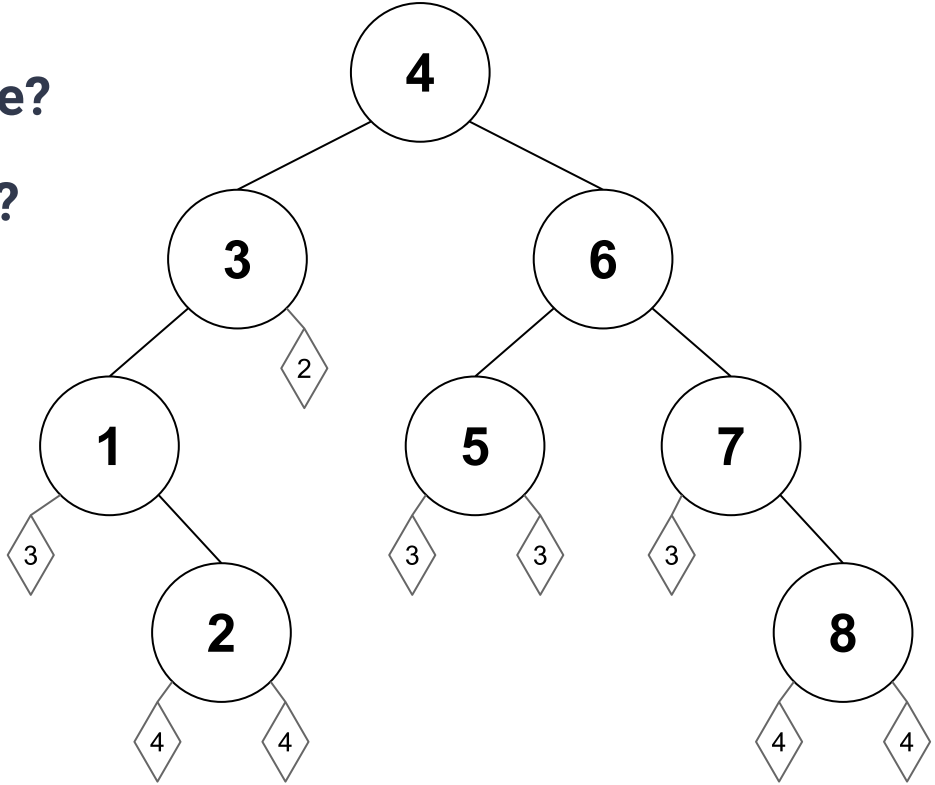


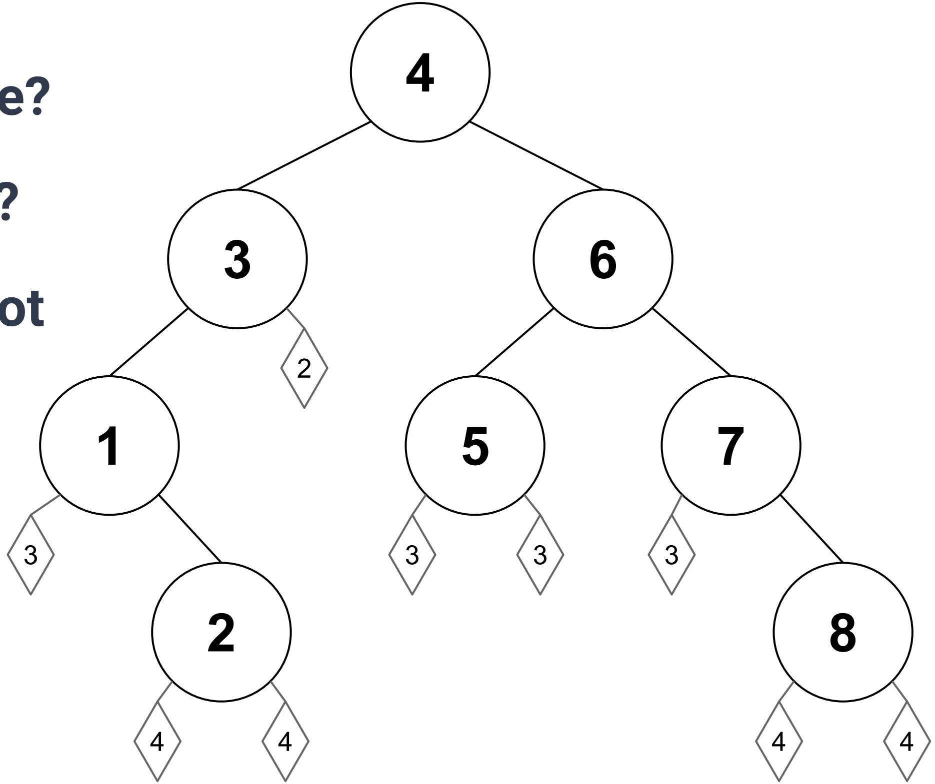
Clarification on Red-Black Trees

Is the following a valid Red-Black Tree?

What depth are the EmptyTree nodes?

By just considering depth from the root this tree seems to fit the necessary constraints…

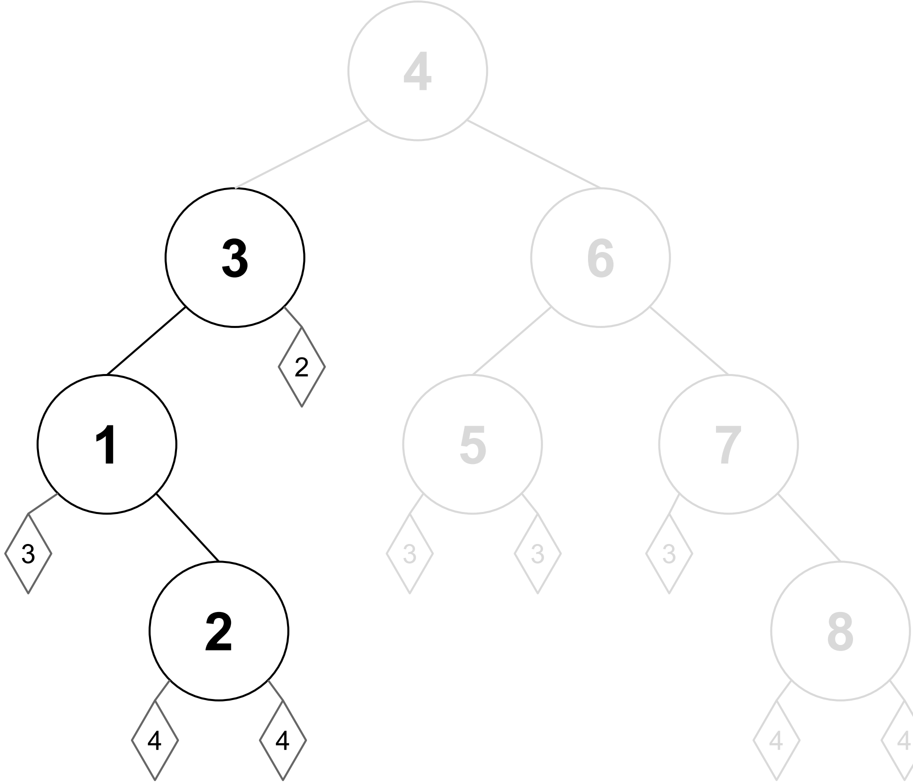
But just like AVL trees, this property must hold for ALL nodes in the tree…



Clarification on Red-Black Trees

Consider the subtree rooted at 3

What are the depth of the EmptyTree nodes with respect to 3?



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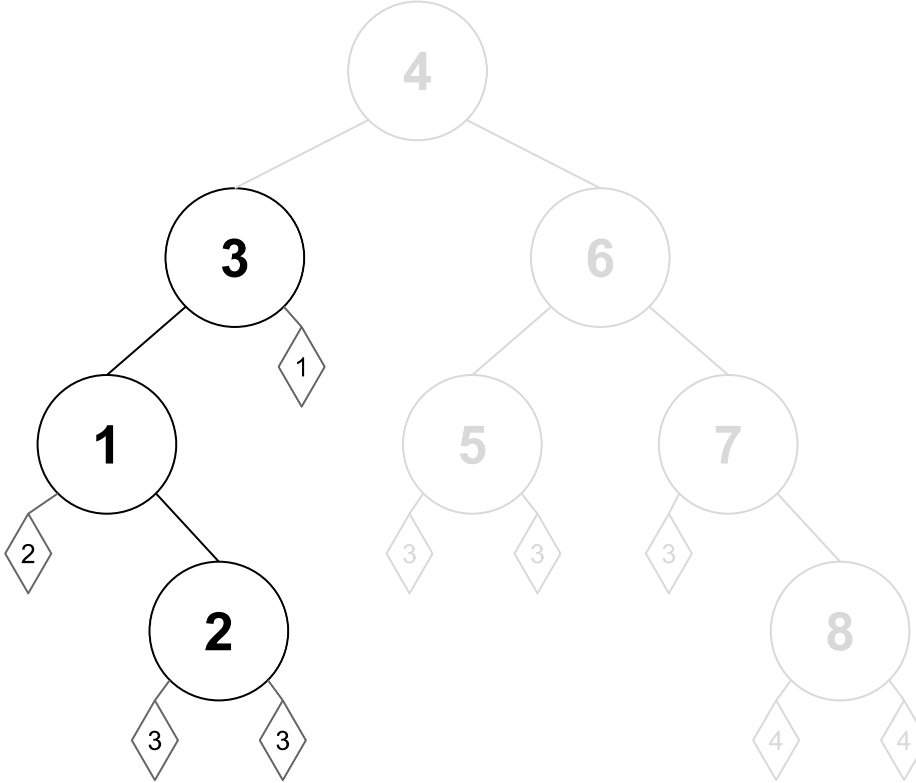


