CSE 250

Data Structures

Dr. Eric Mikida
epmikida@buffalo.edu
208 Capen Hall

More Spatial Data Structures

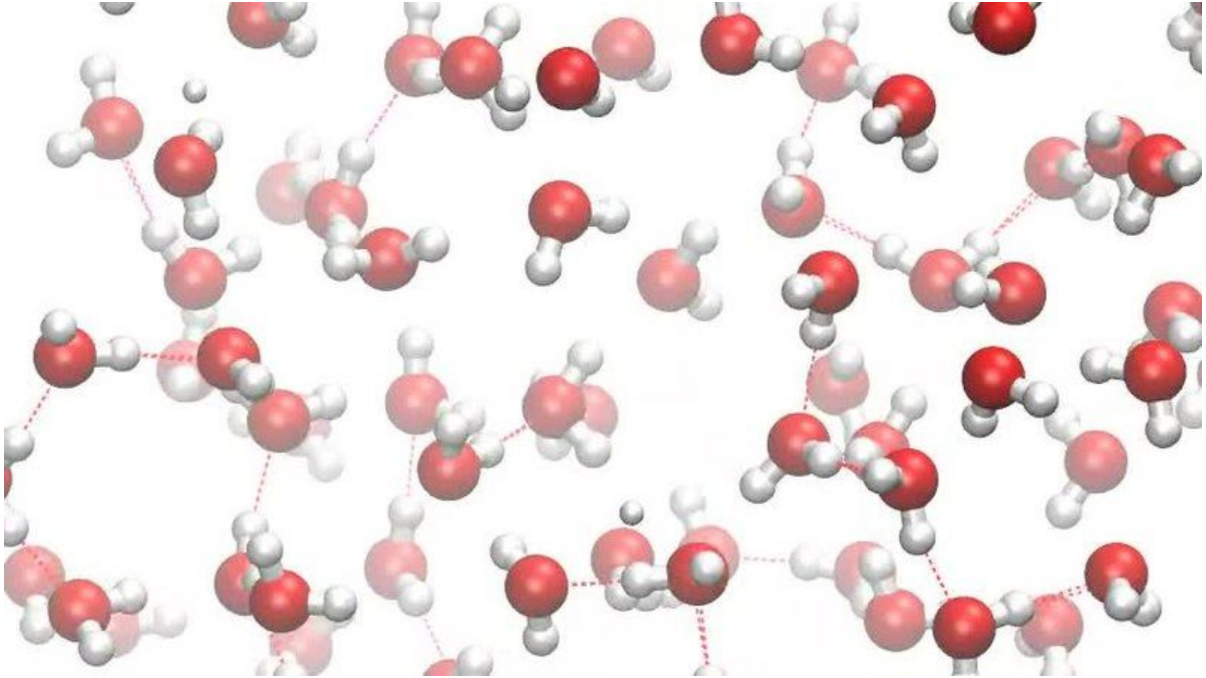
Announcements

- PA4 due Sunday

Some Problems are REALLY Big

ESA/Hubble and NASA: http://www.spacetelescope.org/images/potw1006a/

Some Problems are REALLY Small



Molecular Dynamics Simulation of Liquid Water
https://commons.wikimedia.org/wiki/File:A_Molecular_Dynamics_Simulation_of_Liquid_Water_at_298_K.webm

Some Problems are REALLY Detailed

This is **NOT** a photo. It is a computer generated image.

https://en.wikipedia.org/wiki/Ray_tracing_%28graphics%29#/media/File:Glasses_800_edit.png

Other Problems: N-Body Problem

What if we want to compute interactions between one body and every other body? How long would we expect that to take?

Other Problems: N-Body Problem

What if we want to compute interactions between one body and every other body? How long would we expect that to take?

Naively, this would take ***O*(n^2)**...but likely we don't care as much about interactions with bodies that are very very far away.

