CSE 250

Data Structures

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B+ Trees

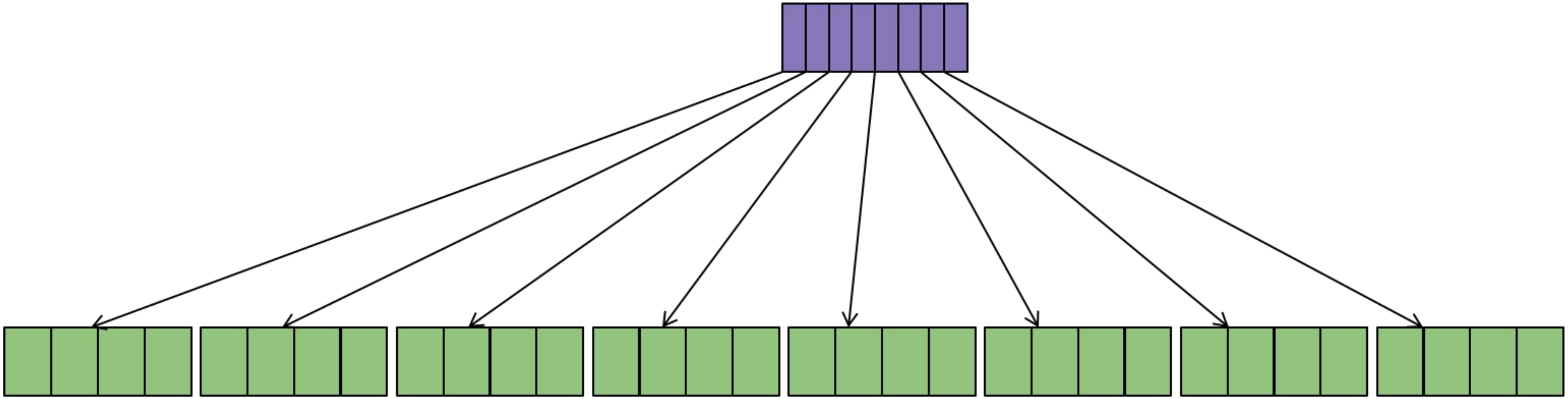
Announcements and Feedback

- Course evaluations are now open!
  - All feedback welcome, please fill them out
  - If 85% of the class has filled it out by the start of lecture on the last day of classes, I will release one of the final exam questions

Improving on Fence Pointers

Fence pointer array (in memory)

Page of actual data



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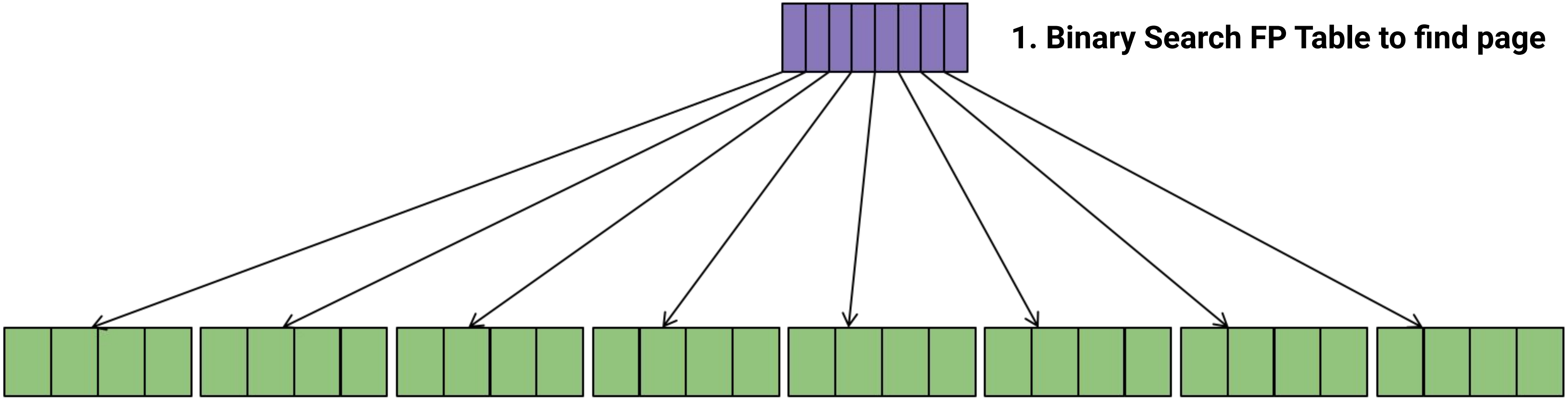

1. Binary Search FP Table to find page

Improving on Fence Pointers

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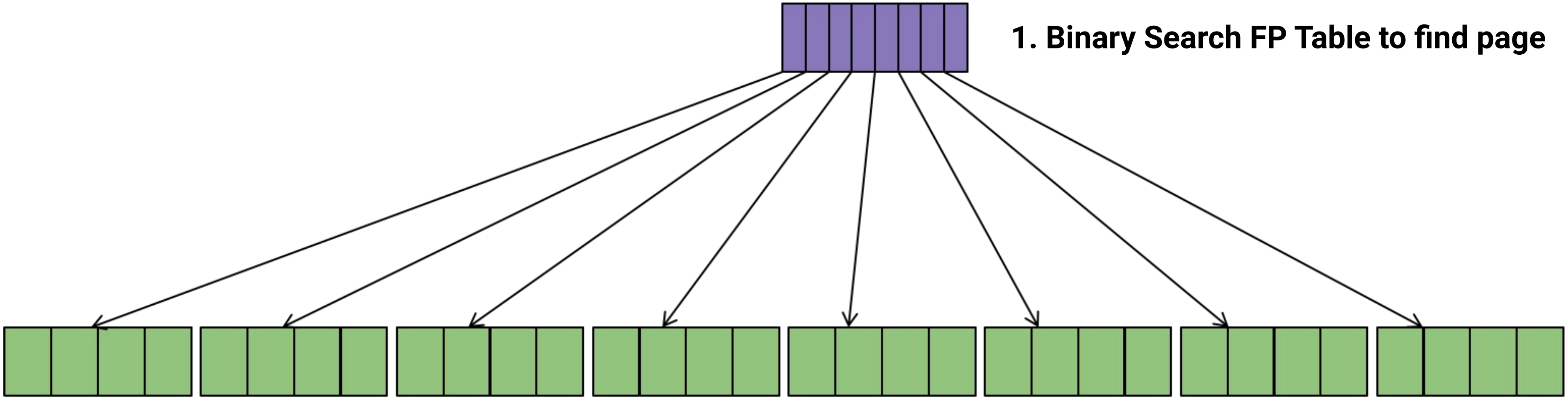
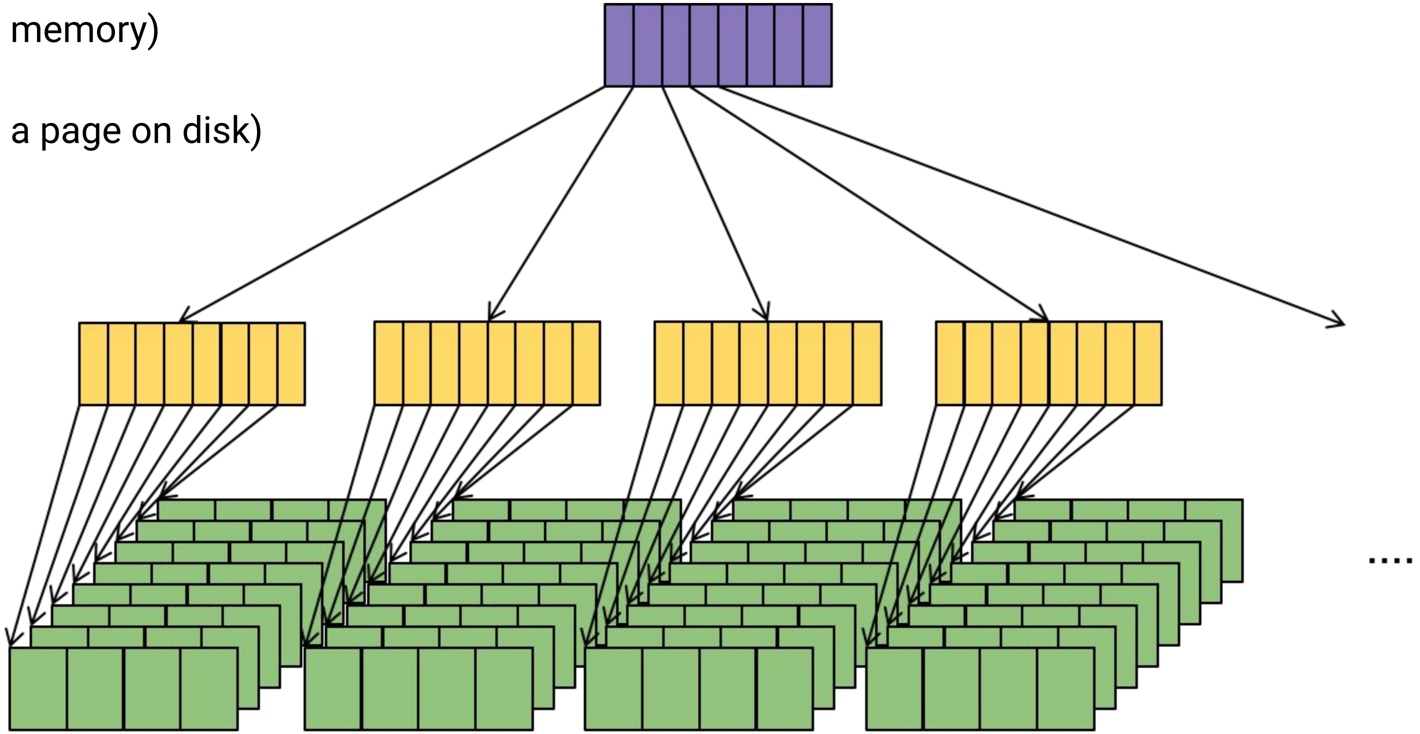


1. Binary Search FP Table to find page

2. Load page and binary search for record

Improving on Fence Pointers

- Fence pointer array (in memory)
- Fence pointer array (in a page on disk)
- Page of actual data



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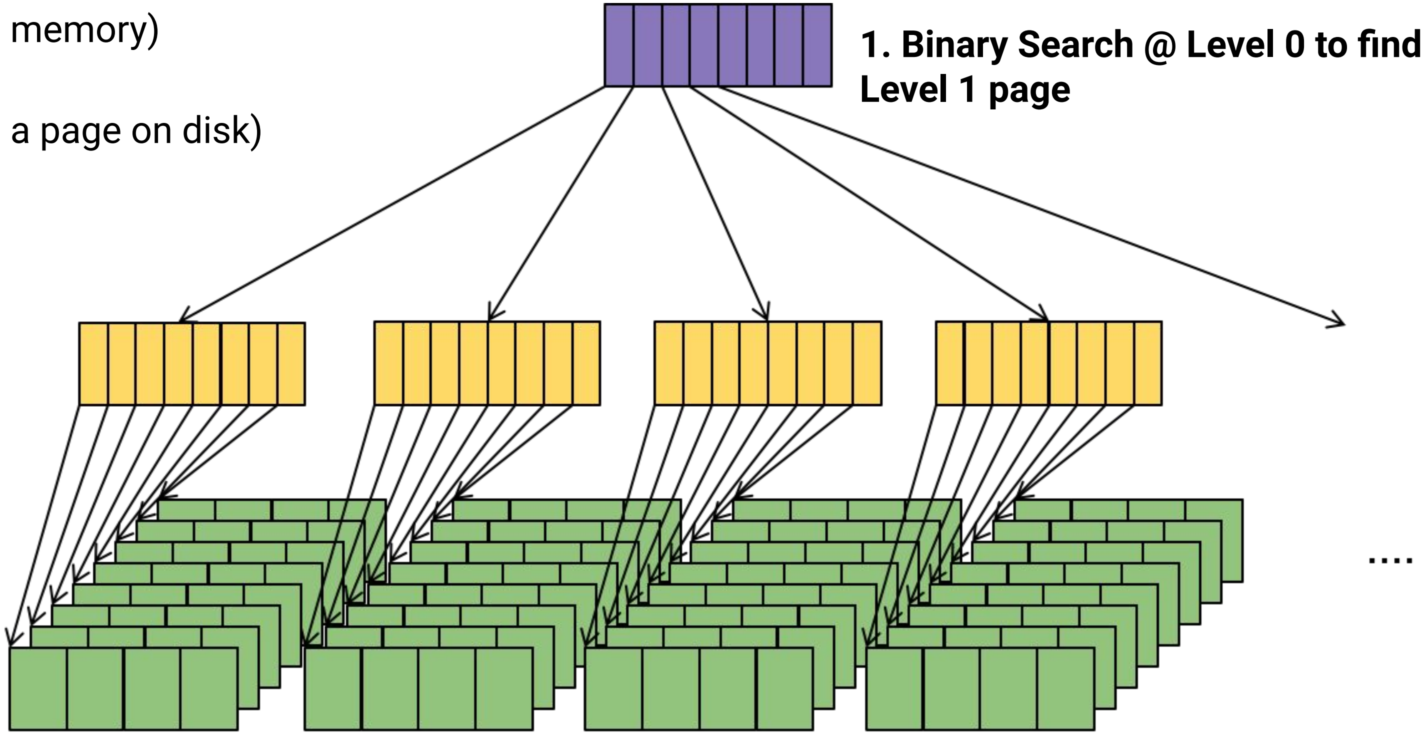