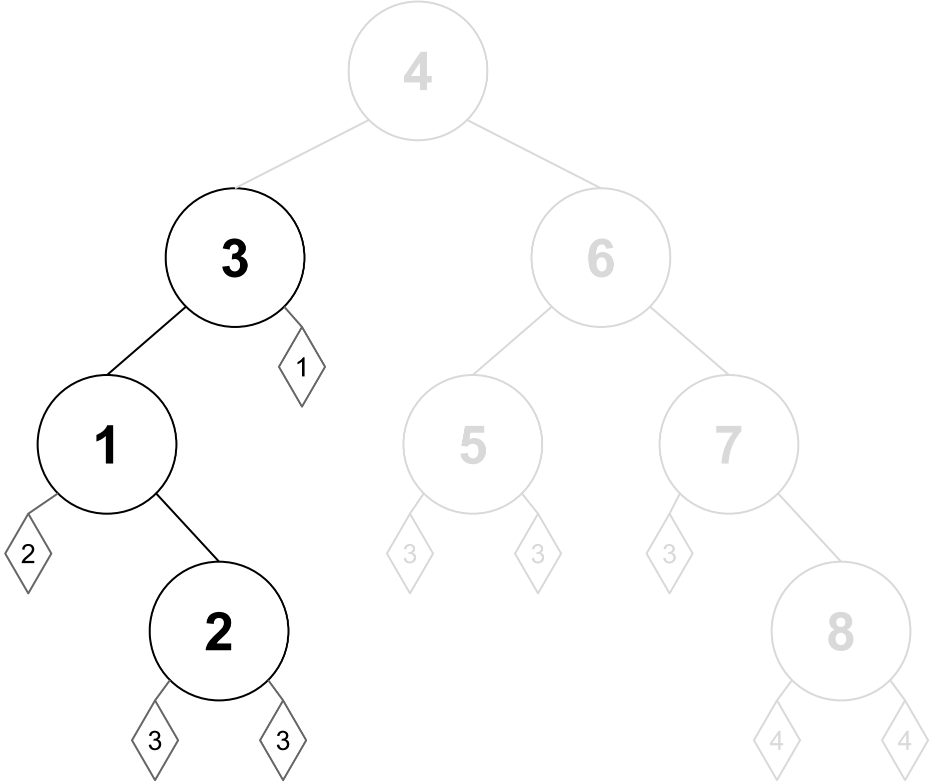
Clarification on Red-Black Trees

Consider the subtree rooted at 3

What are the depth of the EmptyTree nodes with respect to 3?

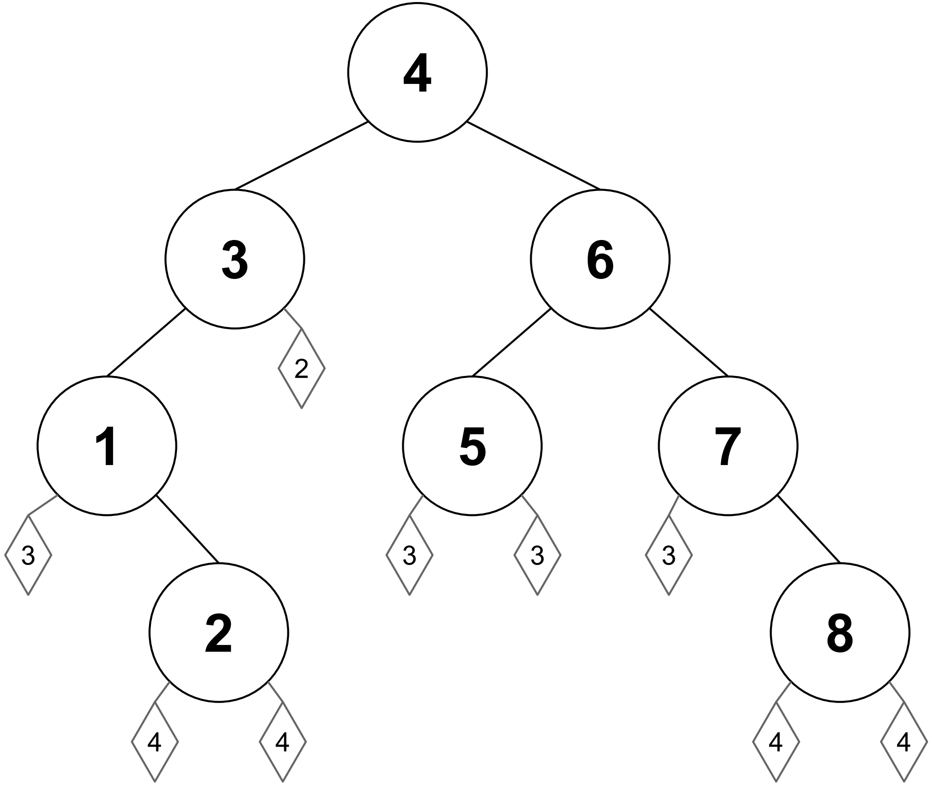
This does not meet Red-Black constraints on depth

…and we can see it is impossible to color these nodes by the rules



Clarification on Red-Black Trees

Since we are unable to color the subtree rooted at 3, we are unable to color the entire tree



Back to HashTables…

`Map` Implementations

`Map[K,V]` as a **Sorted Sequence**

- `apply` **$O(\log(n))$ for Array, $O(n)$ for Linked List**
- `add` $O(n)$
- `remove` $O(n)$

`Map[K,V]` as a **balanced Binary Search Tree**

- `apply` $O(\log(n))$
- `add` $O(\log(n))$
- `remove` $O(\log(n))$

Finding Items

For most of these operations, the expensive part is **finding** the record…

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So…let's skip the search

Assigning Bins

Idea: What if we could assign each record to a location in an Array

- Create and array of size **N**
- Pick an ***O*(1)** function to assign each record a number in **[0,N)**
    - ie: If our records are names, first letter of name → [0,26)

Assigning Bins

A	B	…	F	G	H	…	Z

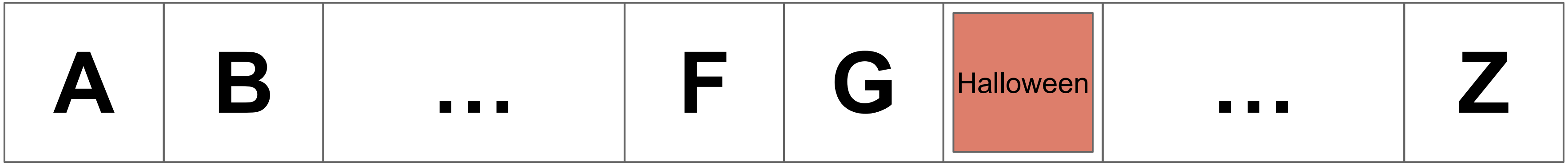
Assigning Bins

```
add("Halloween")
```

A	B	...	F	G	H	...	Z

Assigning Bins

`add("Halloween")` → "Halloween"[0] == "H" == 7

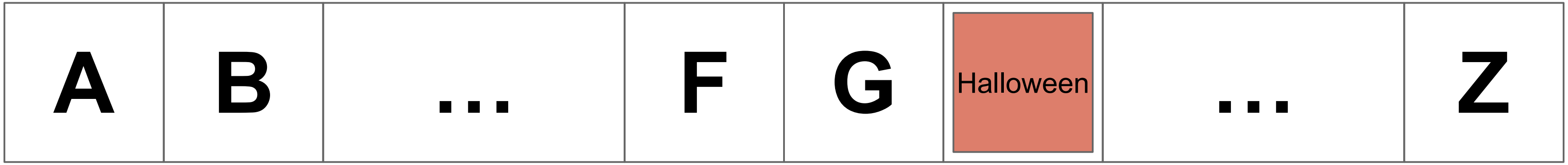


A	B	…	F	G	Halloween	…	Z

Assigning Bins

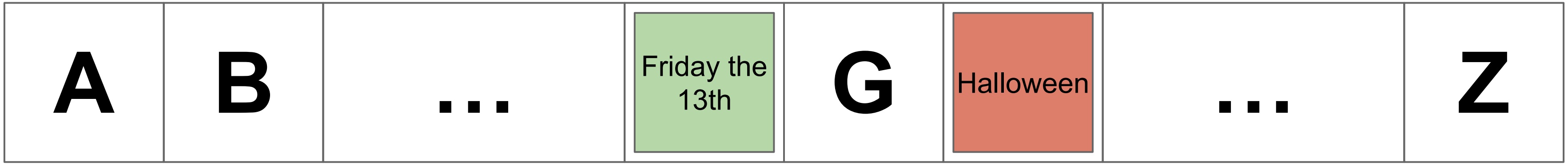
`add("Halloween")` → "Halloween"[0] == "H" == 7