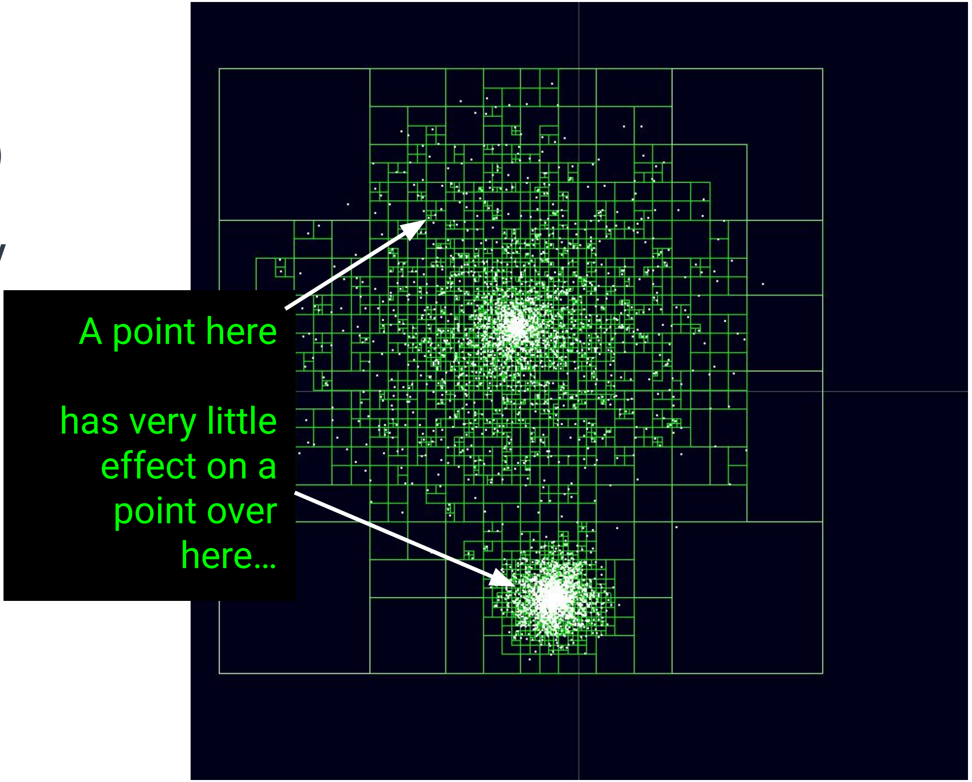
Other Problems: N-Body Problem

Idea: Divide our points into a quadtree (or octree in 3 dimensions)

Do full calculation for points closeby (in the same box)

Compute a summary (ie total force and center of mass) for each box that can be applied to far away boxes