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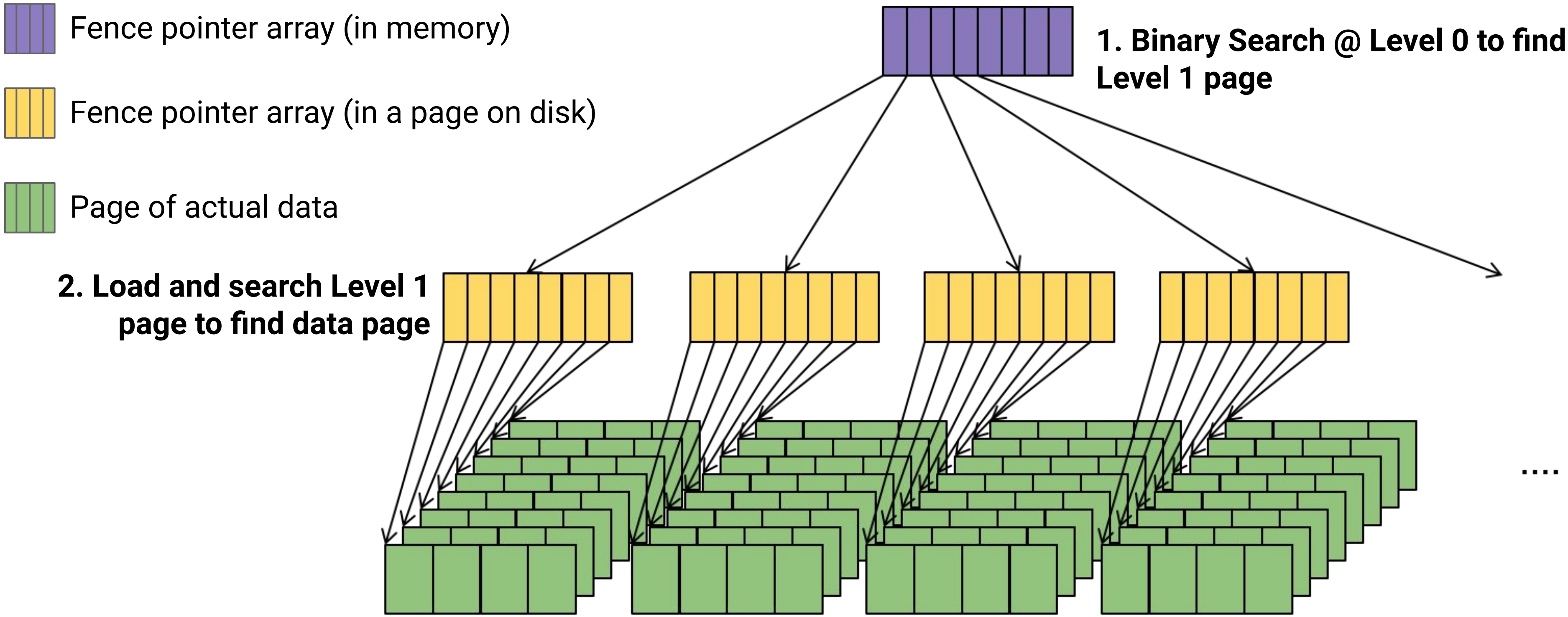

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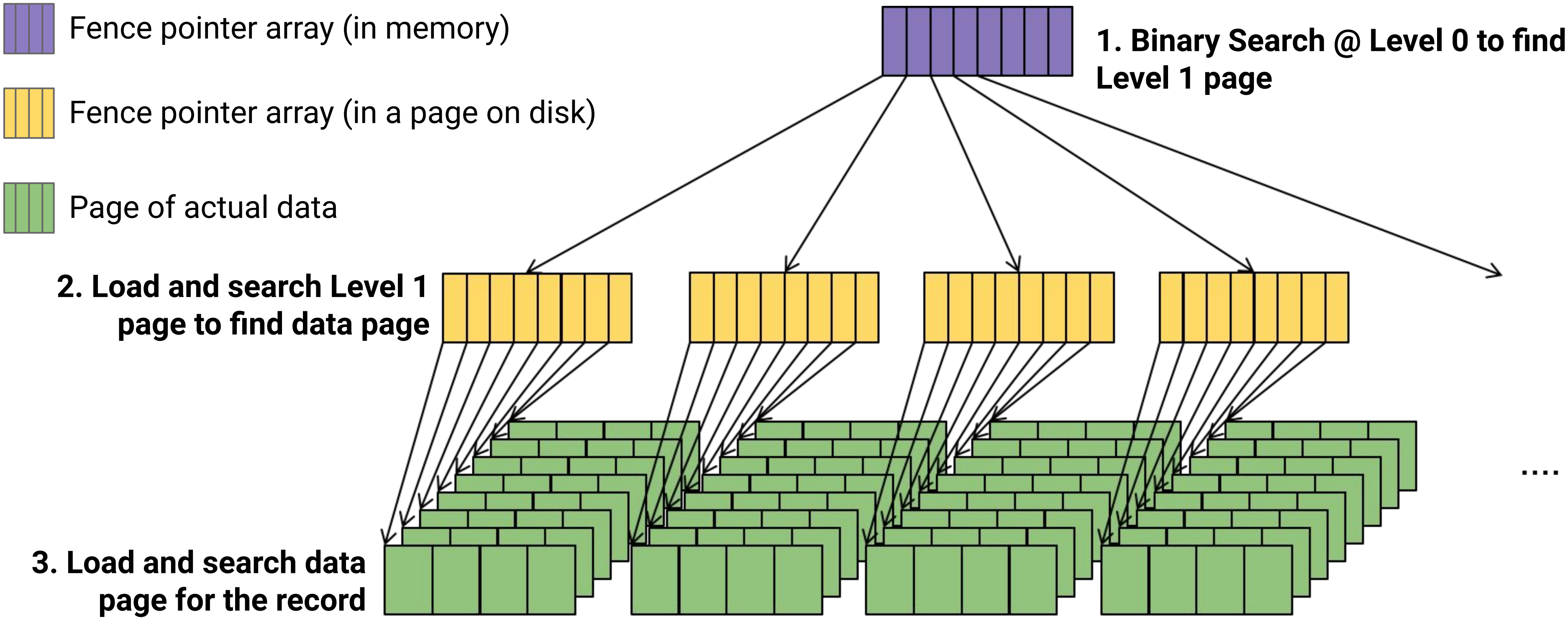

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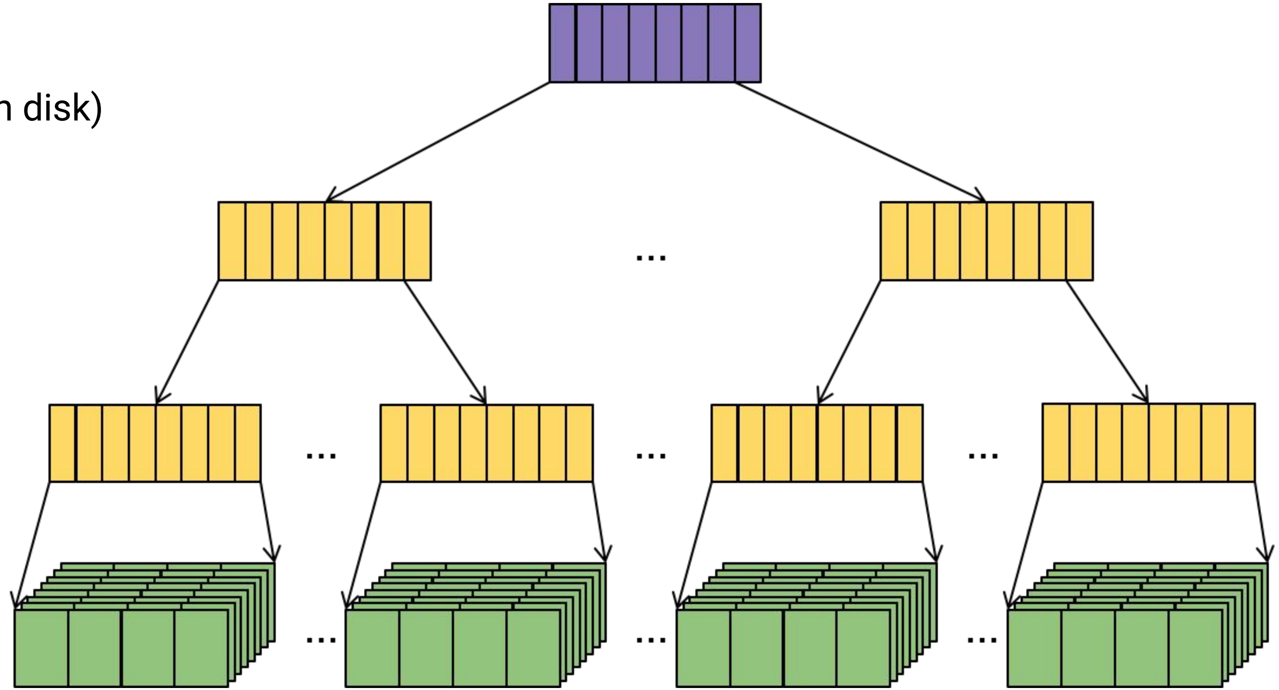
Page of actual data

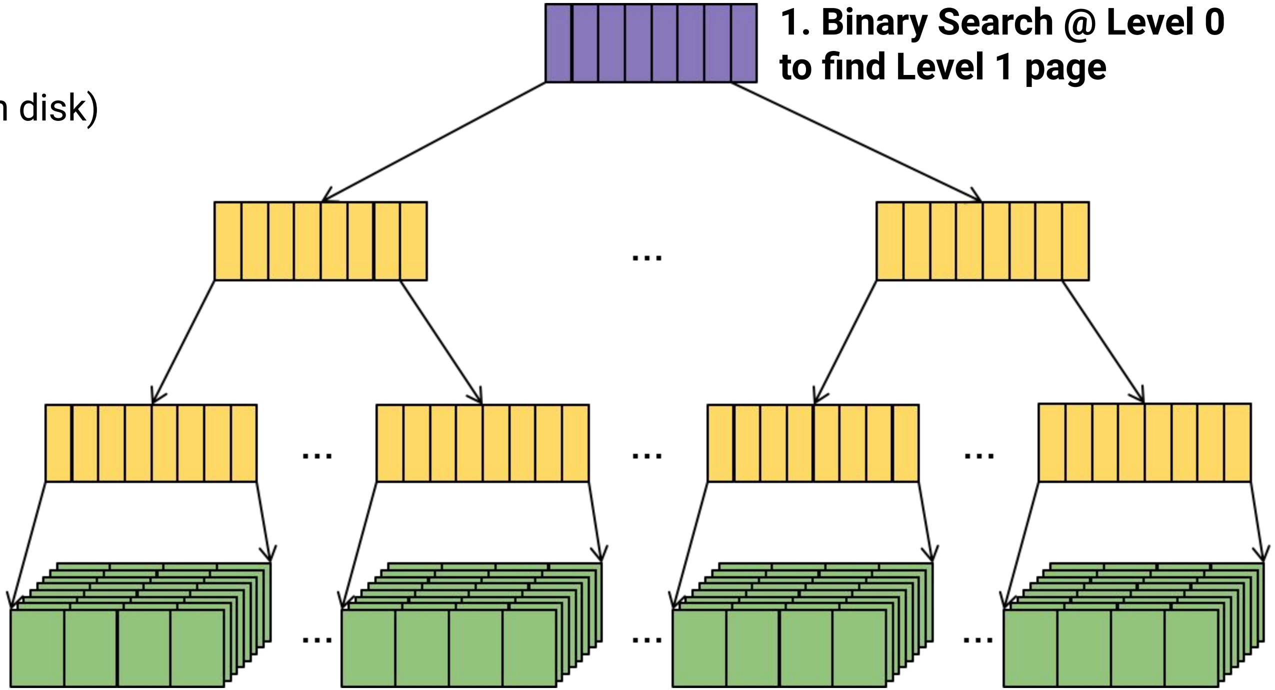


Improving on Fence Pointers

Fence pointer array (in memory)

Fence pointer array (in a page on disk)

Page of actual data



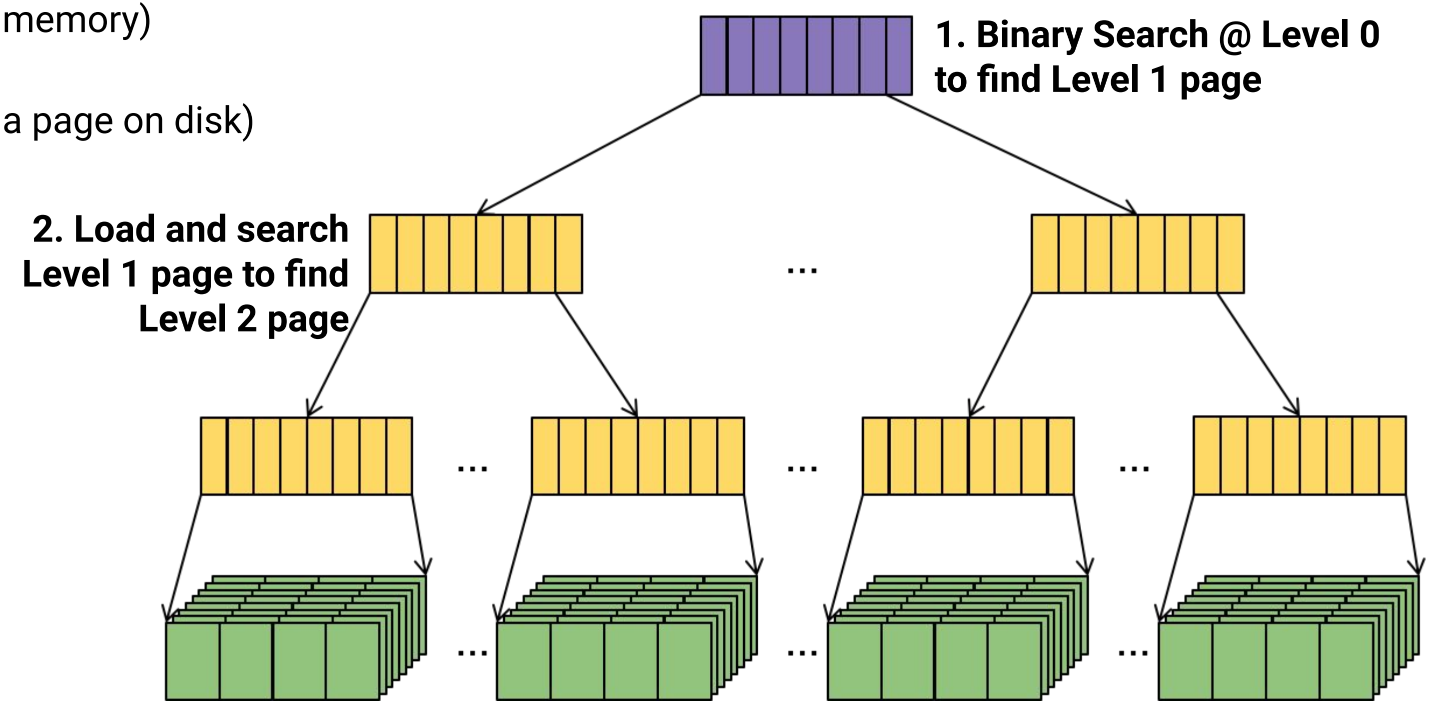
1. Binary Search @ Level 0 to find Level 1 page

Improving on Fence Pointers

Fence pointer array (in memory)

Fence pointer array (in a page on disk)