This computation is ***O*(1)**



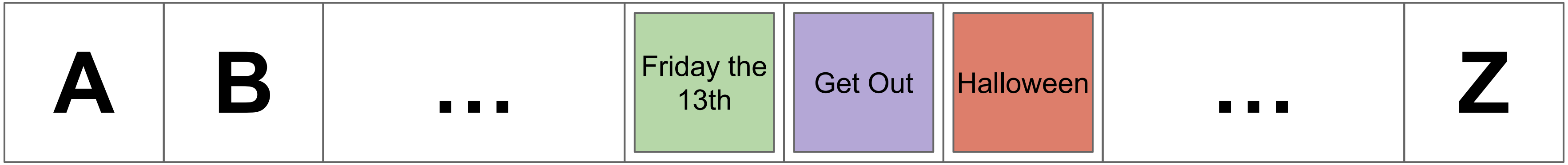
Assigning Bins

`add("Friday the 13th")` → "Friday the 13th"[0] == "F" == 5



Assigning Bins

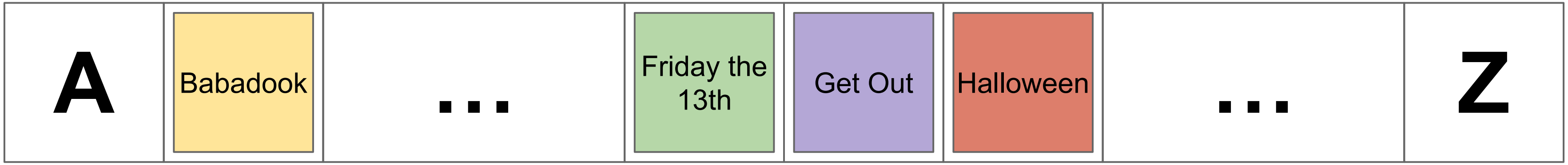
`add("Get Out")` → "Get Out"[0] == "G" == 6



A	B	…	Friday the 13th	Get Out	Halloween	…	Z

Assigning Bins

`add("Babadook")` → "Babadook"[0] == "B" == 1

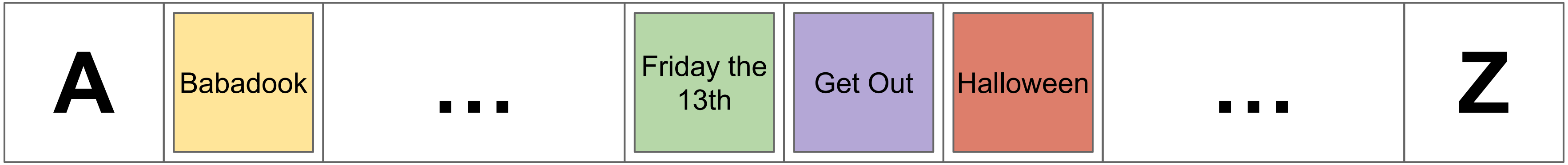


A	Babadook	…	Friday the 13th	Get Out	Halloween	…	Z

Assigning Bins

`find("Get Out")` → "Get Out"[0] == "G" == 6

Find in constant time!



A

Babadook

...

Friday the 13th

Get Out

Halloween

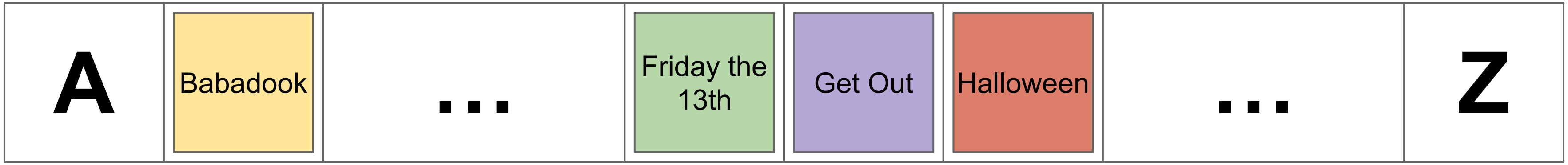
...

Z

Assigning Bins

`find("Scream")` → "Scream"[0] == "S" == 18

Determine that "Scream" is not in the Set in constant time!



A

Babadook

...

Friday the 13th

Get Out

Halloween

...

Z

Assigning Bins

Pros

- ***O***(1) insert
- ***O***(1) find
- ***O***(1) remove

Cons

- Wasted space (4/26 slots used in the example, will we ever use "Z"?)
- Duplication (What about inserting **F**rankenstein)

Bin-Based Organization

Wasted Space

- Not ideal…but not wrong
- ***O*(1)** access time might be worth it
- Also depends on the choice of function

Duplication

- We need to be able to handle duplicates

Bin-Based Organization

Wasted Space

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- $O(1)$ access time might be worth it
- Also depends on the choice of function

Duplication

- We need to be able to handle duplicates

What about "buckets" instead of "bins" (store multiple items per location)

Handling "Duplicates"

How can we store multiple items at each location?

Bigger Buckets

Fixed Size Buckets (*B* elements)

Pros

- Can deal with up to ***B*** dupes
- Still ***O***(1) find

Cons

- What if more than ***B*** dupes?

Arbitrarily Large Buckets (List)

Pros

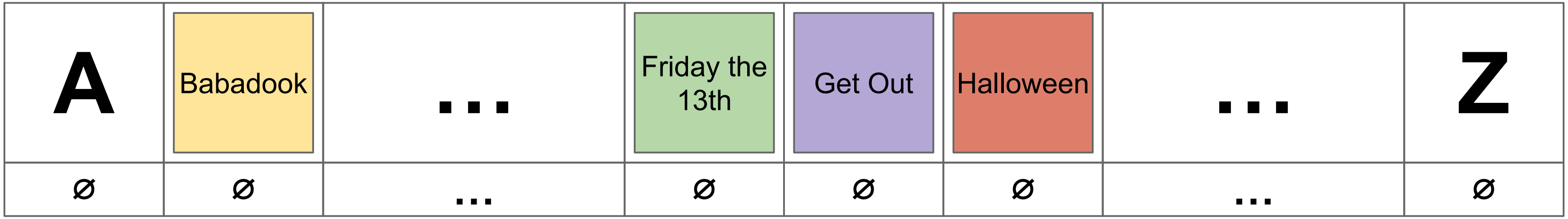
- No limit to number of dupes

Cons

- ***O***(***n***) worst-case find

Assigning Bins

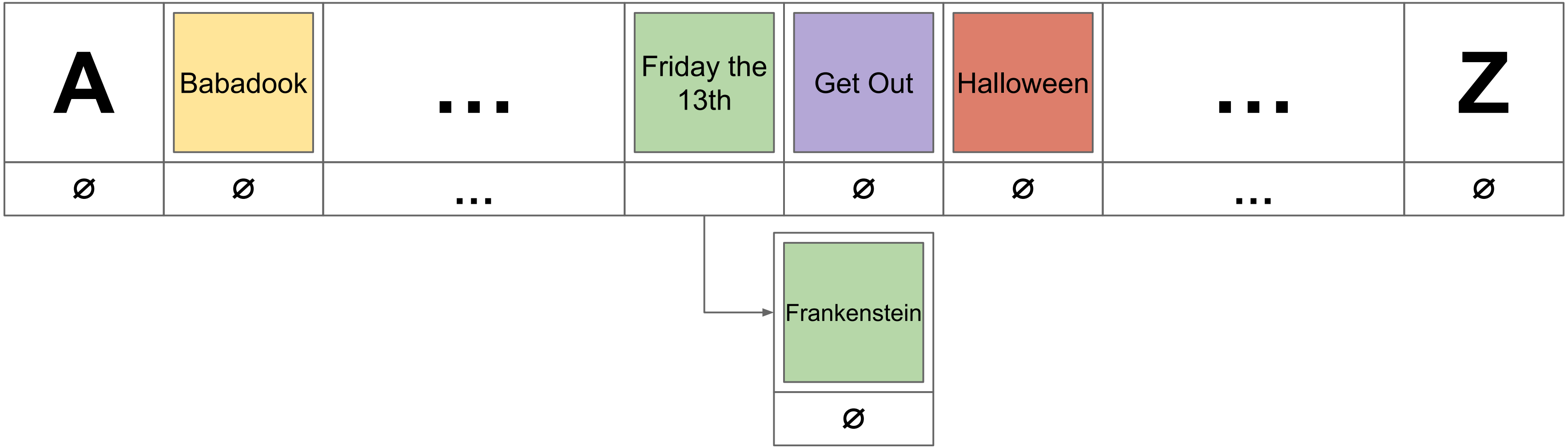
`add("Frankenstein")?`



A	Babadook	...	Friday the 13th	Get Out	Halloween	...	Z
∅	∅	...	∅	∅	∅	...	∅

Assigning Bins

`add("Frankenstein")?`



LinkedList Bins

Now we can handle as many duplicates as we need. But are we losing our constant time operations?