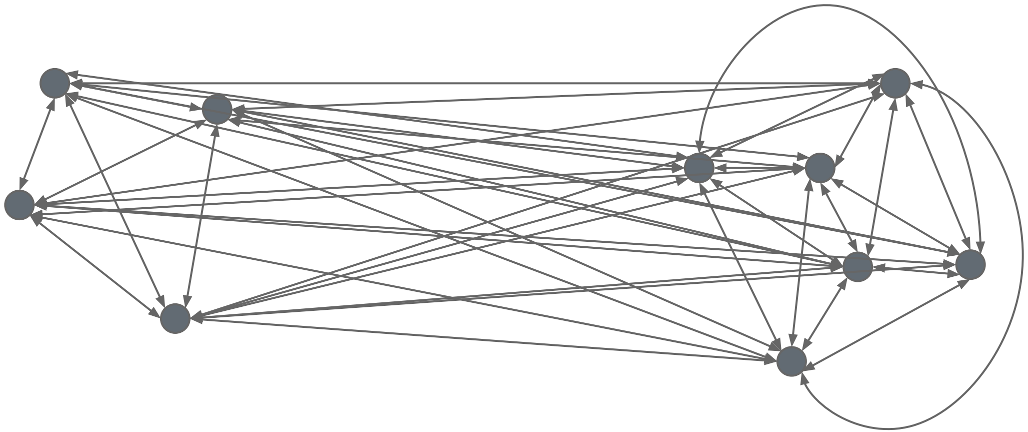
Target runtime: ~***O*(*n*log(*n*))**



Example

This diagram contains 10 bodies interacting with one another…

***O*(*n*^{2}) = ~**100 interactions (arrows)

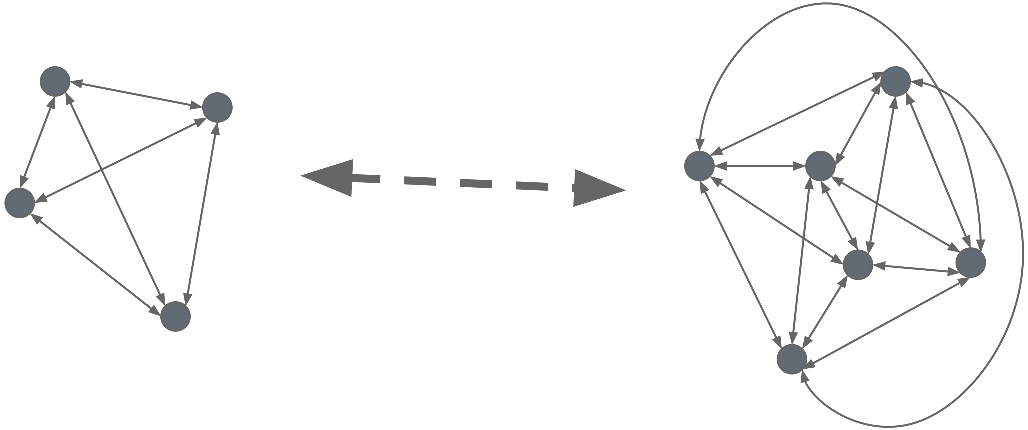


Example

This diagram contains 10 bodies interacting with one another…

***O*(*n*²) = ~**100 interactions (arrows)

Idea: Estimate the interactions between far away points



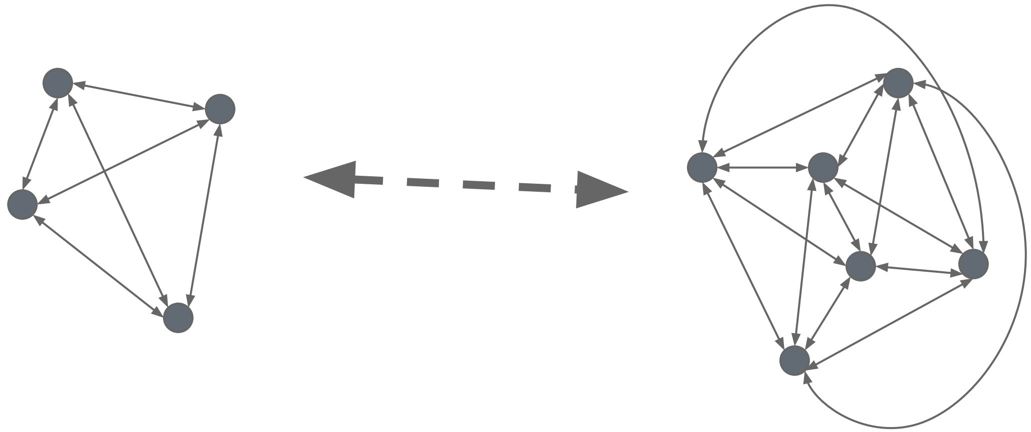
Example

This diagram contains 10 bodies interacting with one another…

$O(n^2)$ = ~100 interactions (arrows)

How can we do this systematically?

Idea: Estimate the interactions between far away points



Quad/Oct Trees Revisited

Idea: Let's organize the data (spatially) in a tree structure

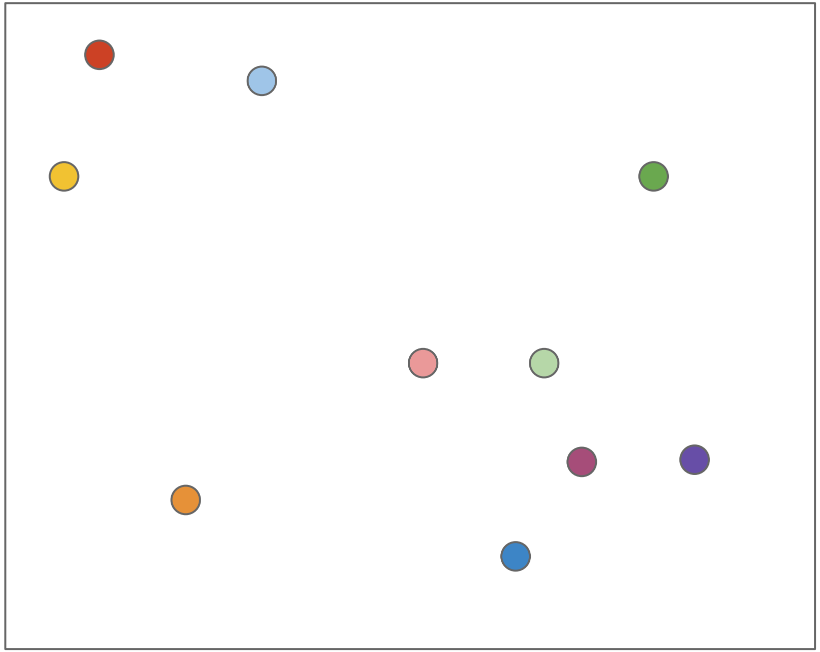
- 2D space → use a quad tree
- 3D space → use an oct tree (each node has at most 8 children)

Unlike last time, let's partition the space we are simulating, rather than the points in the space

Space Partitioning - 2D Example

Create a quad-tree by recursively partitioning the space

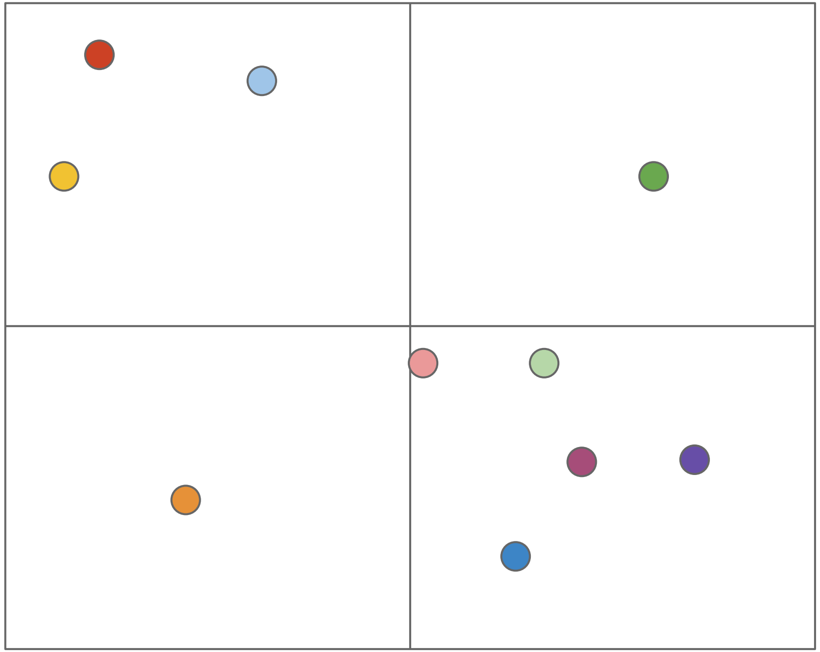
- Divide the space evenly until there is only one element per partition
- Internal tree nodes represent the partitions, leaves are the actual elements