Page of actual data

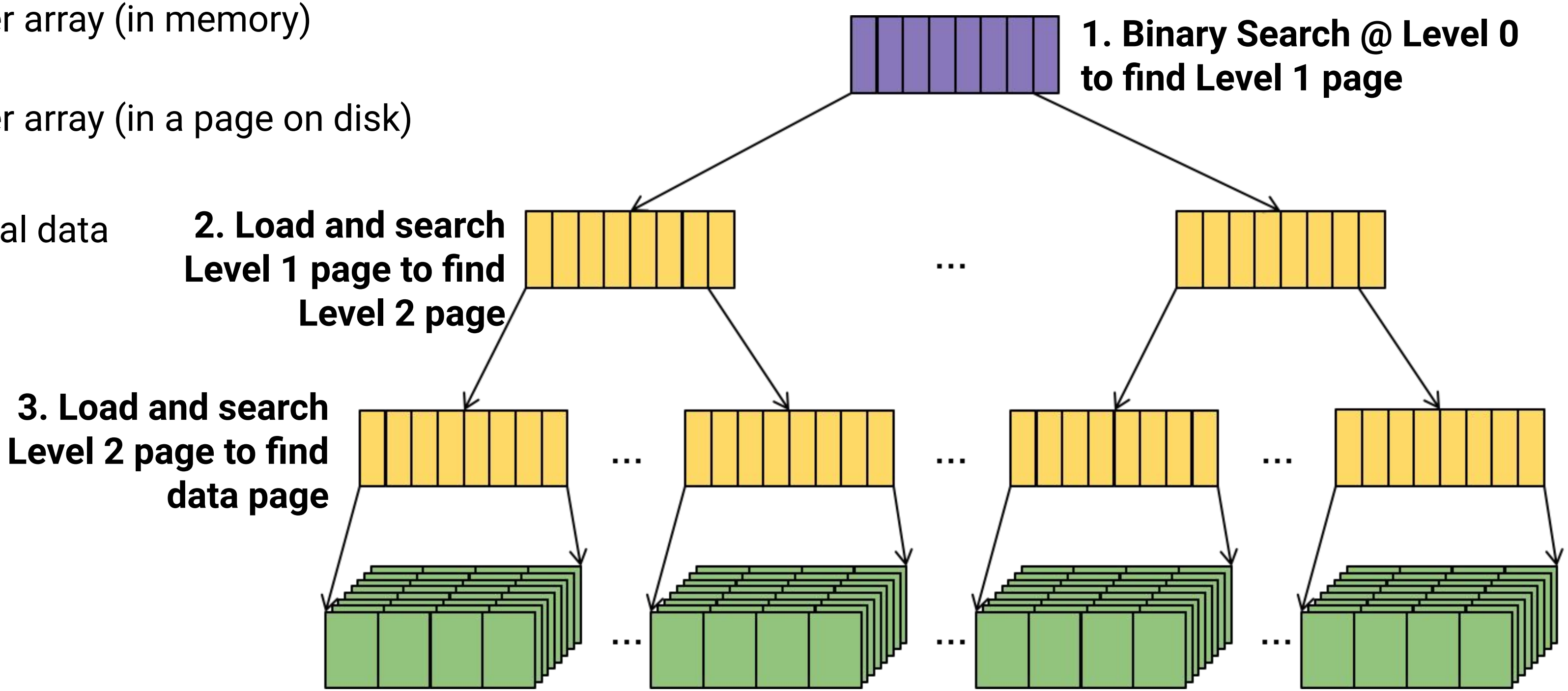


Improving on Fence Pointers

Fence pointer array (in memory)

Fence pointer array (in a page on disk)

Page of actual data



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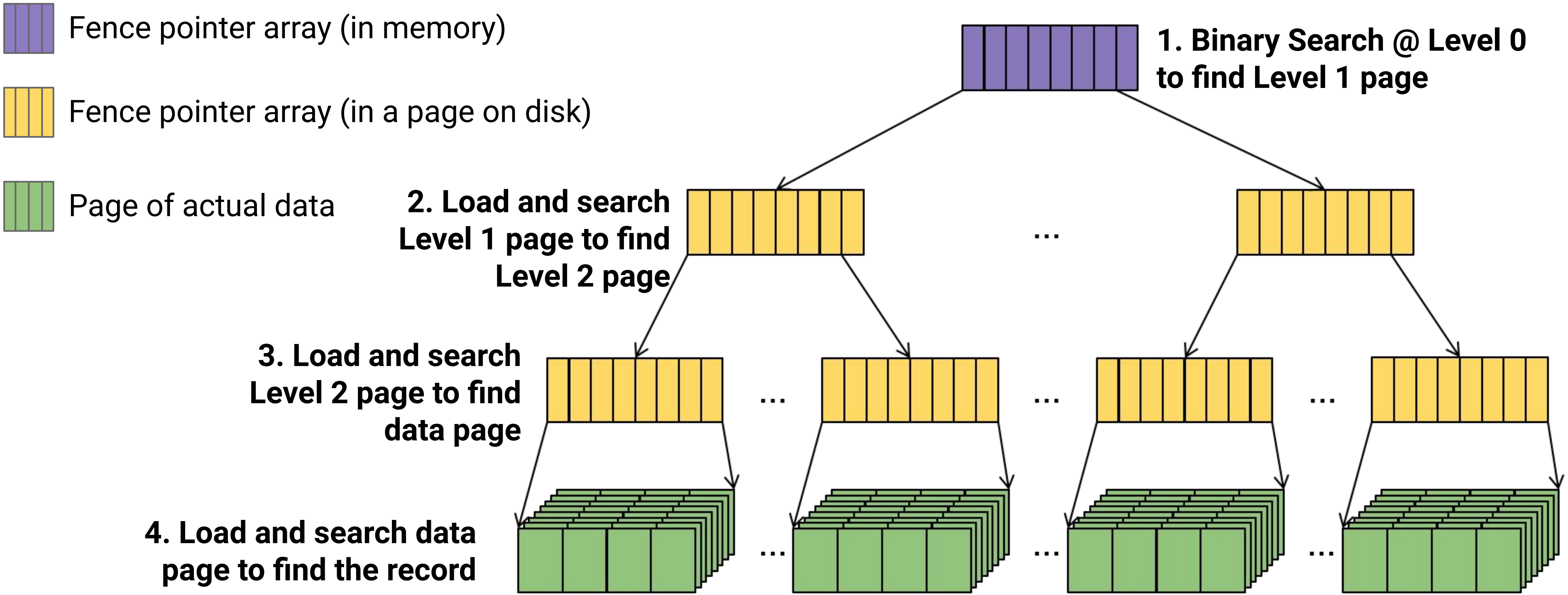
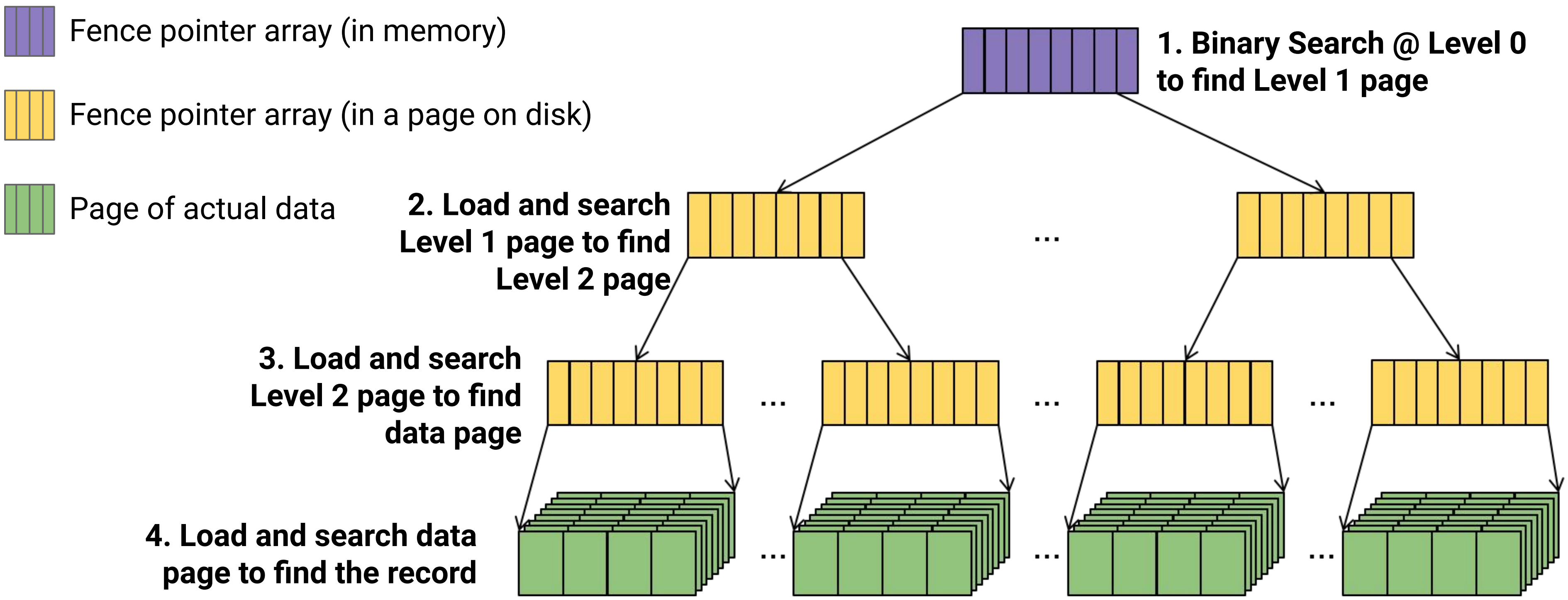


~~Improving on Fence Pointers~~ ISAM Index



ISAM Index

IO Complexity:

- 1 read at L0 (or assume already in memory)
- 1 read at L1
- 1 read at L2
- …
- 1 read at L_{max}
- 1 read at data level

ISAM Index

How many levels will there be (this isn't a binary tree…)

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- Level 1: Up to $\boldsymbol{C_{key}}$ pages w/ $\boldsymbol{C_{key}}^2$ keys

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How many levels will there be (this isn't a binary tree…)

- Level 0: 1 page w/$\boldsymbol{C_{key}}$ keys
- Level 1: Up to $\boldsymbol{C_{key}}$ pages w/$\boldsymbol{C_{key}^{2}}$ keys
- Level 2: Up to $\boldsymbol{C_{key}^{2}}$ pages w/$\boldsymbol{C_{key}^{3}}$ keys
- …

ISAM Index

How many levels will there be (this isn't a binary tree…)

- Level 0: 1 page w/ $\boldsymbol{C_{key}}$ keys
- Level 1: Up to $\boldsymbol{C_{key}}$ pages w/ $\boldsymbol{C_{key}^{2}}$ keys
- Level 2: Up to $\boldsymbol{C_{key}^{2}}$ pages w/ $\boldsymbol{C_{key}^{3}}$ keys
- …
- Level max: Up to $\boldsymbol{C_{key}^{max}}$ pages w/ $\boldsymbol{C_{key}^{max+1}}$ keys

ISAM Index

How many levels will there be (this isn't a binary tree…)

- Level 0: 1 page w/C_{key} keys
- Level 1: Up to C_{key} pages w/C_{key}^{2} keys
- Level 2: Up to C_{key}^{2} pages w/C_{key}^{3} keys
- …
- Level max: Up to C_{key}^{max} pages w/C_{key}^{max+1} keys
- Data Level: Up to C_{key}^{max+1} pages w/$C_{data}C_{key}^{max+1}$ records

ISAM Index

$$n = C_{data} C_{key}^{max+1}$$

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$$\log_{C_{key}}(n) - \log_{C_{key}}(C_{data}) = max + 1$$

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$$\log_{C_{key}} \left(\frac{n}{C_{data}} \right) = max + 1$$

$$\log_{C_{key}}(n) - \log_{C_{key}}(C_{data}) = max + 1$$

Number of Levels: $O\left(\log_{C_{key}}(n)\right)$

ISAM Index

$$n = C_{data} C_{key}^{max+1}$$

$$\frac{n}{C_{data}} = C_{key}^{max+1}$$

$$\log_{C_{key}}\left(\frac{n}{C_{data}}\right) = max + 1$$

$$\log_{C_{key}}(n) - \log_{C_{key}}(C_{data}) = max + 1$$

Note this isn't base 2!

Number of Levels: $O\left(\boxed{\log_{C_{key}}}(n)\right)$

ISAM Index

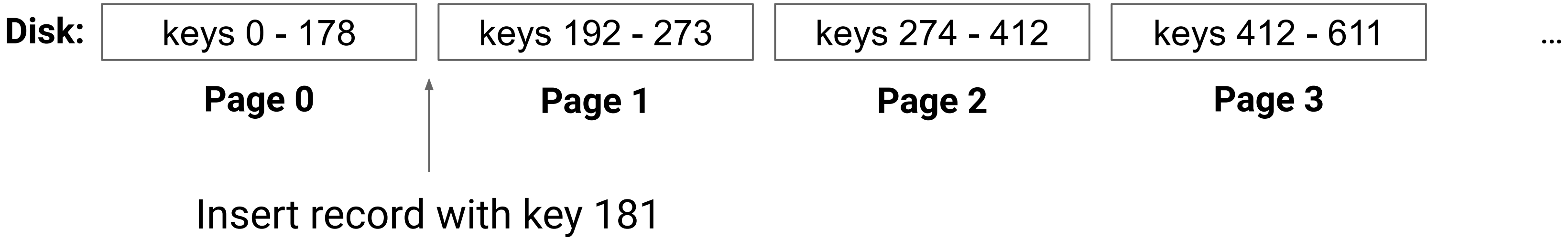
Like Binary Search, but "Cache-Friendly"

- Still takes $\boldsymbol{O(\log(n))}$ steps
- Still requires $\boldsymbol{O(1)}$ memory (1 page at a time)
- Now requires $\boldsymbol{\log_{C_{key}}(n)}$ loads from disk ($\boldsymbol{\log_{C_{key}}(n) \ll \log_2(n)}$)

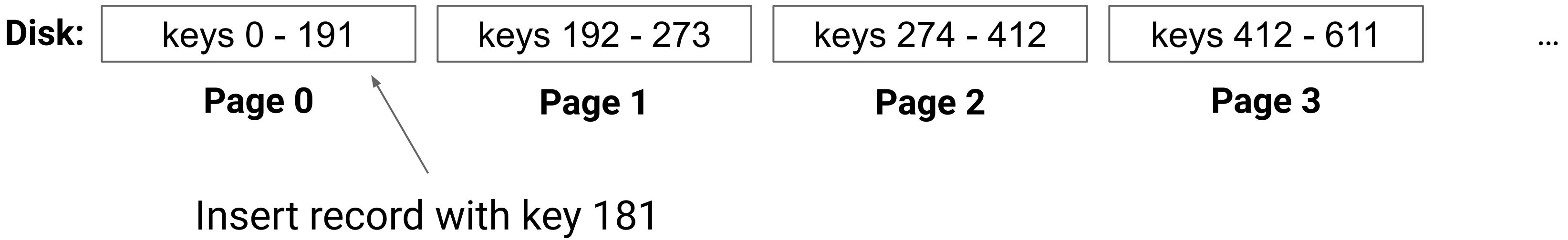
ISAM Index

What if the data changes?