How many elements are we expecting to end up in each bucket?

LinkedList Bins

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How many elements are we expecting to end up in each bucket?

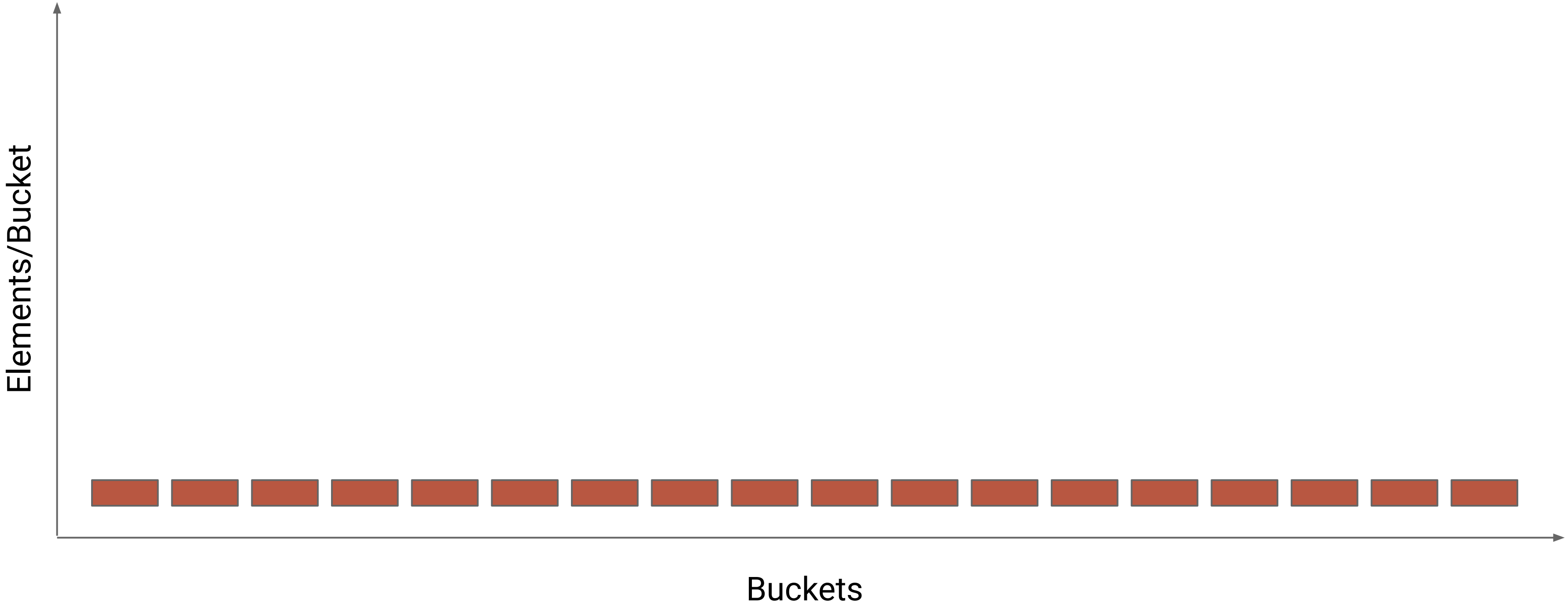
Depends partially on our choice of Hash Function

Picking a Hash Function

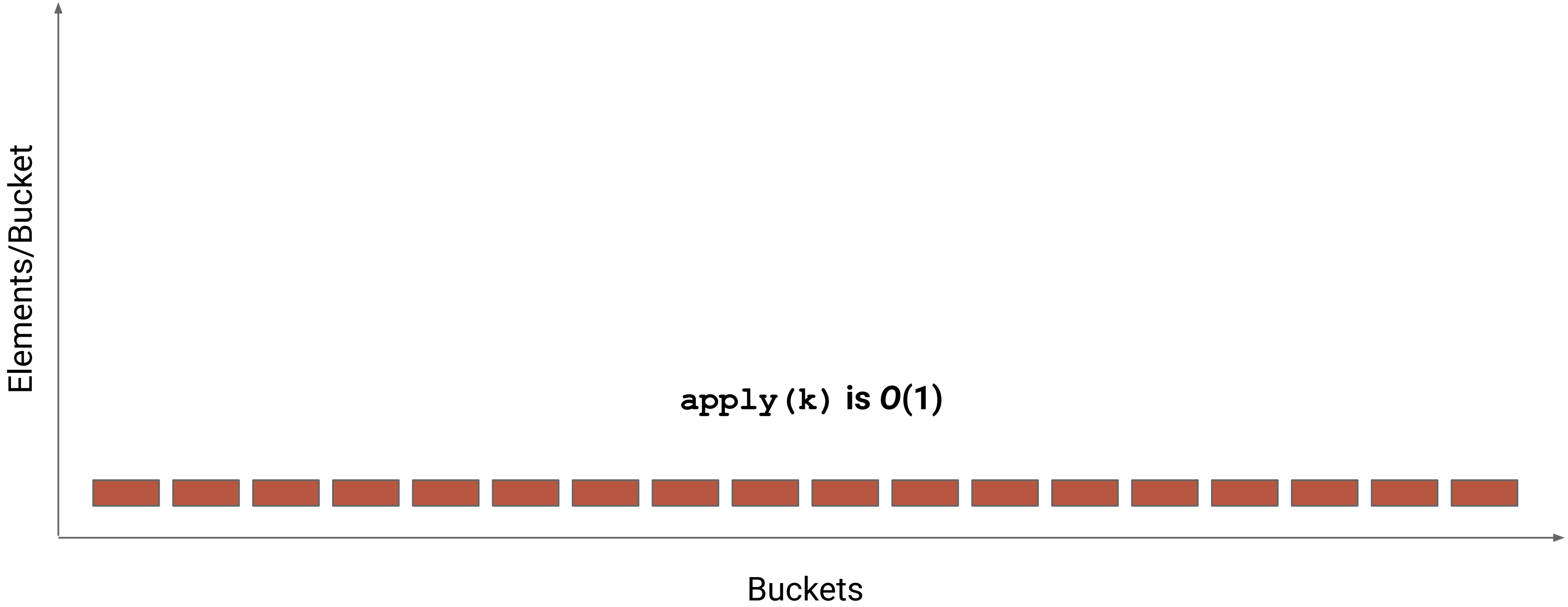
Desirable features for *h*(*x*):