Space Partitioning - 2D Example

Create a quad-tree by recursively partitioning the space

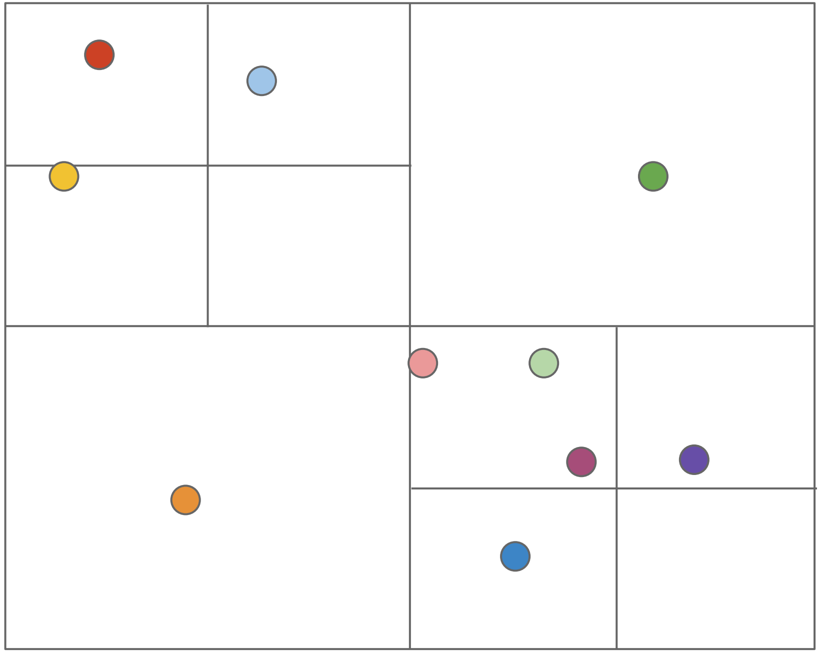
- Divide the space evenly until there is only one element per partition
- Internal tree nodes represent the partitions, leaves are the actual elements



Space Partitioning - 2D Example

Create a quad-tree by recursively partitioning the space

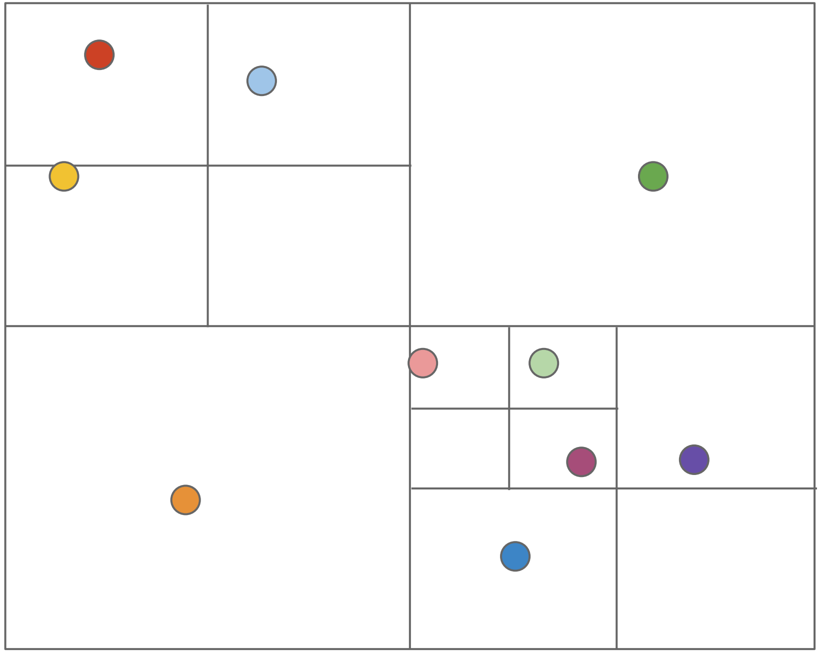
- Divide the space evenly until there is only one element per partition
- Internal tree nodes represent the partitions, leaves are the actual elements