Inserting New Records



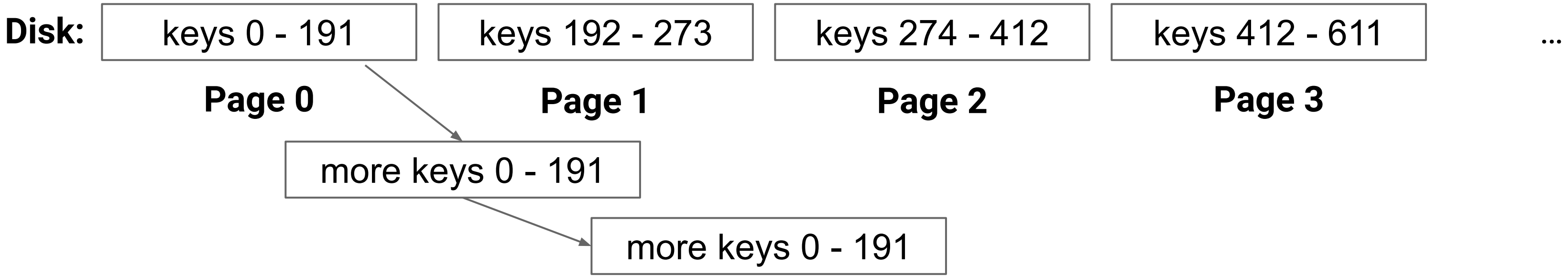
Inserting New Records



Idea: Keep "free" space on each page for new records

... what happens when it fills up?

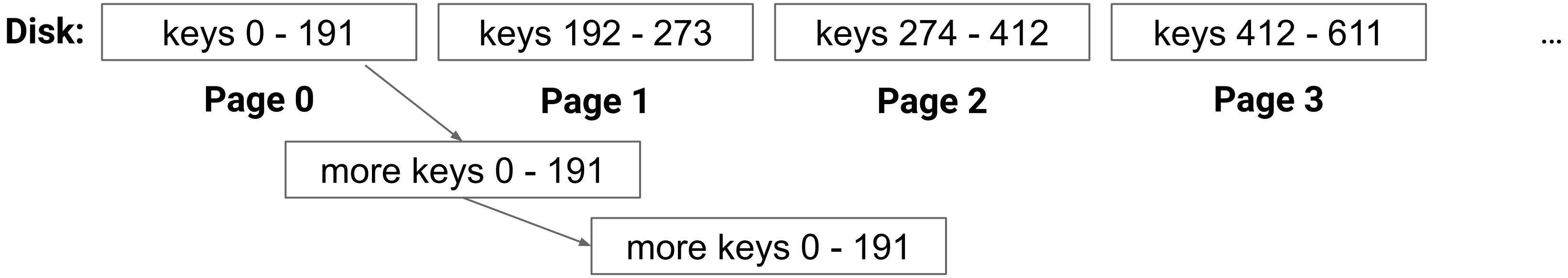
Inserting New Records



Idea: Linked lists to store overflow

…but now our I/O complexity is ***O*(*n*)** again…

Inserting New Records



Idea: We'll have to rearrange the tree

Dynamic Page Allocation

Treat the disk as an ADT:

`allocate(): PageID`

- Allocates a page in the data file and returns its position

`load[T](page: PageID): T`

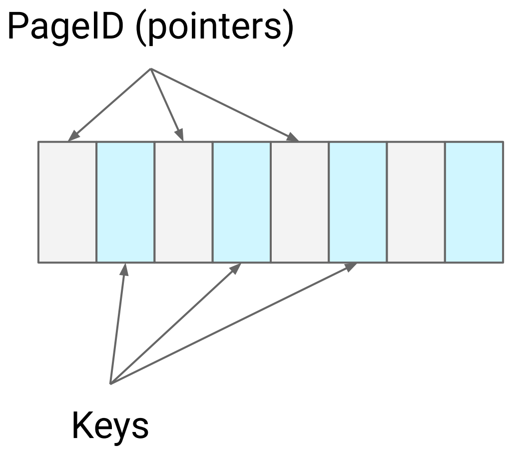
- Reads in a 4k chunk of data

`write[T](page: PageID, data: T)`

- Writes a 4k chunk of data to the page

Pointers to Pages

Our pages are now dynamic, need "pointers" instead of indices



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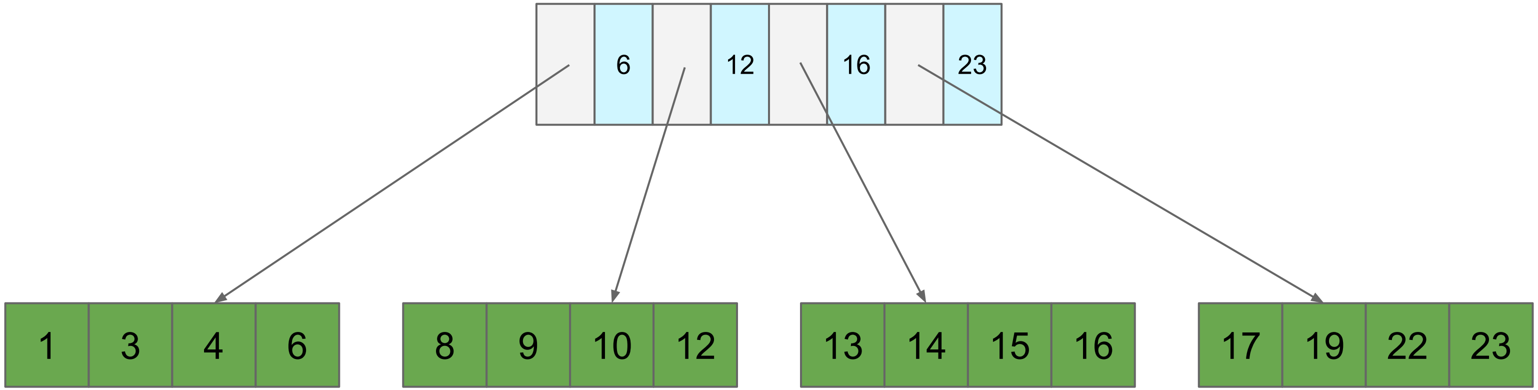
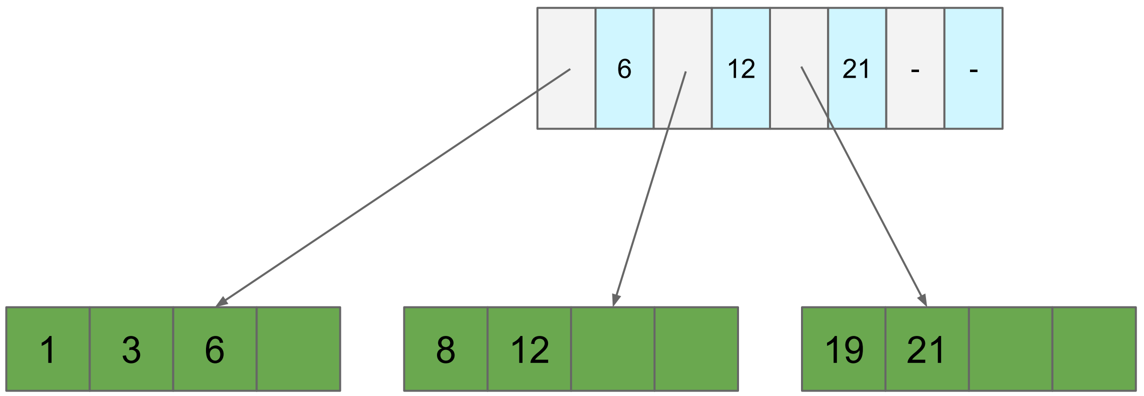
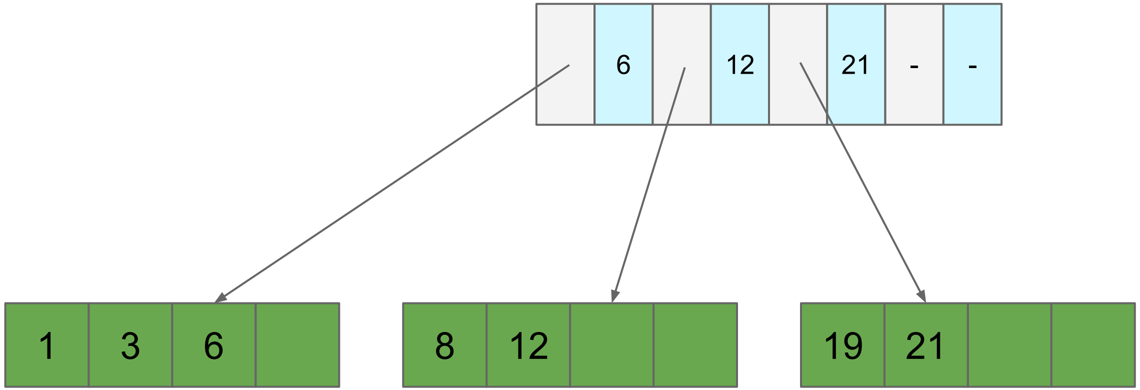


Free Space Revisited



Free Space Revisited

Add 9



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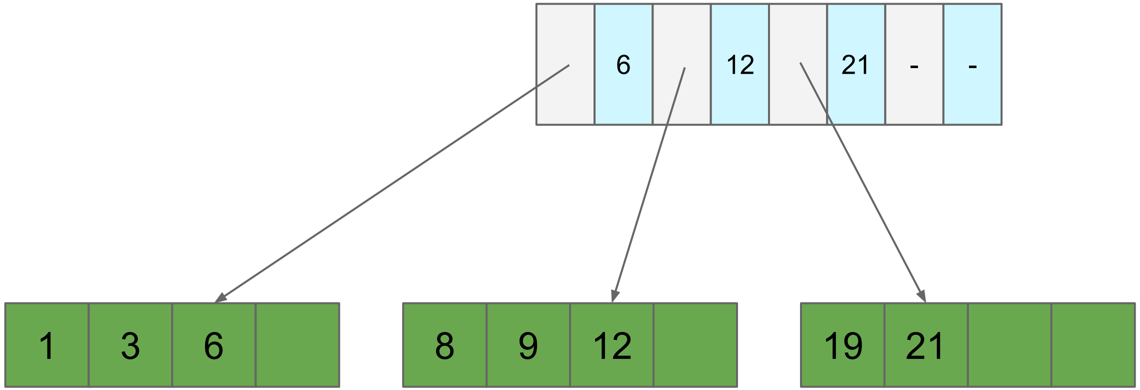
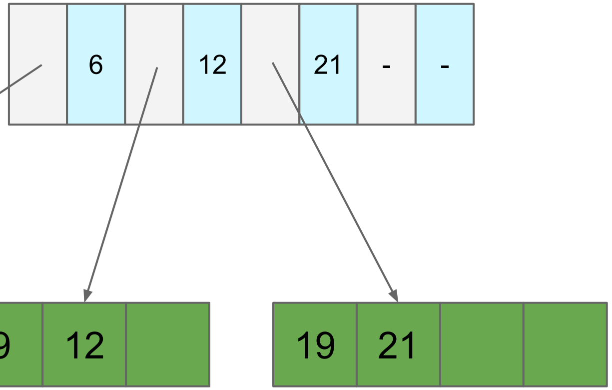


Free Space Revisited

Add 14



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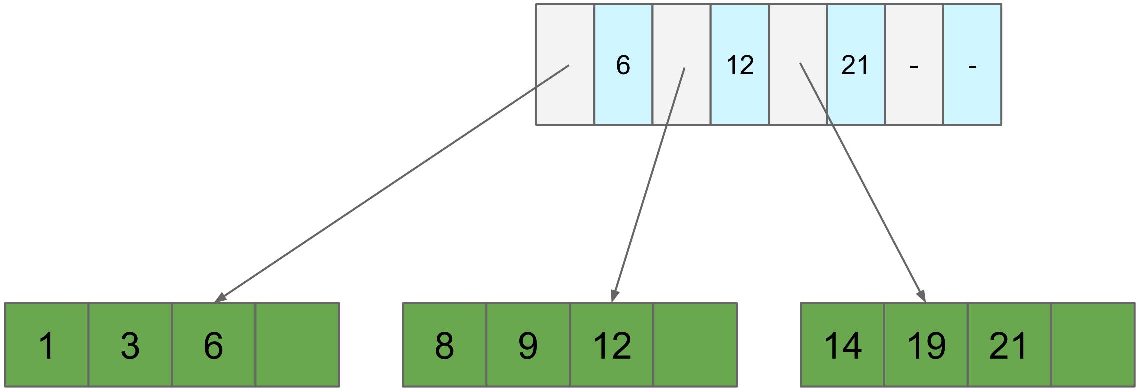
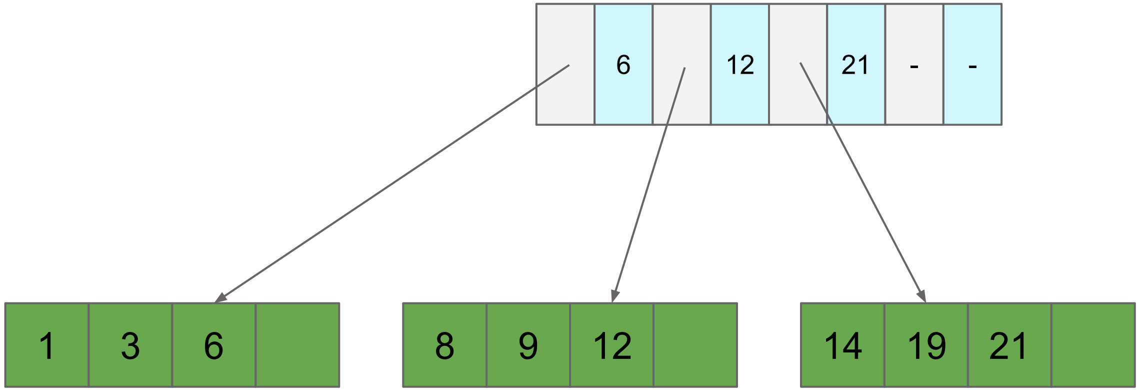