- Fast — needs to be ***O*(1)**
- "Unique" – As few duplicate bins as possible

Picking a Hash Function

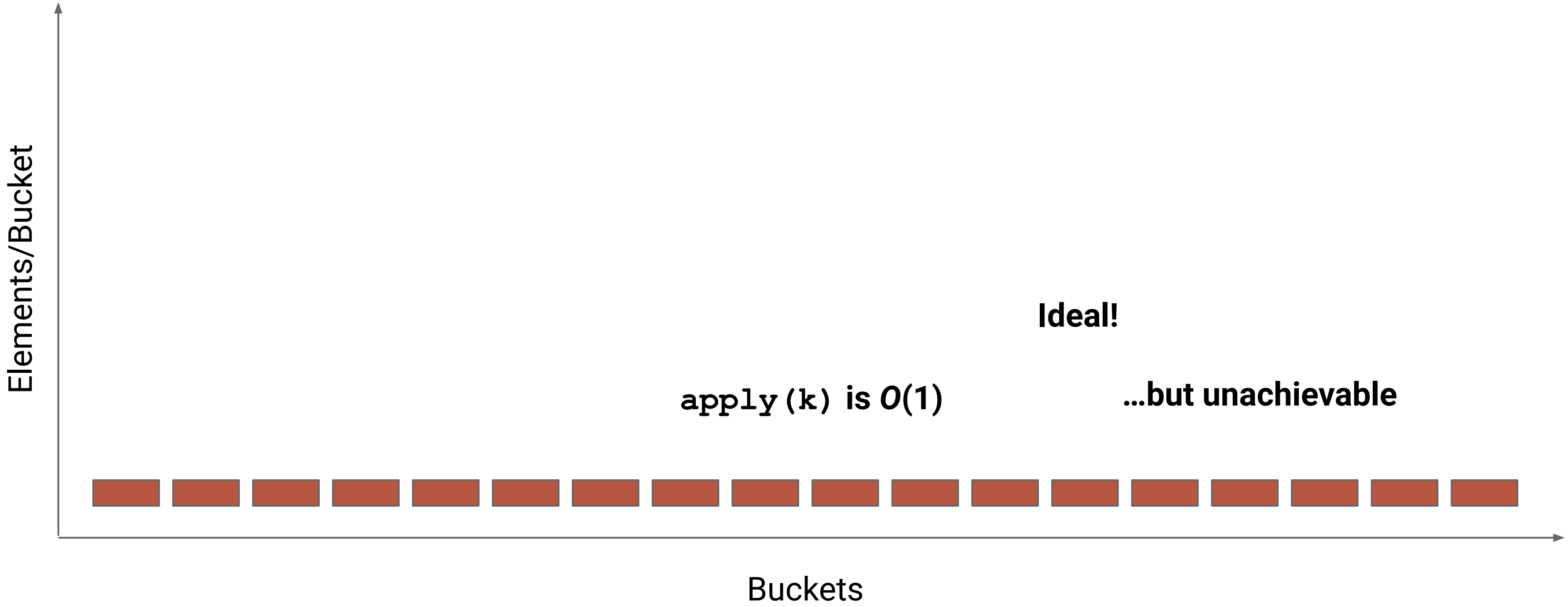


Picking a Hash Function



`apply(k)` is $O(1)$

Picking a Hash Function

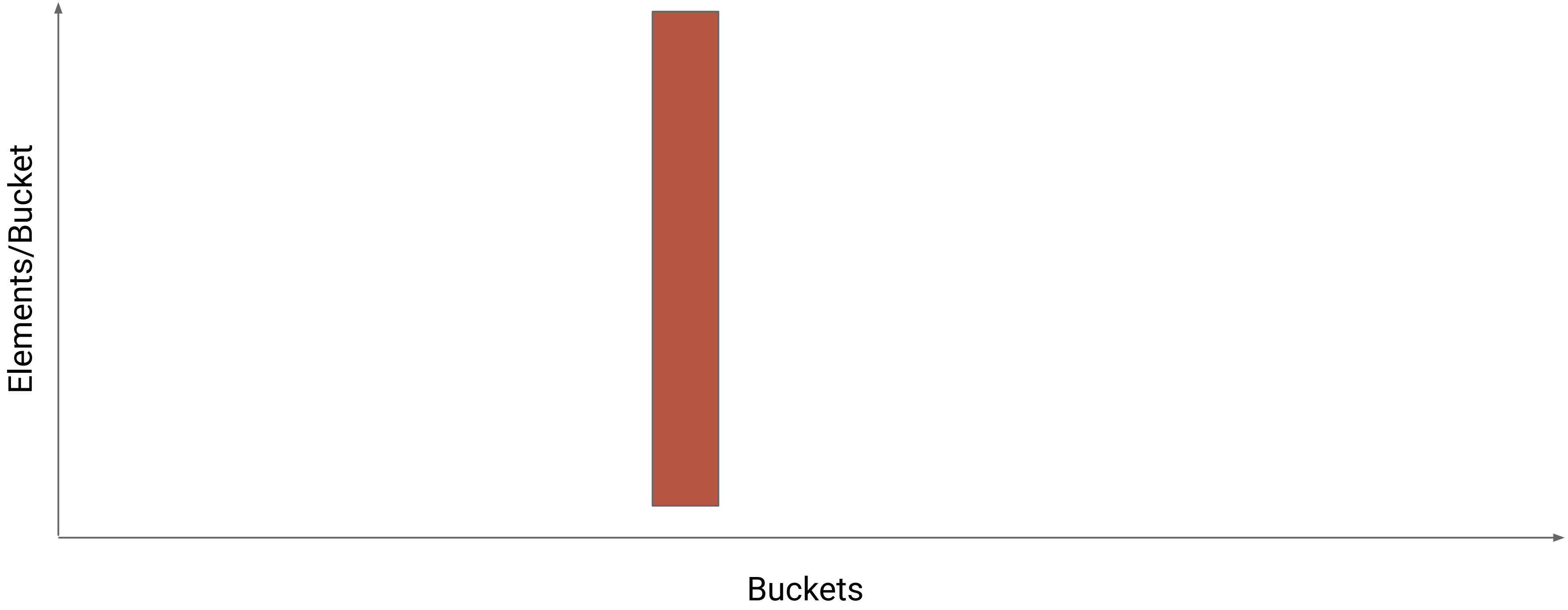


Ideal!

`apply(k)` is $O(1)$

…but unachievable

Picking a Hash Function



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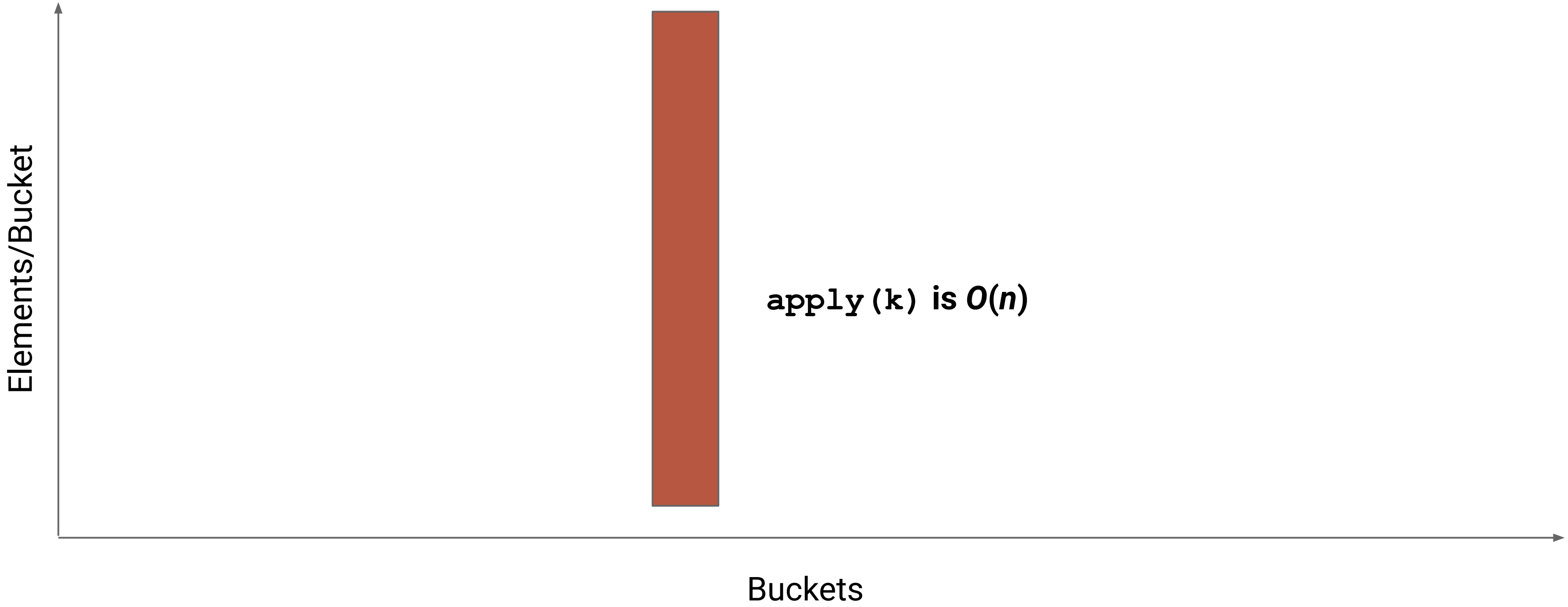