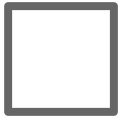
Space Partitioning - 2D Example

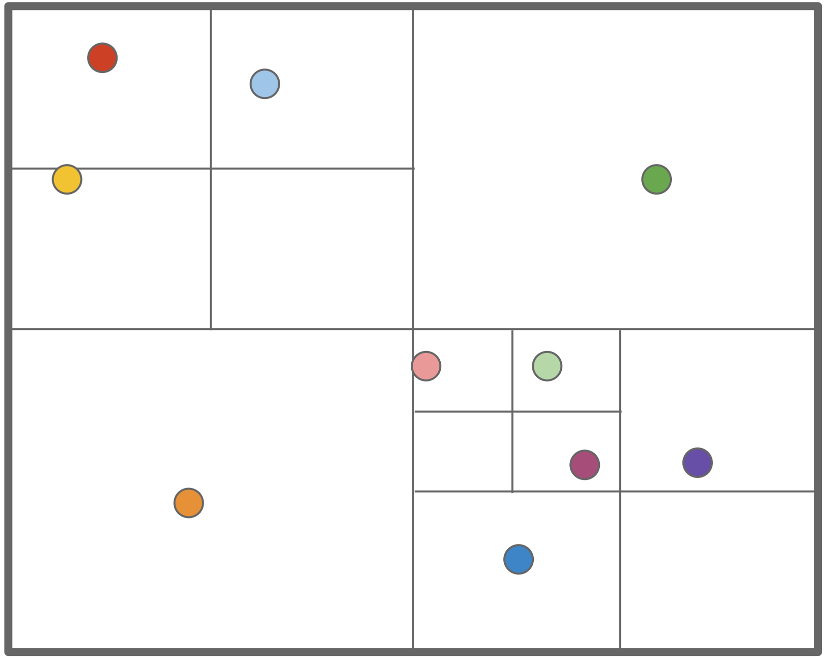
Create a quad-tree by recursively partitioning the space

- Divide the space evenly until there is only one element per partition
- Internal tree nodes represent the partitions, leaves are the actual elements