Free Space Revisited

Add 10



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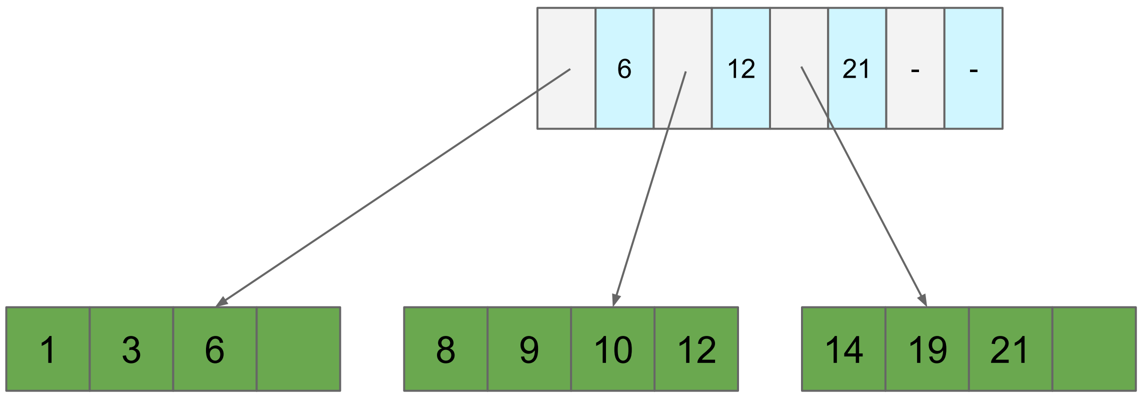
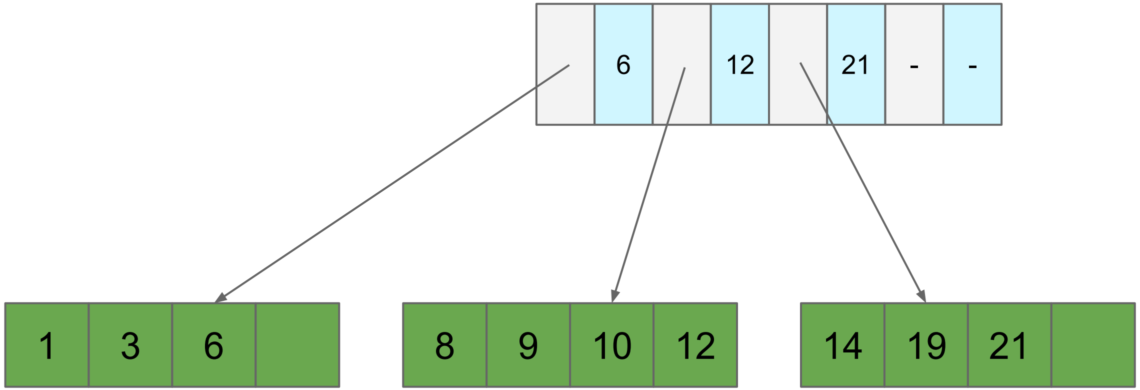


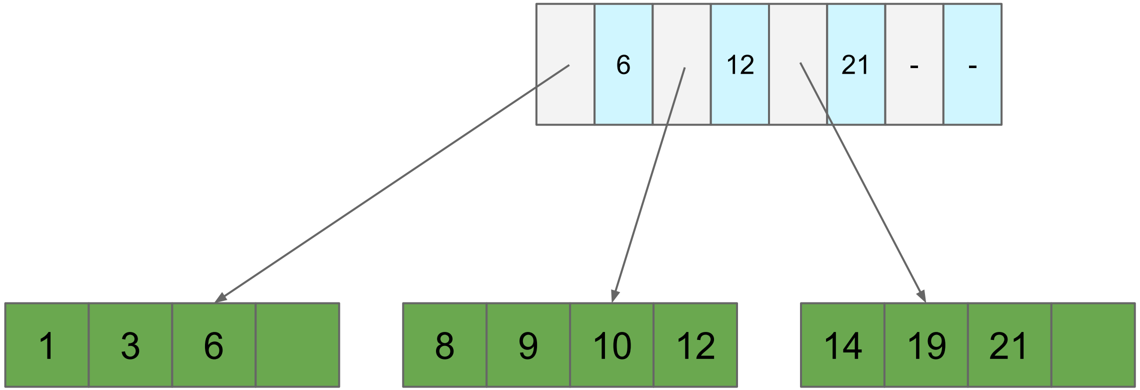
Free Space Revisited

Add 11? Where does it go?



Free Space Revisited

Add 11? Where does it go? Split the page!



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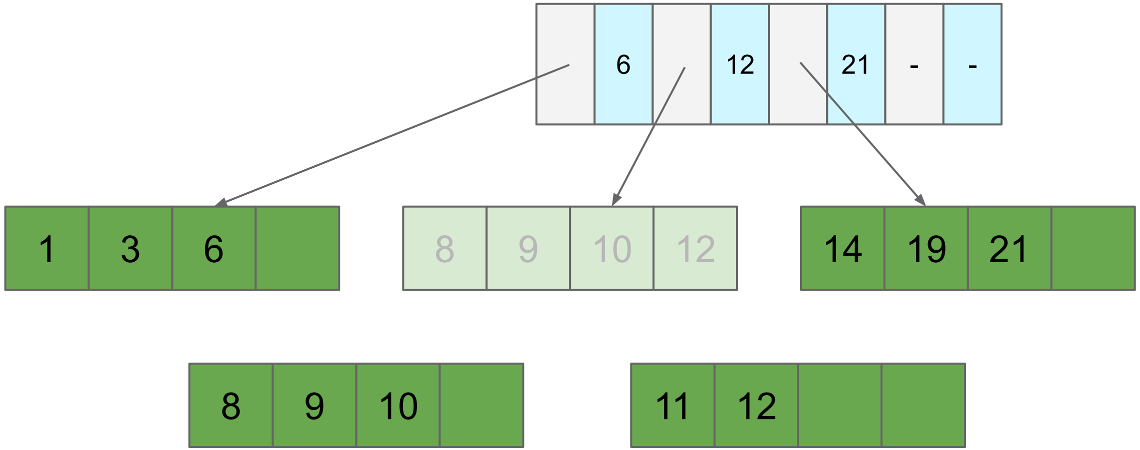

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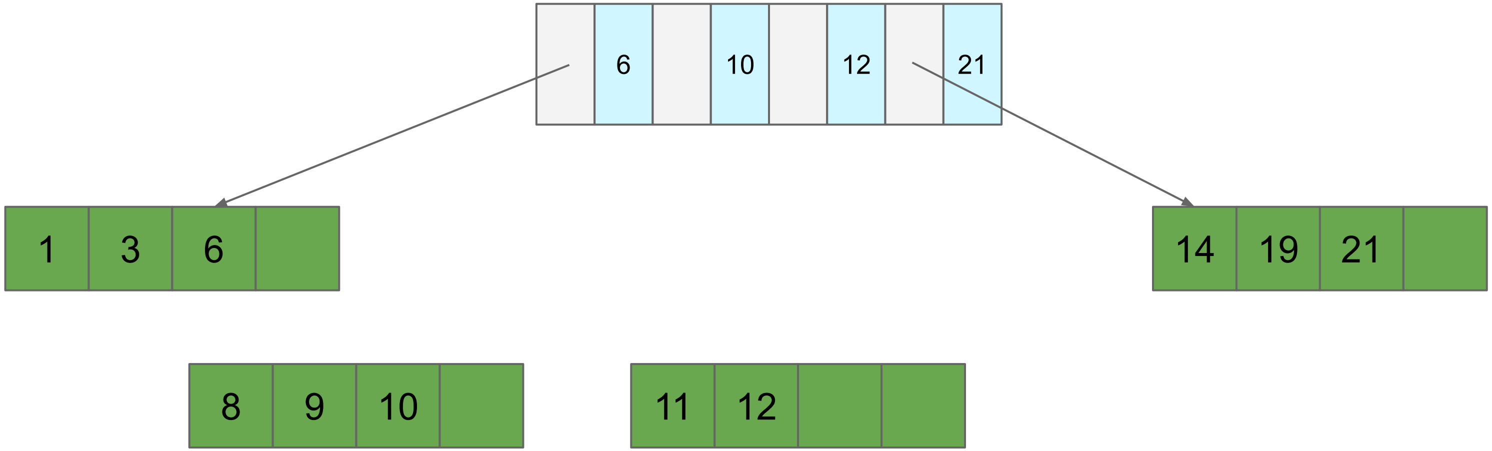

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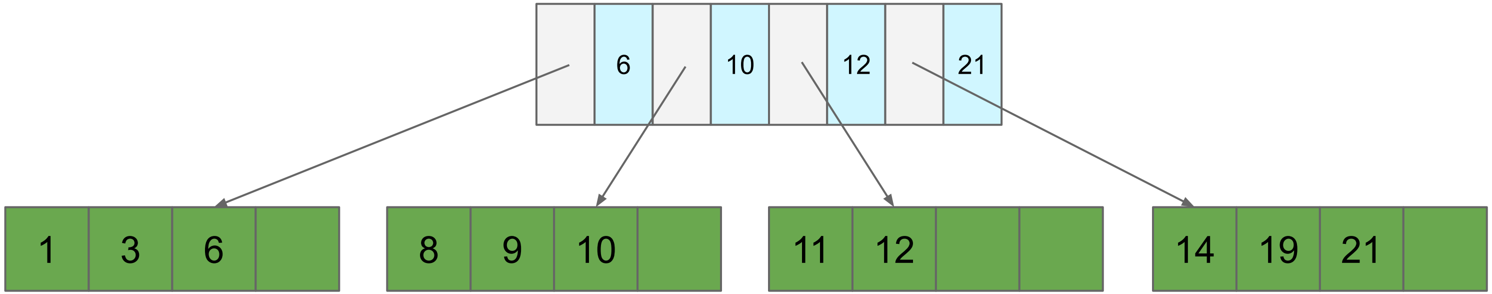
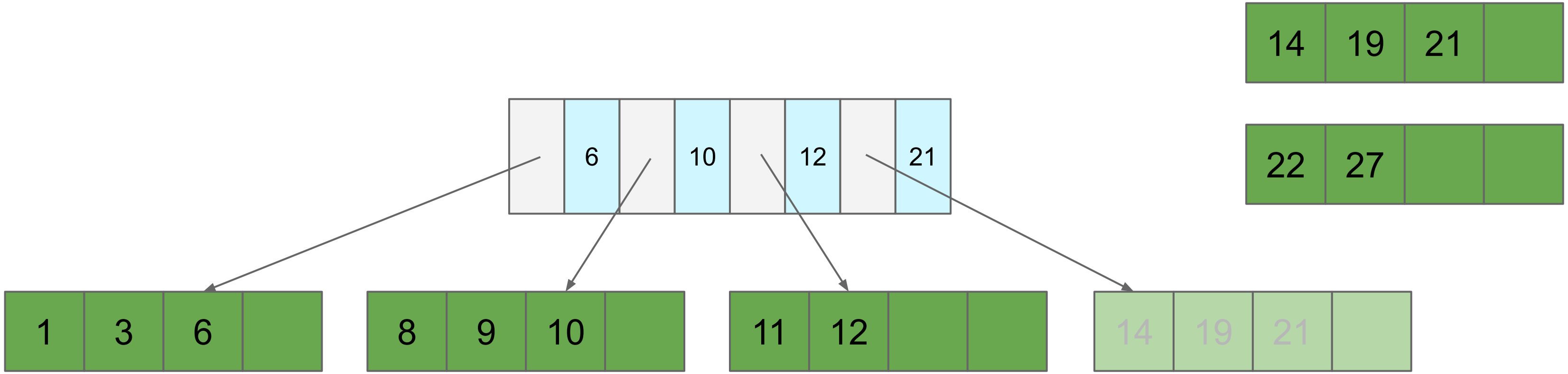


Free Space Revisited

Add 22, 27?



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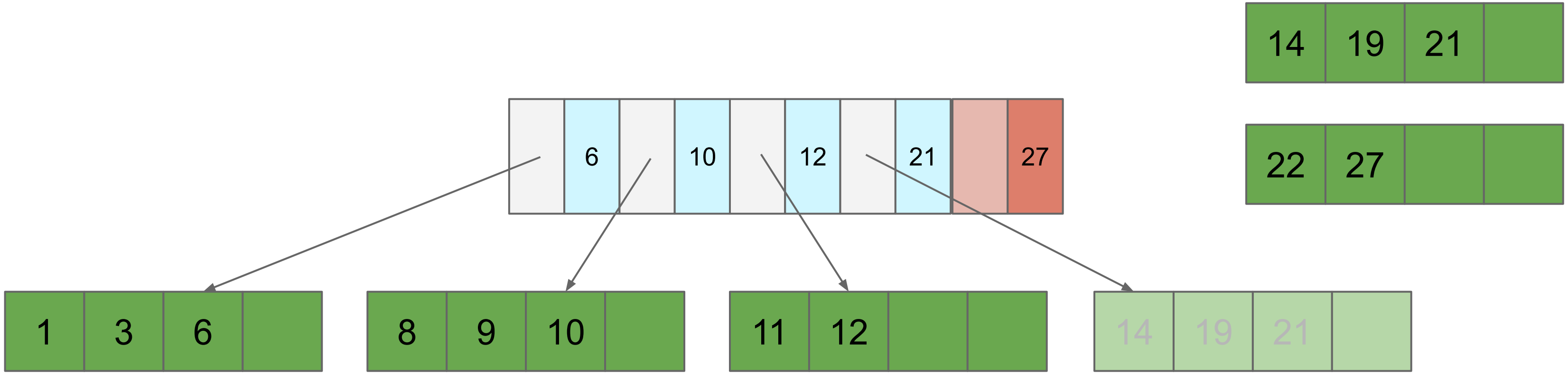
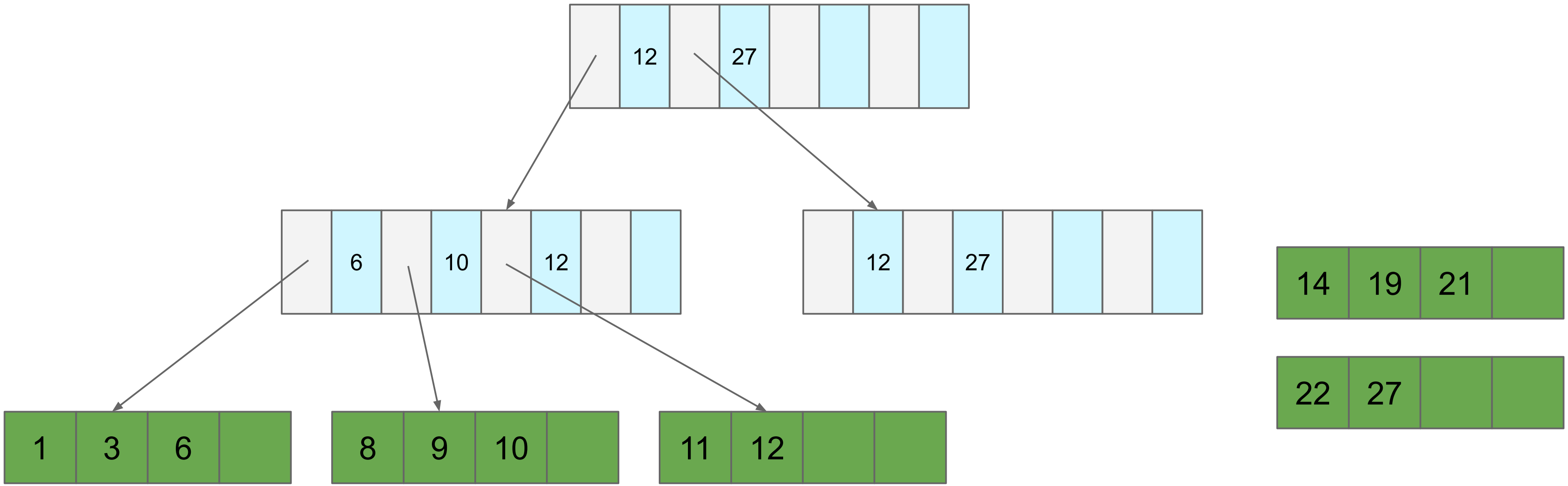


Free Space Revisited

Add 22, 27? Split the page of pointers!