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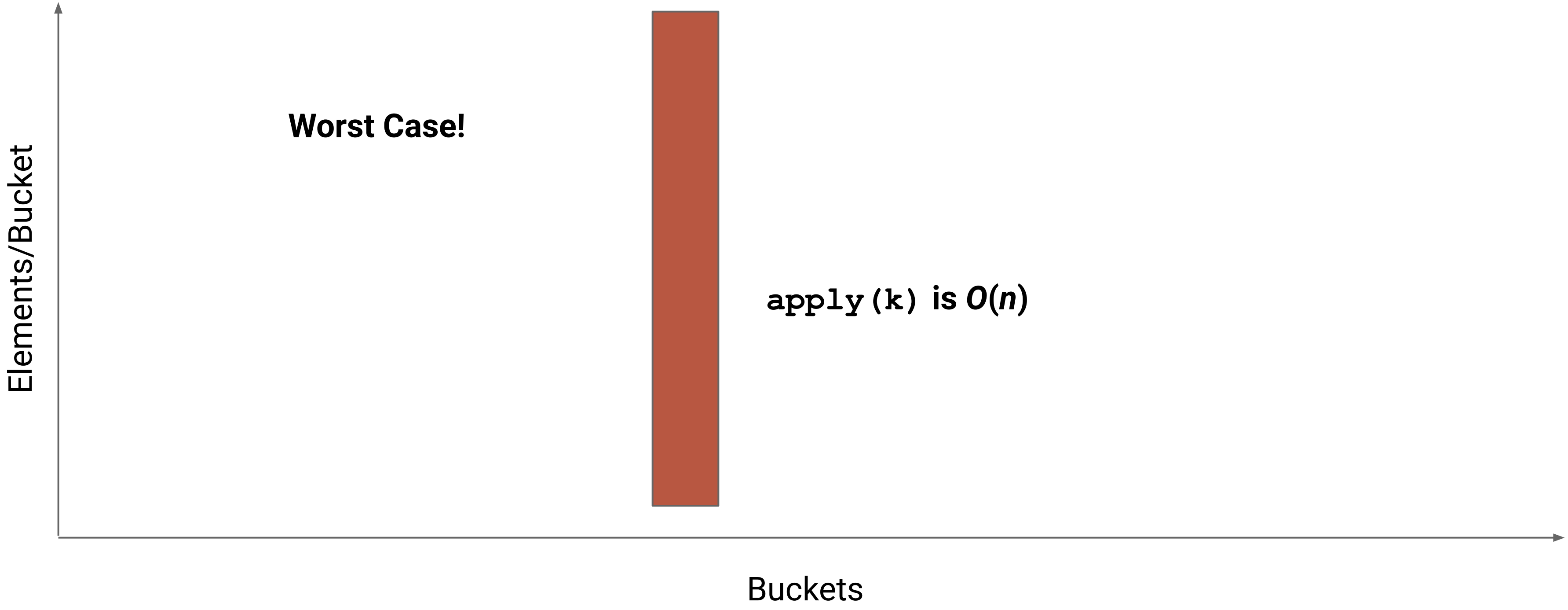
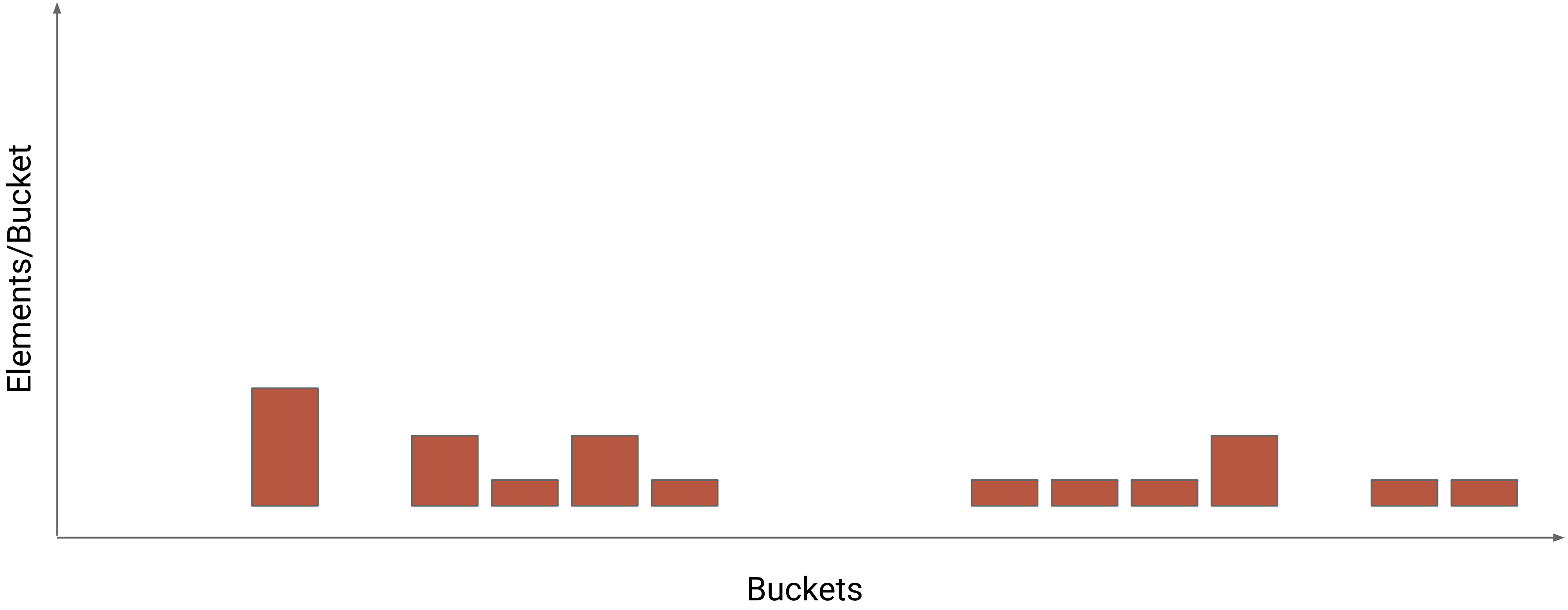


Worst Case!

`apply(k)` is *O*(*n*)

Picking a Hash Function



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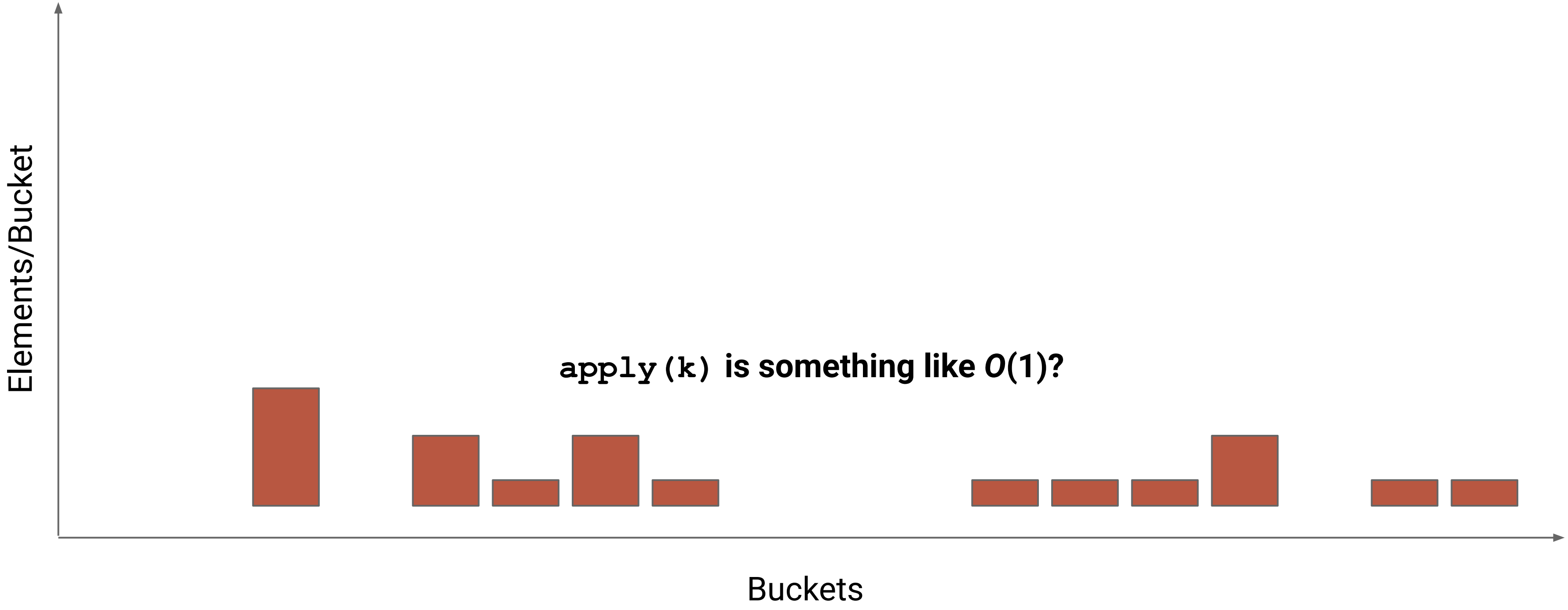
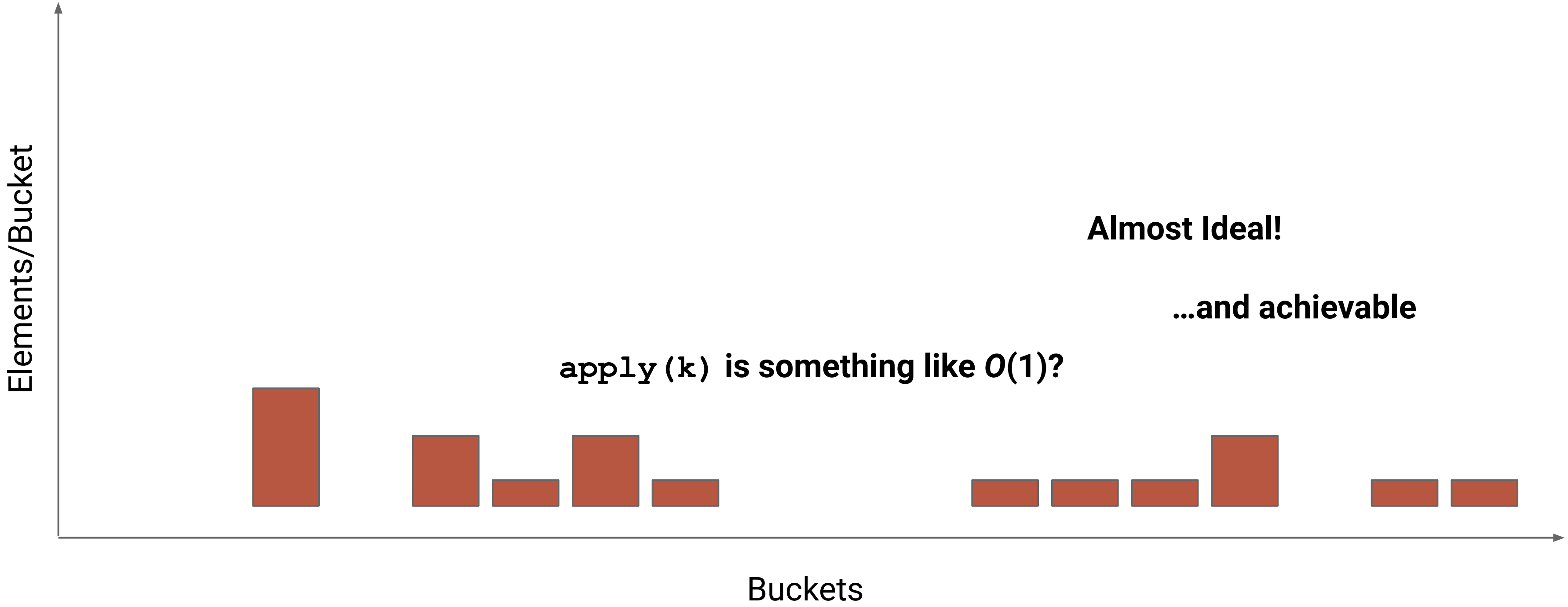