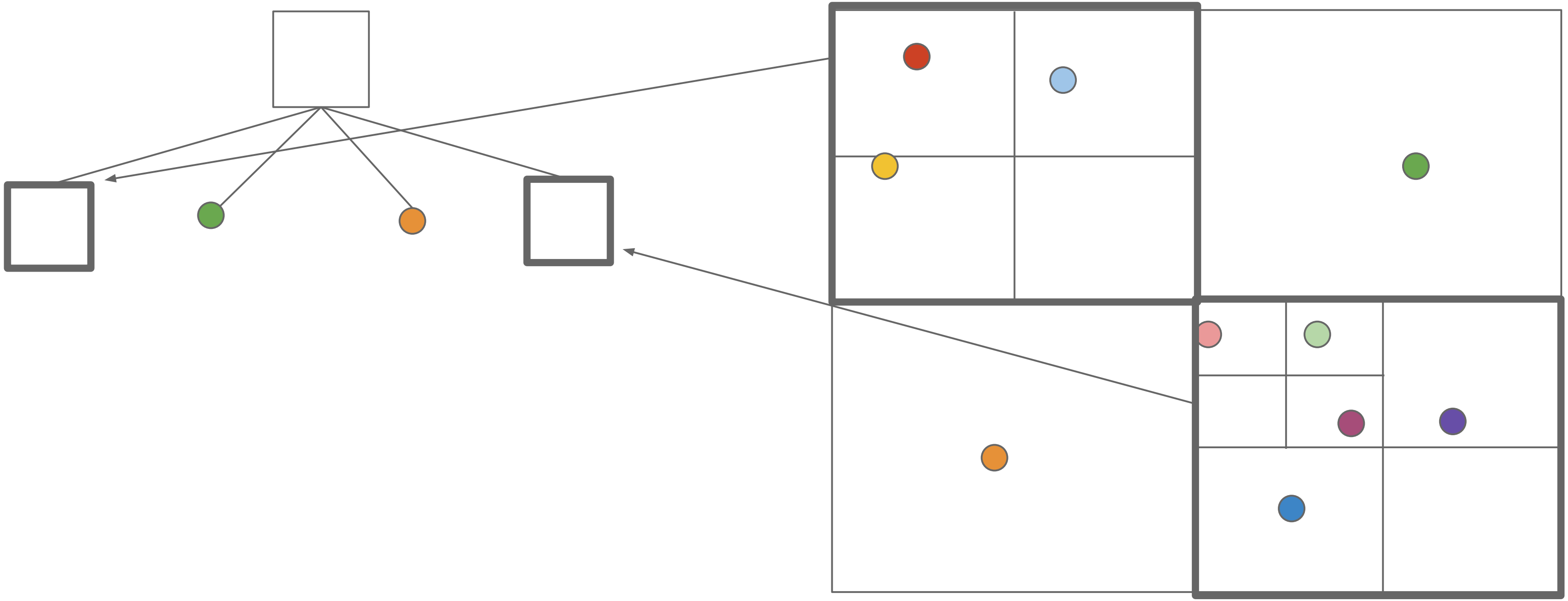


Space Partitioning - 2D Example

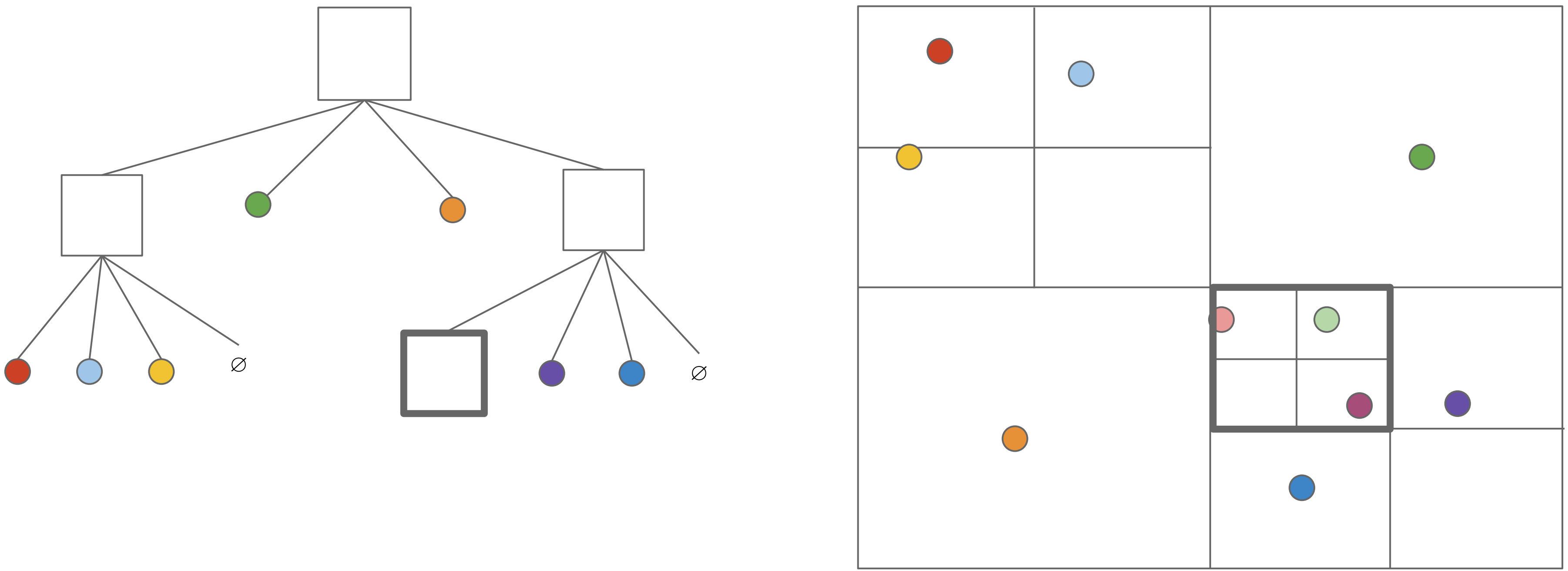




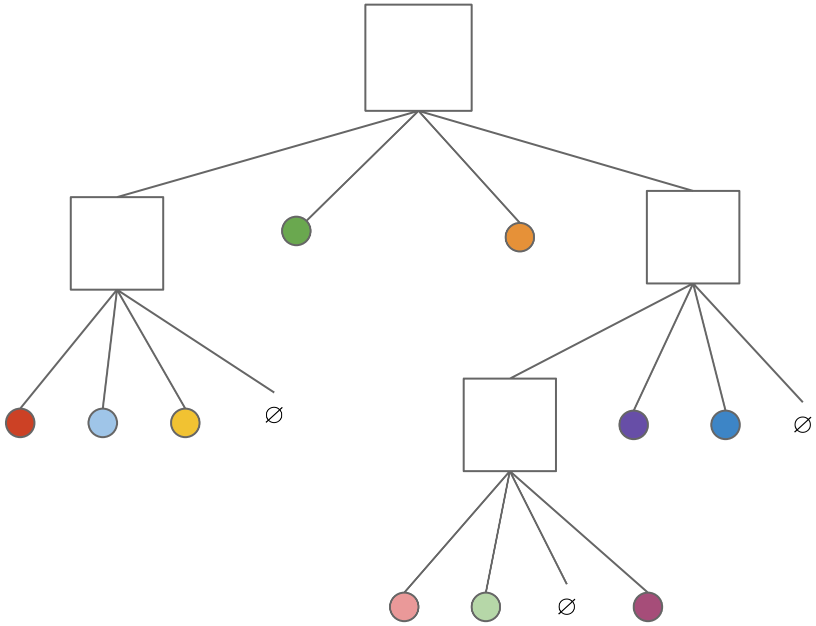
Space Partitioning - 2D Example

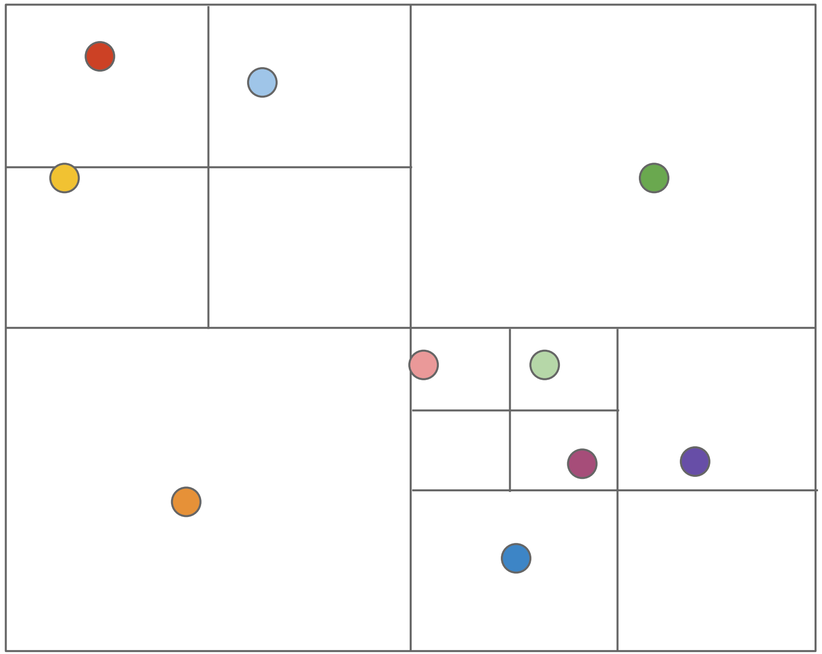


Space Partitioning - 2D Example

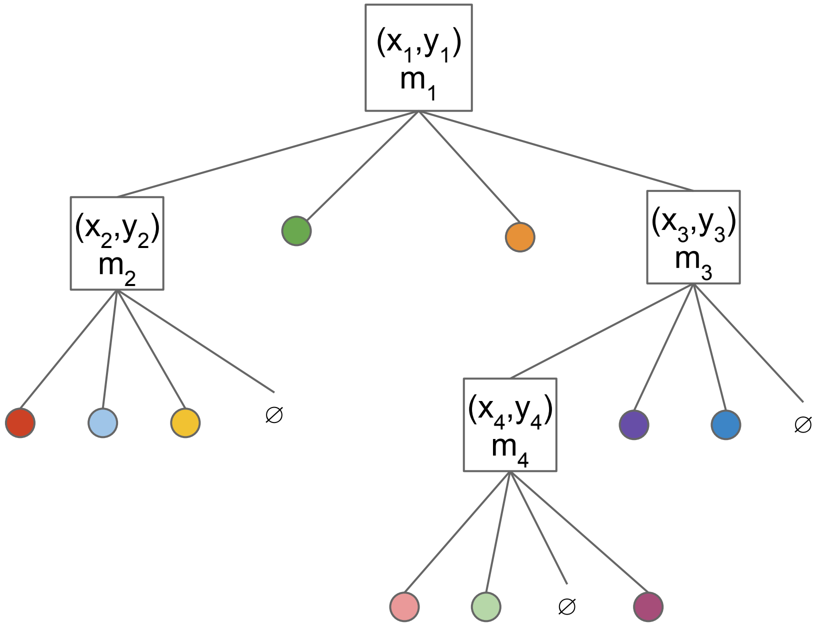


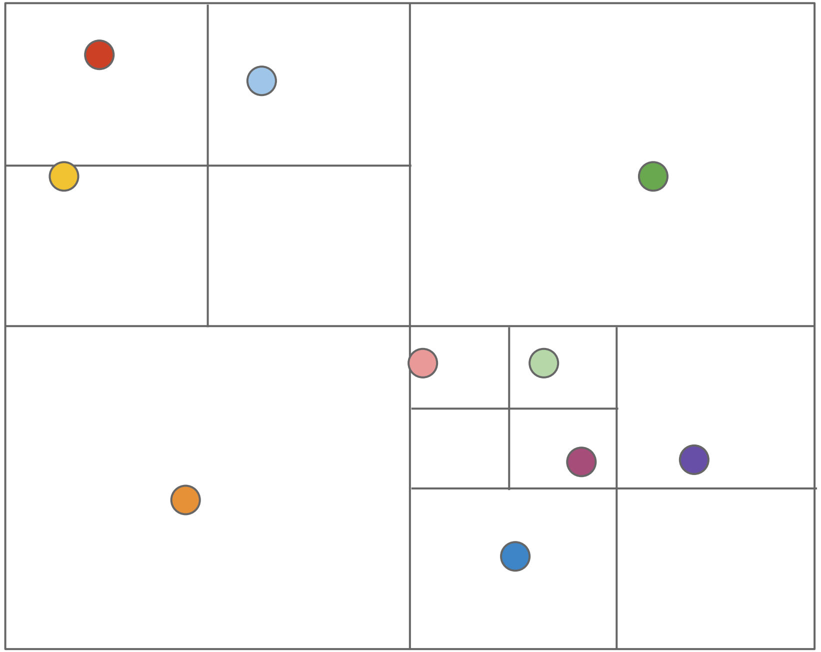
Space Partitioning - 2D Example





Space Partitioning - 2D Example





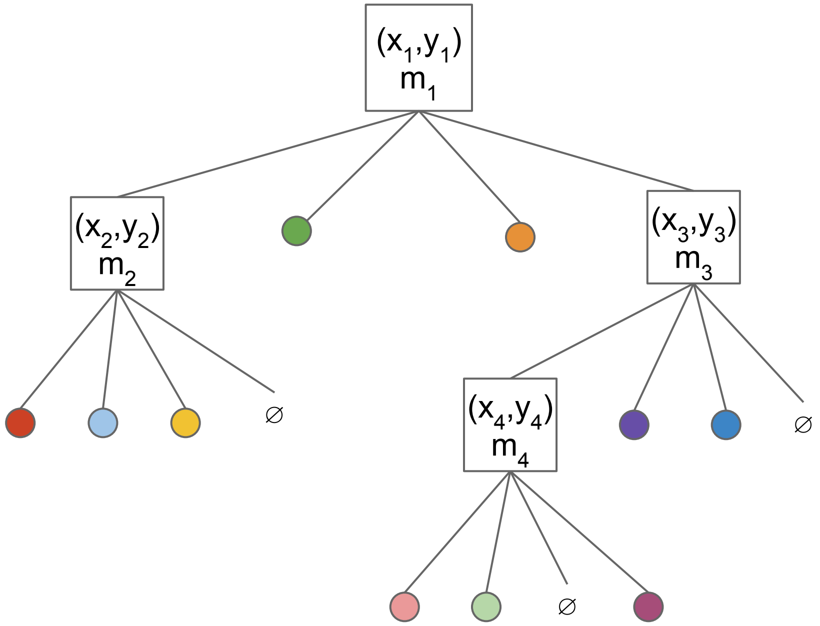
For each internal node, we can compute the center of mass and total mass

Barnes-Hut Algorithm (simplified)

Now to use the tree:

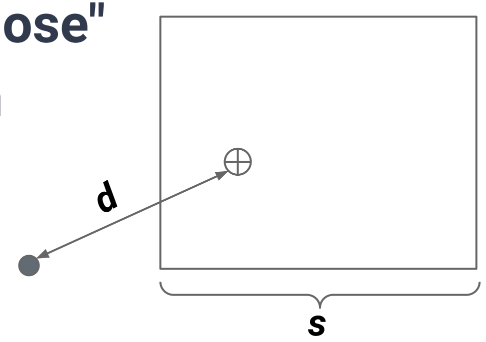