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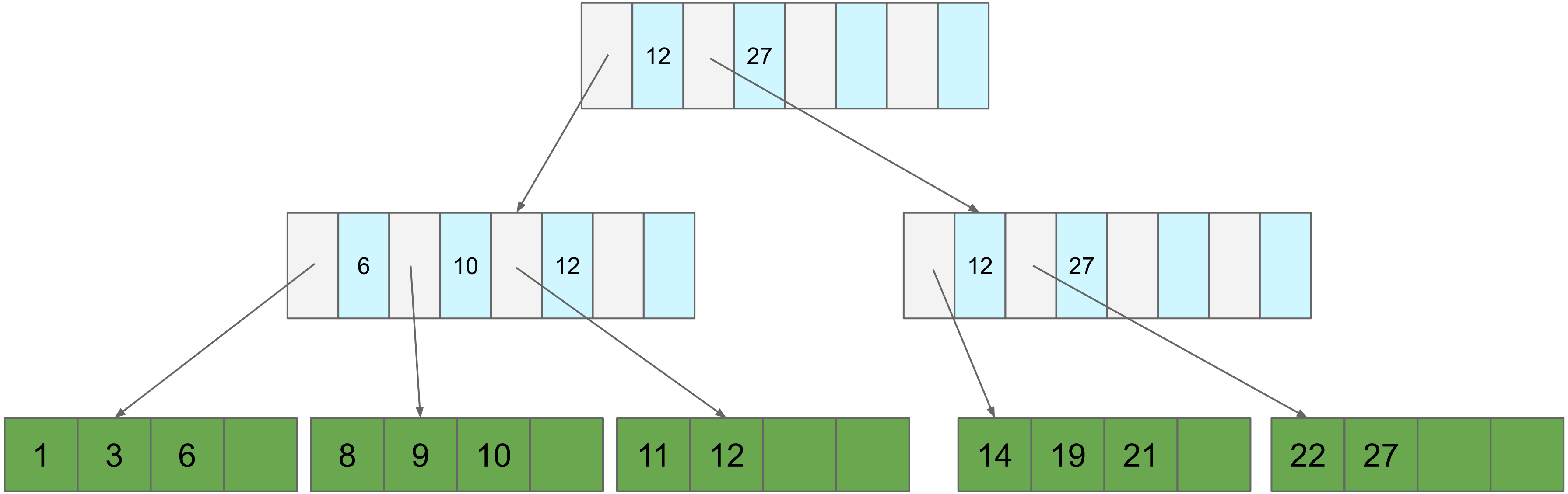


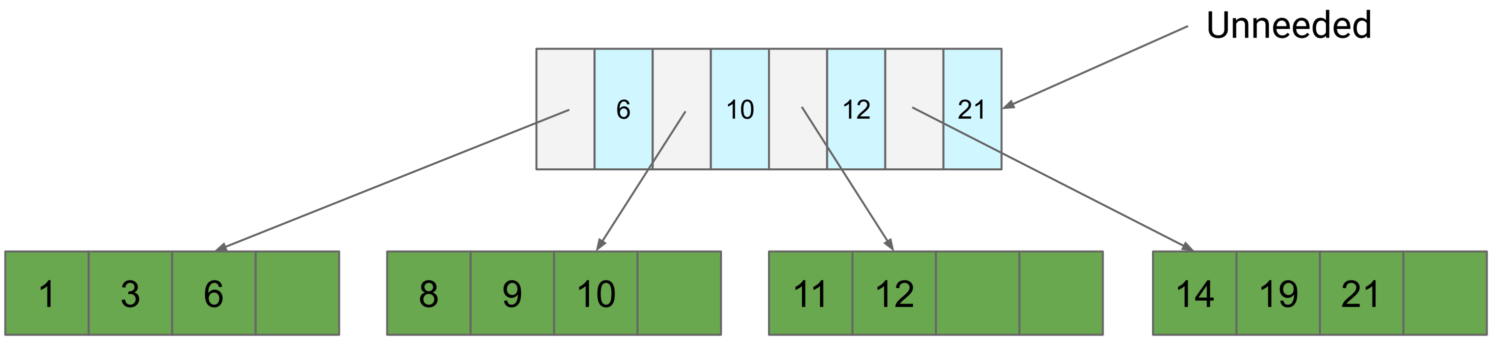
B+ Tree (Almost)

`Insert`

1. Find the page the record belongs on
2. Insert record there
3. If full, "split" the page
    a. Insert additional separator in the parent directory
    b. If full, split the parent directory and repeat
        i. If root is split, create a new root

B+ Trees

Observation: Don't need the largest key



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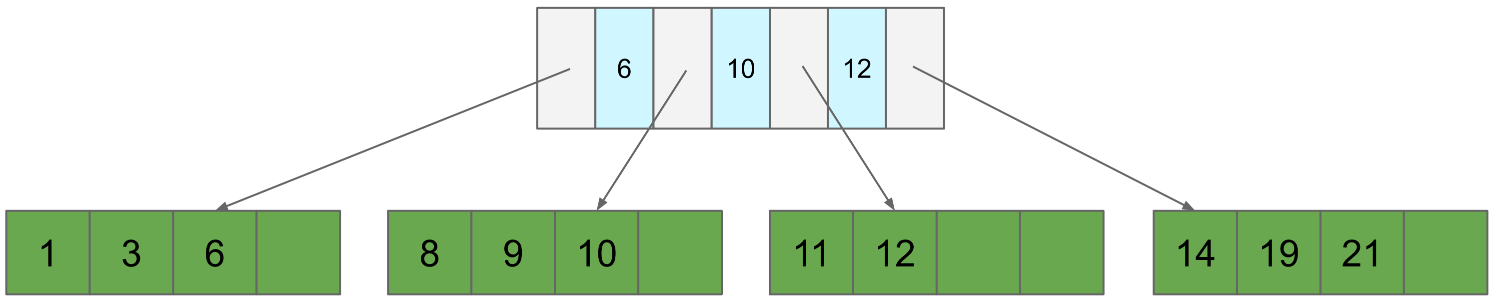
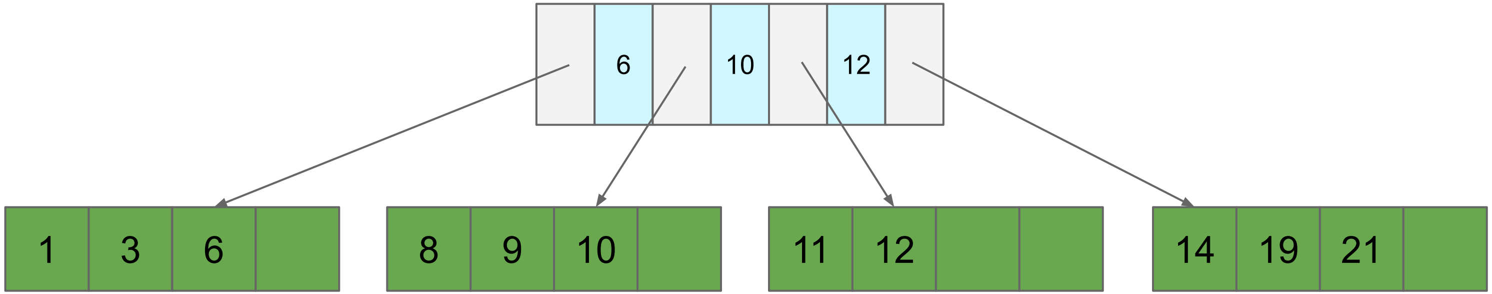


B+ Trees

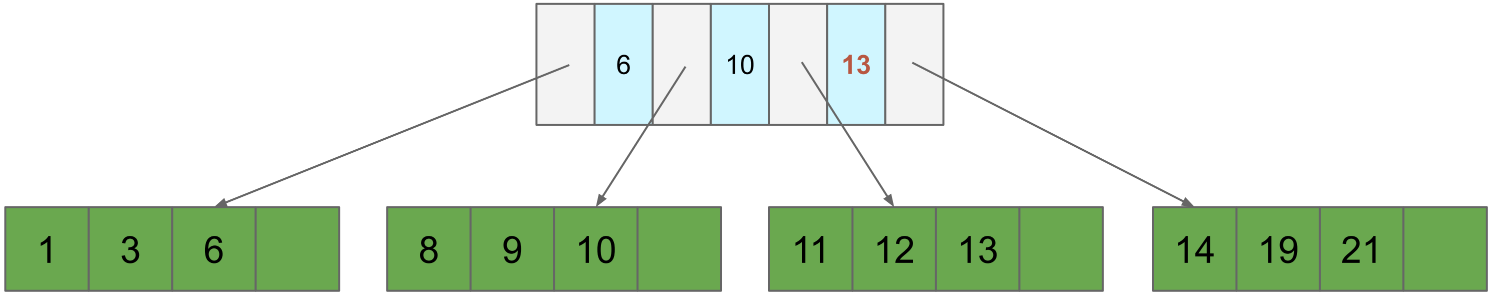
Question: What if separators are mispositioned? What if we insert 13?



B+ Trees

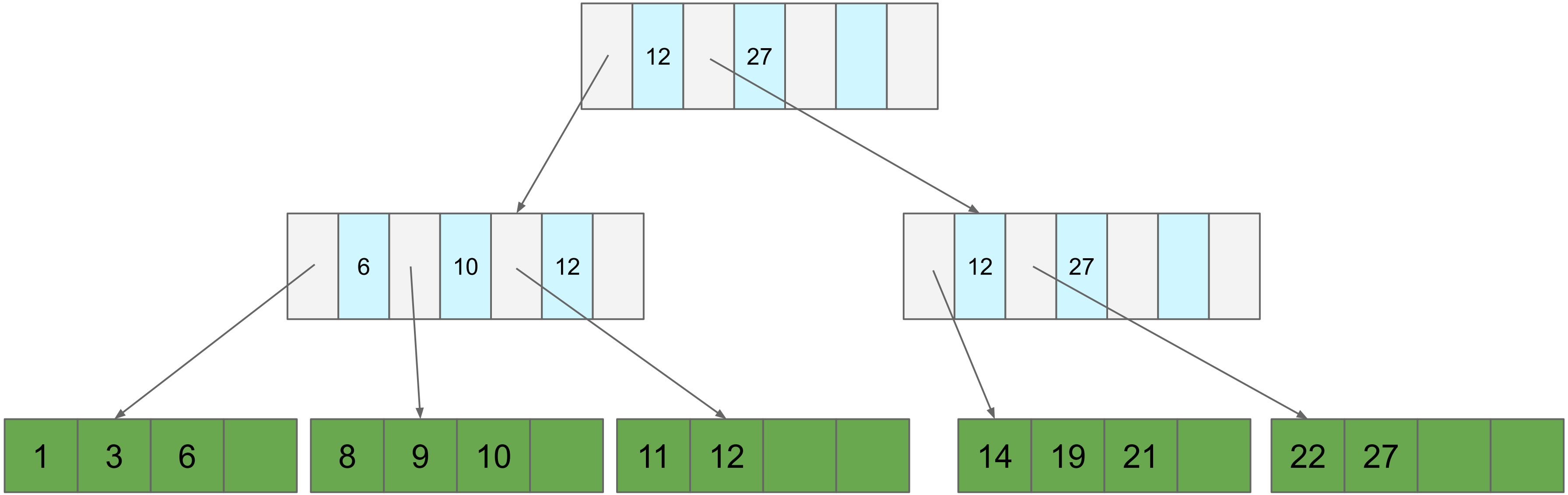
Question: What if separators are mispositioned? What if we insert 13?

Idea: Steal space from neighbor (and update separator)



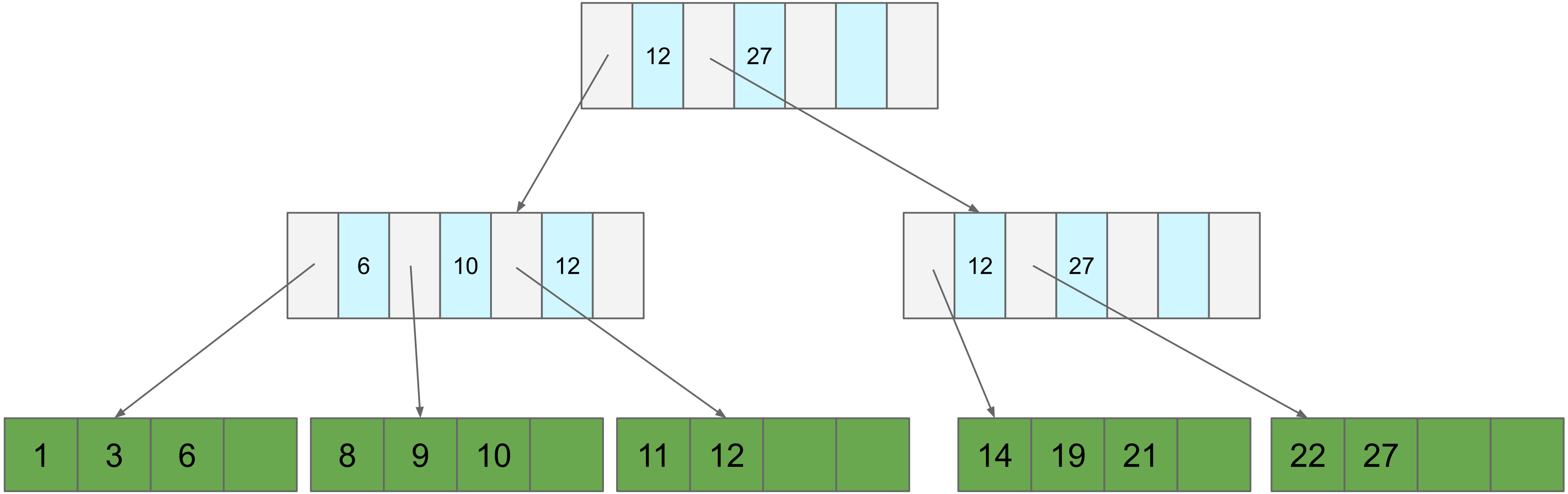
B+ Trees

Question: What if we delete records?