`apply(k)` **is something like *O*(1)?**

Picking a Hash Function



Almost Ideal!

…and achievable

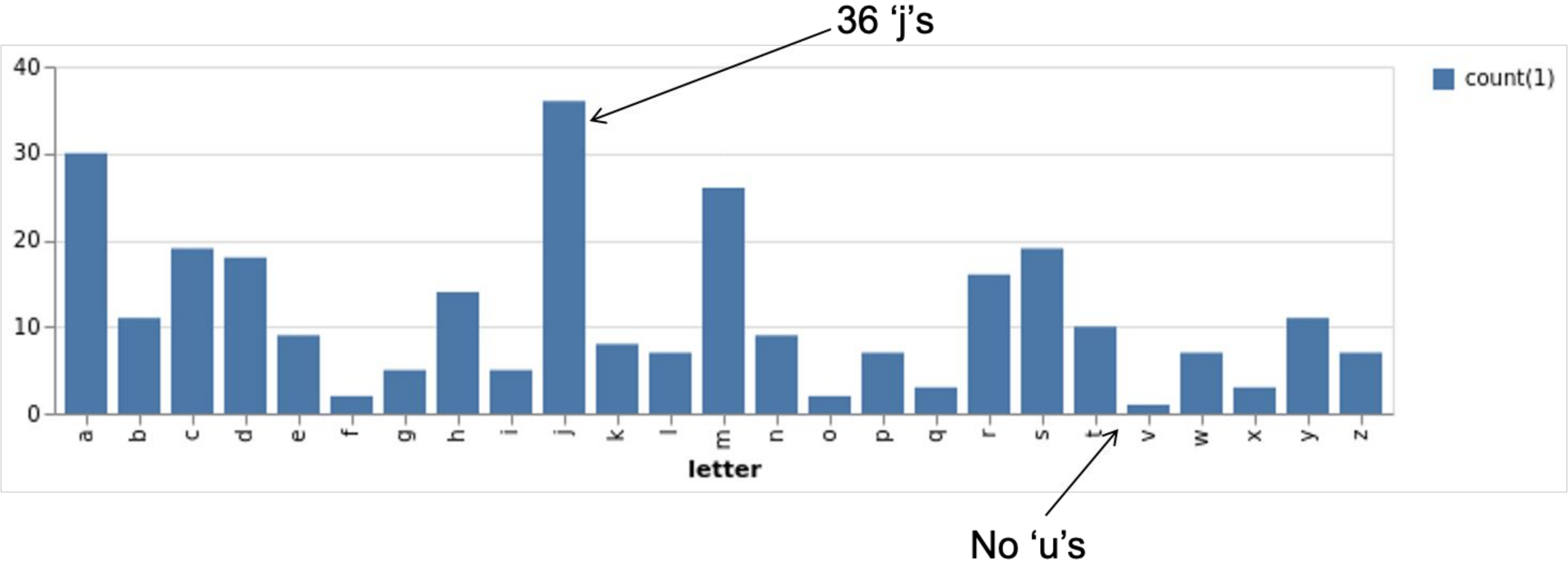
`apply(k)` is something like *O*(1)?

Other Functions

First Letter of UBIT Name

- Unevenly distributed, $O(n)$ worst case apply

First Letter of UBIT Name


36 'j's

No 'u's

Other Functions

First Letter of UBIT Name

- Unevenly distributed, ***O*(*n*)** worst case apply

Identity Function on UBIT #

- Need a 50m+ element array

Other Functions

First Letter of UBIT Name

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Identity Function on UBIT #

- Need a 50m+ element array
- **Problem:** For reasonable N, identity function returns something > N

Other Functions

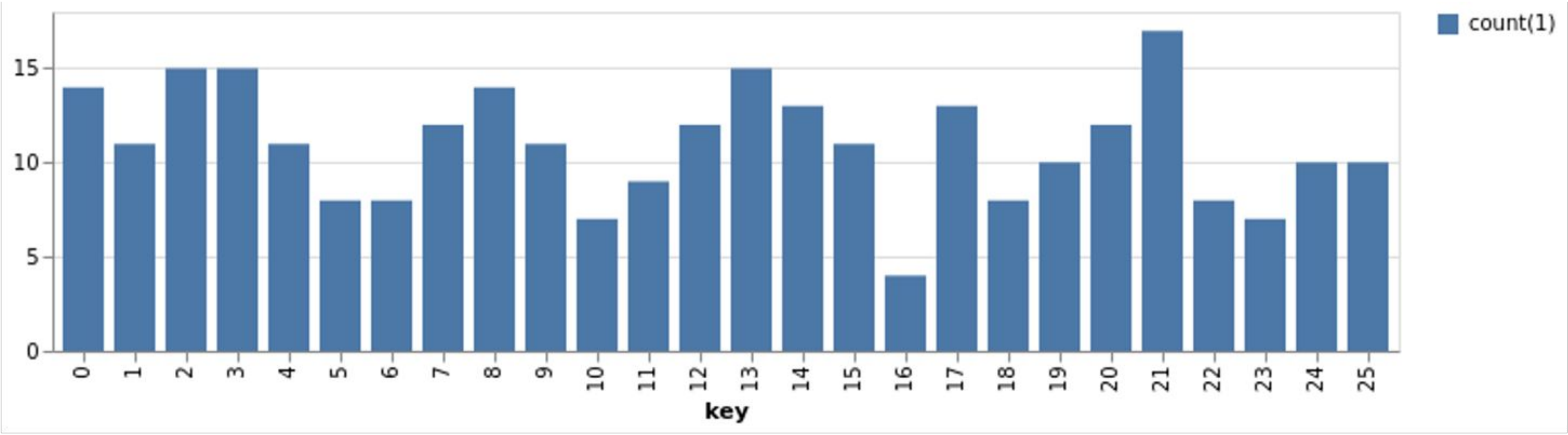
First Letter of UBIT Name

- Unevenly distributed, ***O*(*n*)** worst case apply

Identity Function on UBIT #

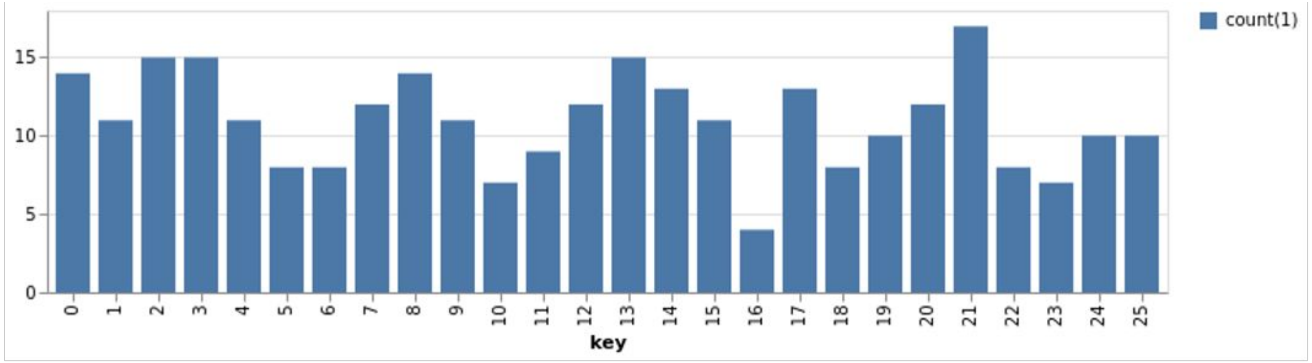
- Need a 50m+ element array
- **Problem:** For reasonable ***N*,** identity function returns something > ***N***
- **Solution:** Cap return value of function to ***N*** with modulus
  - (x: Int) => x % N

Identity of UBIT # mod 26

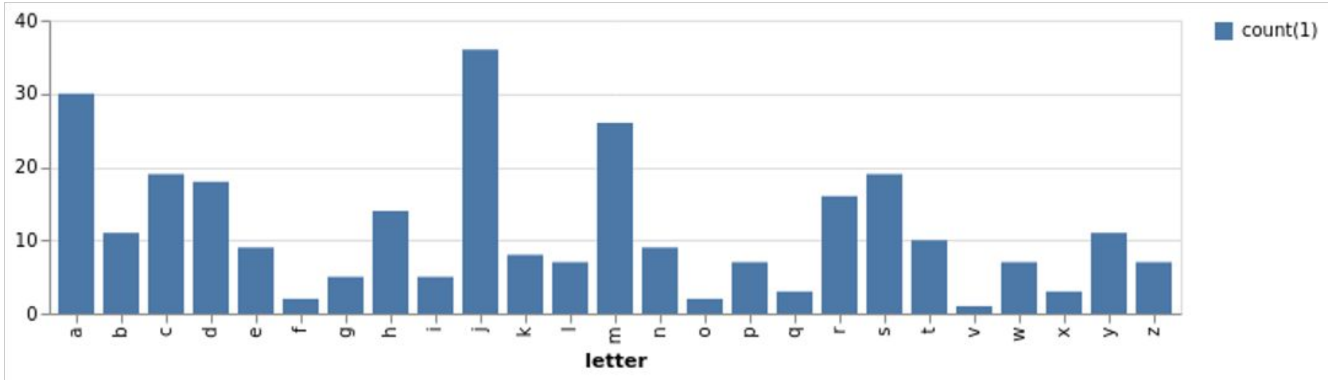


Comparison

UBIT # % 26



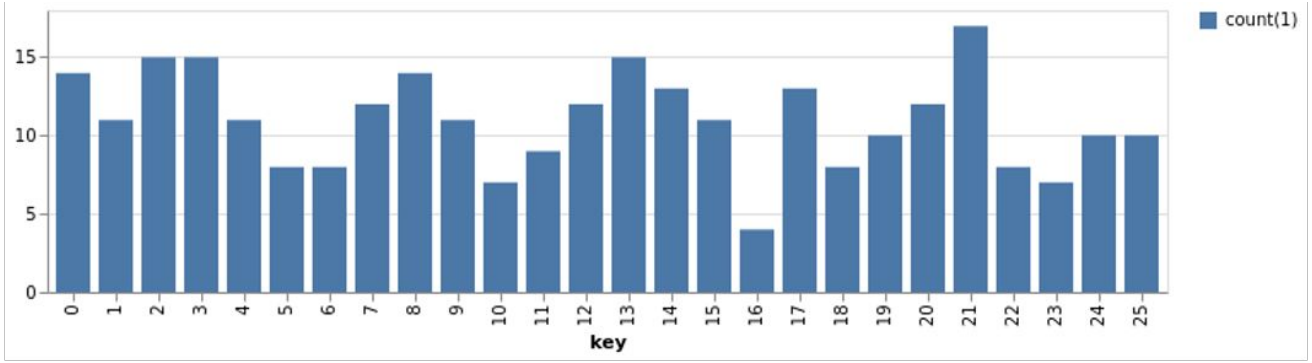
substr(UBITName, 0, 1)



Comparison

UBIT # % 26

This still relies on UBIT # being "randomly distributed"



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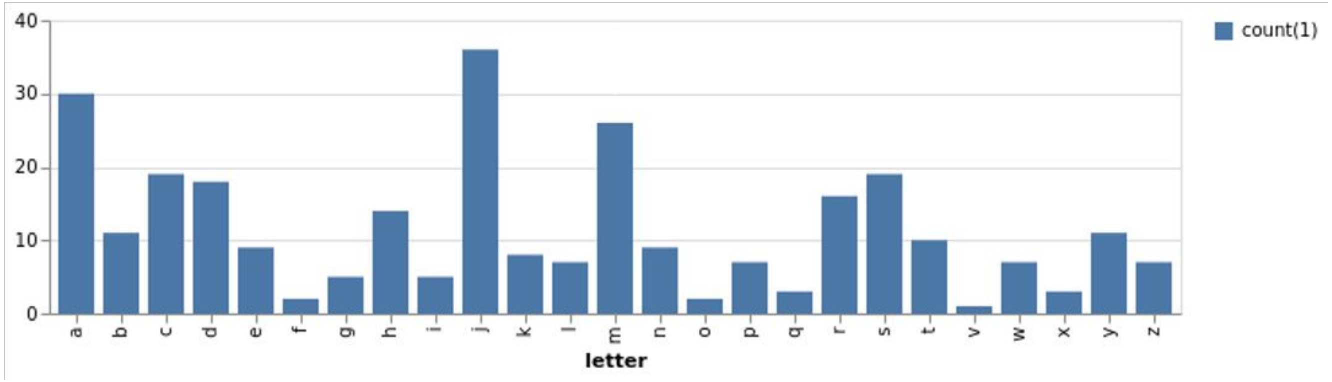

Picking a Hash Function

What else could we use that would evenly distribute values to locations?

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What else could we use that would evenly distribute values to locations?

Wacky Idea: Have $h(x)$ return a random value in **[0,N)**

(This makes apply impossible…but bear with me)

Random Hash Function

$$n = \text{number of elements in any bucket}$$

$$N = \text{number of buckets}$$

$$b_{i,j} = \begin{cases} 1 & \textbf{if} \text{ element } i \text{ is assigned to bucket } j \\ 0 & otherwise \end{cases}$$

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$$\mathbb{E}\left[b_{i,j}\right] = \frac{1}{N}$$

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$$\mathbb{E}\left[\sum_{i=0}^{n} b_{i,j}\right] = \frac{n}{N}$$

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$$b_{i,j} = \begin{cases} 1 & \textbf{if} \text{ element } i \text{ is assigned to bucket } j \\ 0 & otherwise \end{cases}$$

Only true if $b_{i,j}$ and $b_{i',j}$ are uncorrelated for any i ≠ i'

(h(i) can't be related to h(i'))

$$\mathbb{E}\left[\sum_{i=0}^{n} b_{i,j}\right] = \frac{n}{N}$$

The **expected** number of elements in any bucket j

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uncorrelated for any $i \neq i'$

(h(i) can't be related to h(i'))

$$\mathbb{E}\left[\sum_{i=0}^{n} b_{i,j}\right] = \frac{n}{N}$$

The **expected**
number of elements
in any bucket j

...given this information, what do the runtimes of our operations look like?

Random Hash Function

$$n = \text{number of elements in any bucket}$$

$$N = \text{number of buckets}$$

$$b_{i,j} = \begin{cases} 1 & \textbf{if} \text{ element } i \text{ is assigned to bucket } j \\ 0 & otherwise \end{cases}$$

Expected runtime of `insert`, `apply`, `remove`: ***O***(***n***/***N***)

Worst-Case runtime of `insert`, `apply`, `remove`: ***O***(***n***)

Hash Functions In the Real-World

Examples

- SHA256 ← Used by GIT
- MD5, BCRYPT ← Used by unix login, apt
- MurmurHash3 ← Used by Scala

hash(*x*) is pseudo-random

- **hash(*x*)** ~ uniform random value in [0, INT_MAX)
- **hash(*x*)** always returns the same value for the same ***x***
- **hash(*x*)** is uncorrelated with **hash(*y*)** for all x ≠ y

Hash Functions + Buckets

Everything is: $O\left(\frac{n}{N}\right)$

Let's call $\alpha = \frac{n}{N}$ the load factor.

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What do we do when this constraint is violated?

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Let's call $\alpha = \frac{n}{N}$ the load factor.

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Fix an α_{max} and start requiring that $\alpha \leq \alpha_{\text{max}}$

What do we do when this constraint is violated? **Resize!**