B+ Trees

Delete 22,27



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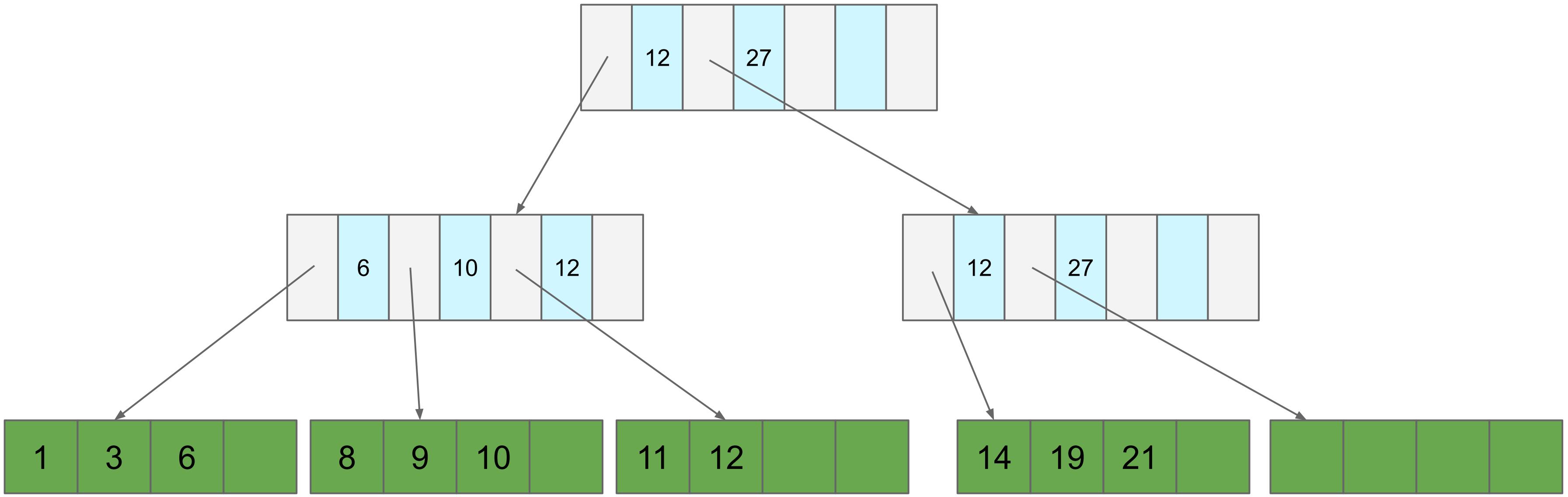

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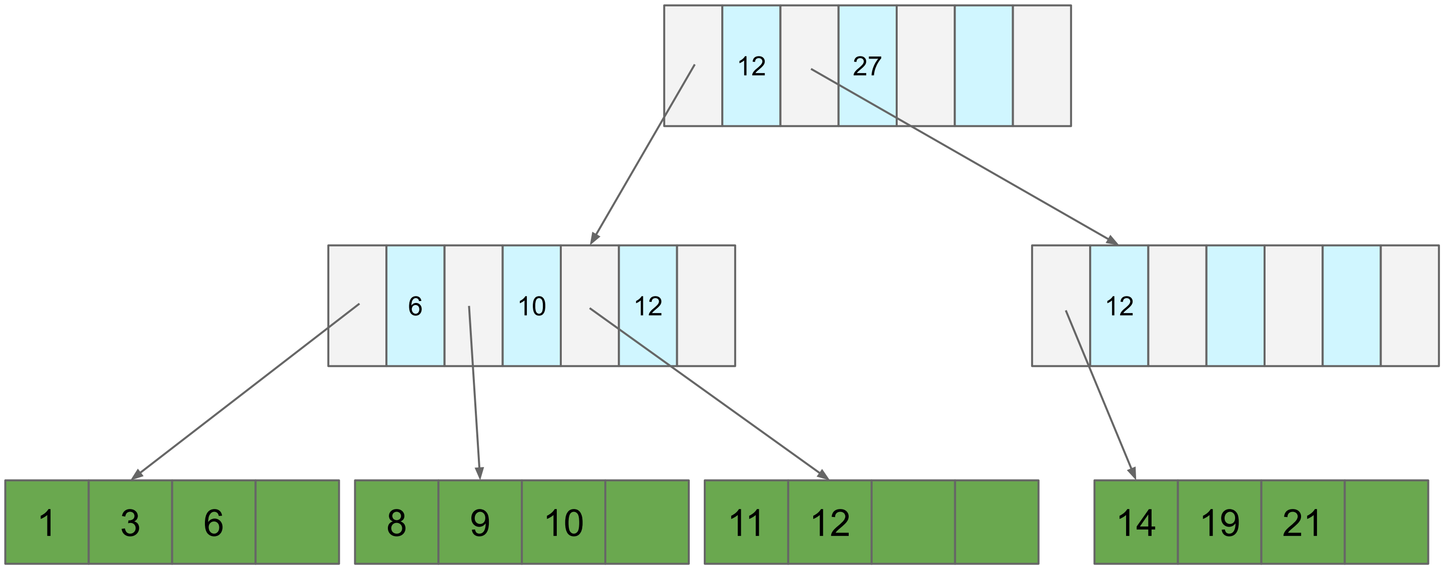
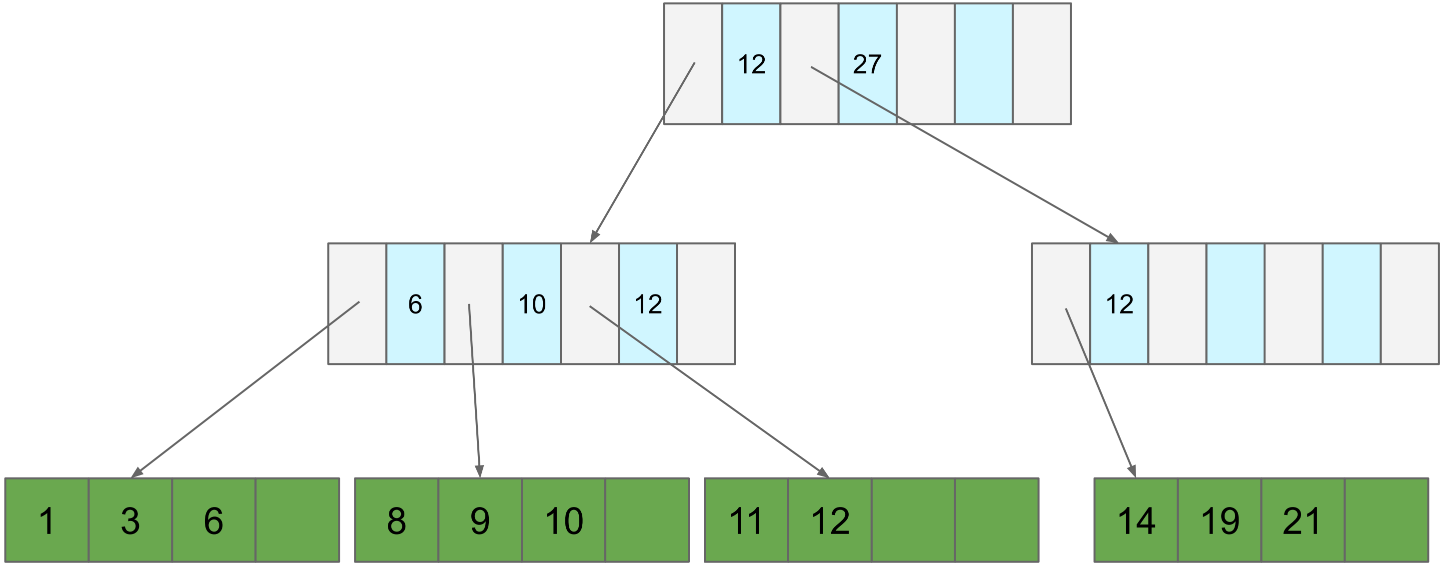


B+ Trees

Delete 8-12



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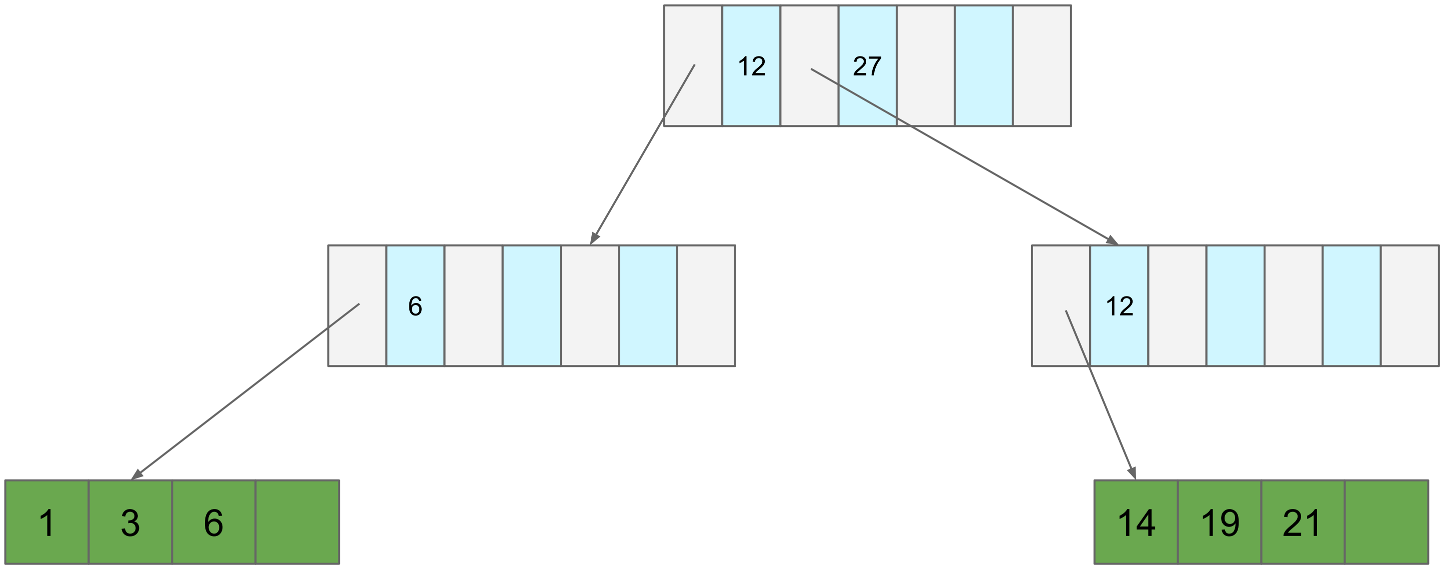
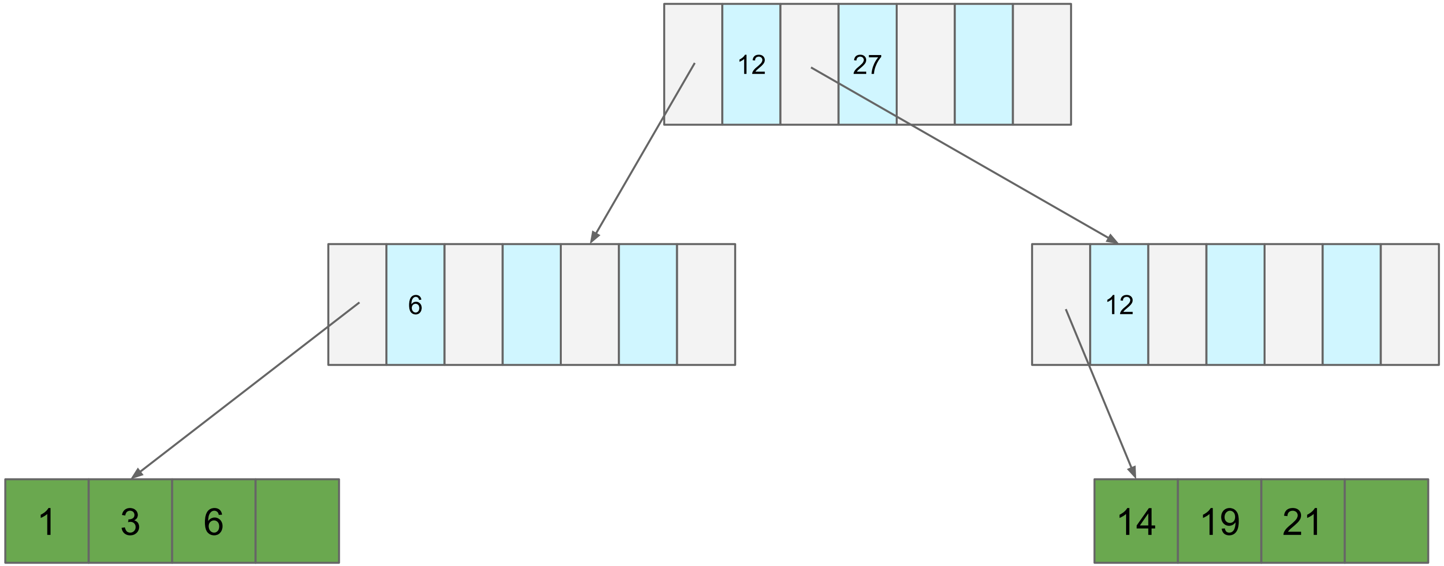


B+ Trees

Problem: We have O(log(n)) reads per search for the biggest n in the tree's history



B+ Trees Minimum Fill

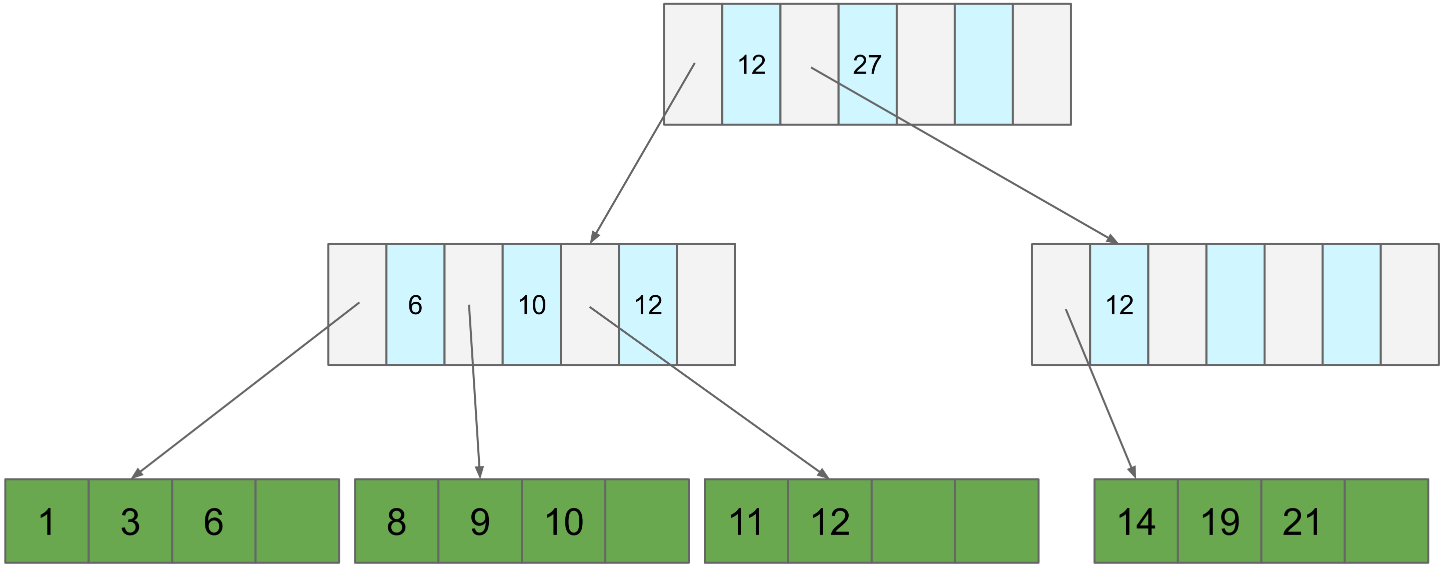
Enforce that each directory and data node must have ≥ c/2 records

- **Exception:** the root

What does this do to tree depth?

- $\boldsymbol{O(\log_{c/2}(n))}$ (as compared to $\mathbf{O(\log_{C}(n))}$ when the tree is static)

B+ Trees Minimum Fill



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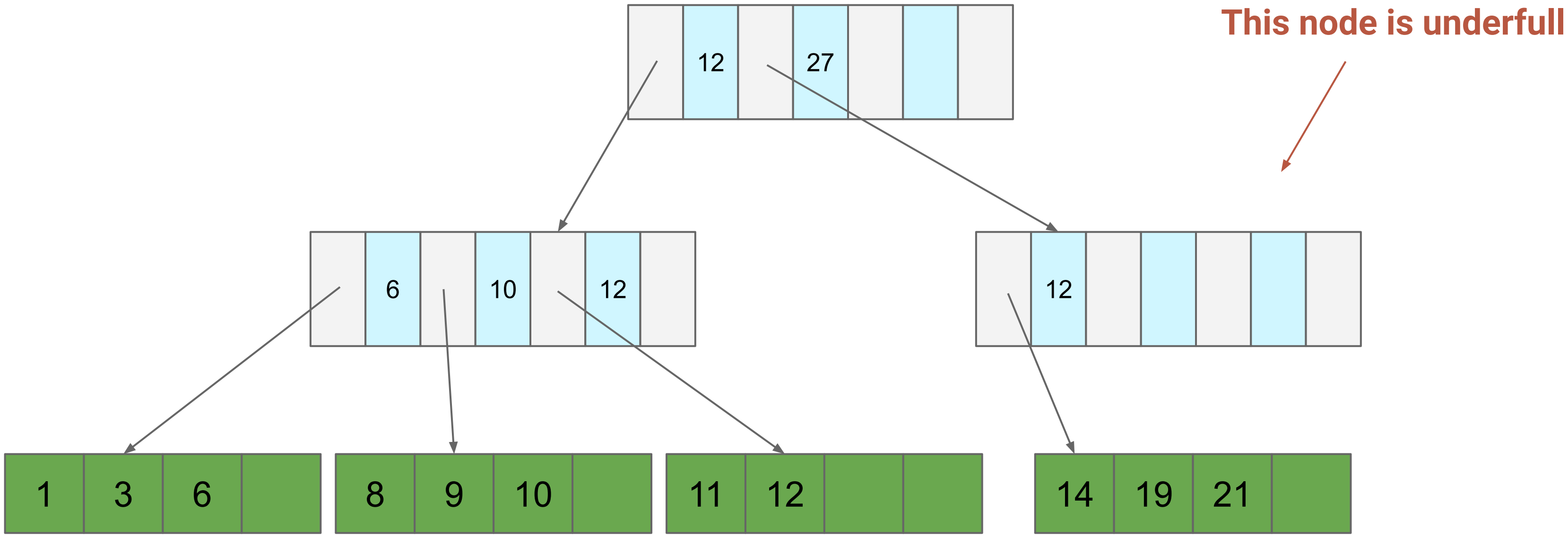

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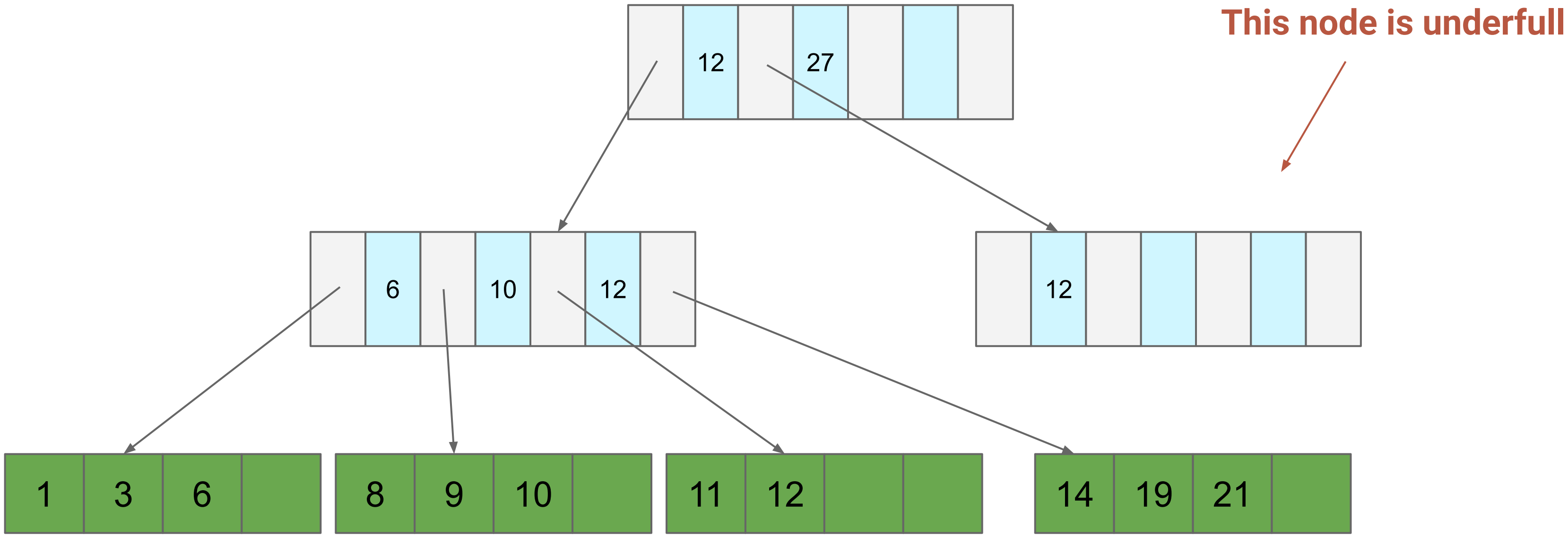

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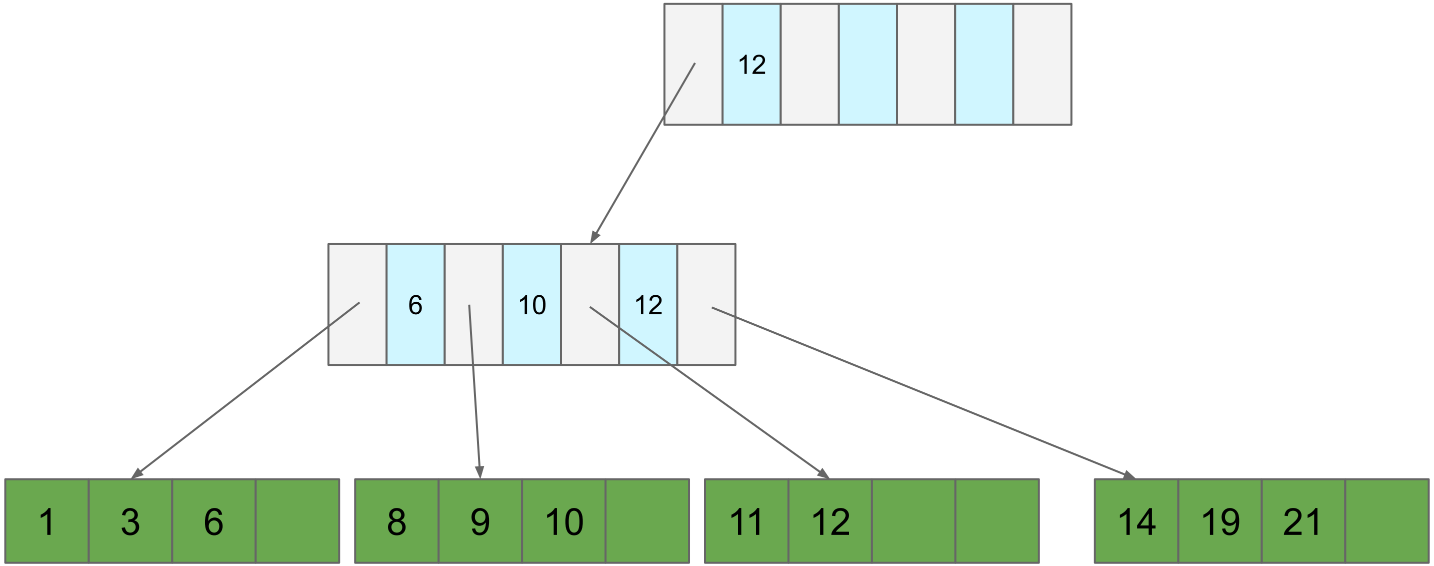
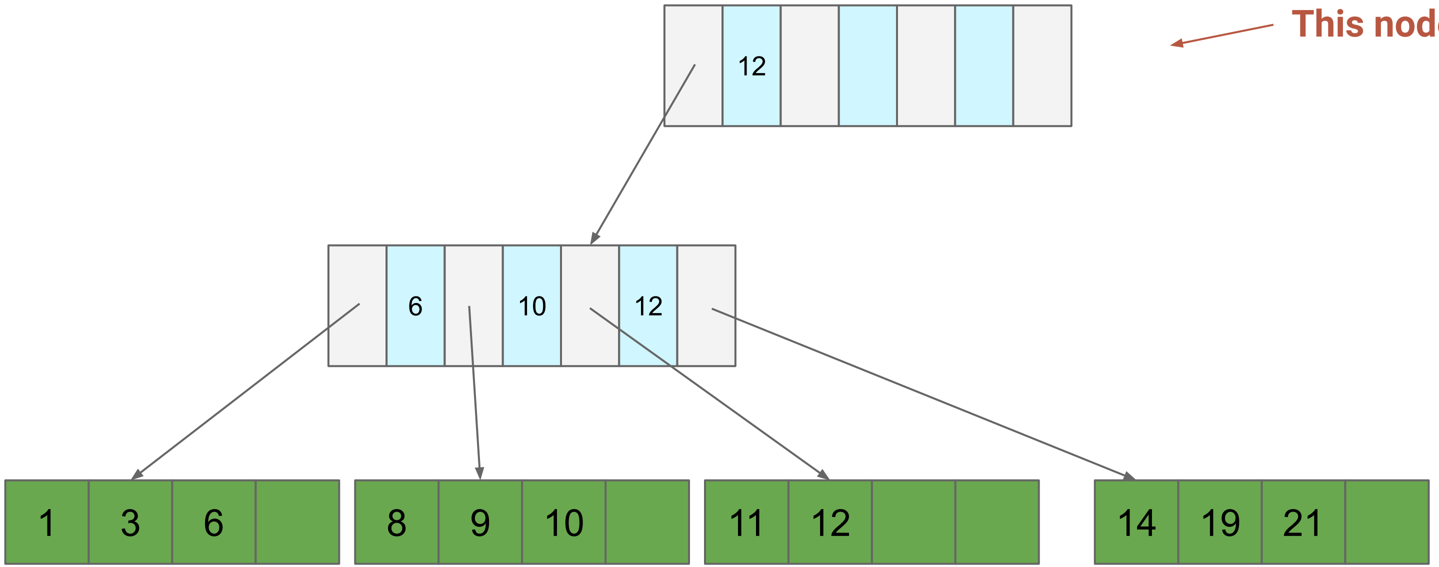
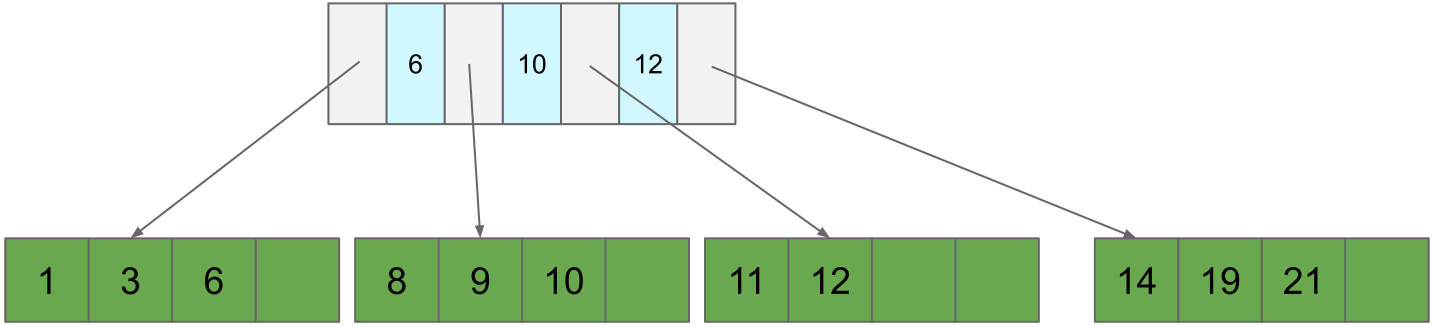


B+ Trees Minimum Fill



This node is underfull

B+ Trees Minimum Fill



B+ Trees

`Delete`

1. Find the page the record is on
2. Delete the record (if present)
3. If underfull, "merge" the page with a neighbor
    a. If either neighbor has > c/2 entries then steal instead
    b. If parent underfull, repeat
        i. If root, then drop the lowest layer