For a body with coordinates (x_b, y_b):

1. Start at the root
2. If the (x_1, y_1) is "far" from (x_b, y_b) then just treat it as a single body with mass m_1
3. If it is "close", then repeat this process with the children



Barnes-Hut Algorithm (simplified)

So what is considered "far", and what is considered "close"



- Find the ratio ***s***/***d*** where ***s*** is the width of the region in question and ***d*** is the distance from the body to the center of mass of that region
- Pick a threshold, θ
  - If ***s***/***d*** > θ then we are close enough to check children in more detail
  - If ***s***/***d*** < θ then we are far away and can treat the region as a single body
- Larger θ means more fudging the numbers, but faster execution (**~*O*(n log n)** to process all ***n*** bodies)
- θ = 0 means finding an exact answer, but at a cost of $O(n^2)$

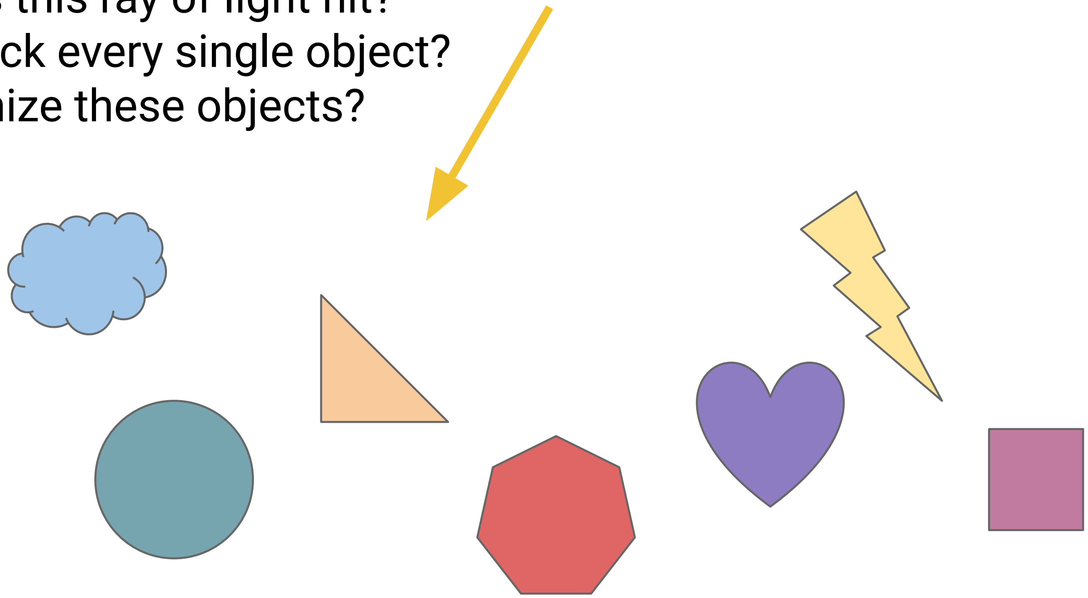
Trees as a Hierarchy

In the n-body problem, we used a tree to ***hierarchically*** organize our data

- When using this hierarchy, for each internal node we could decide whether or not to explore further with a very cheap ***O*(1)** check
  - This allows us to avoid checking all ***n*** elements in a systematic fashion
- This style of algorithm has other applications as well

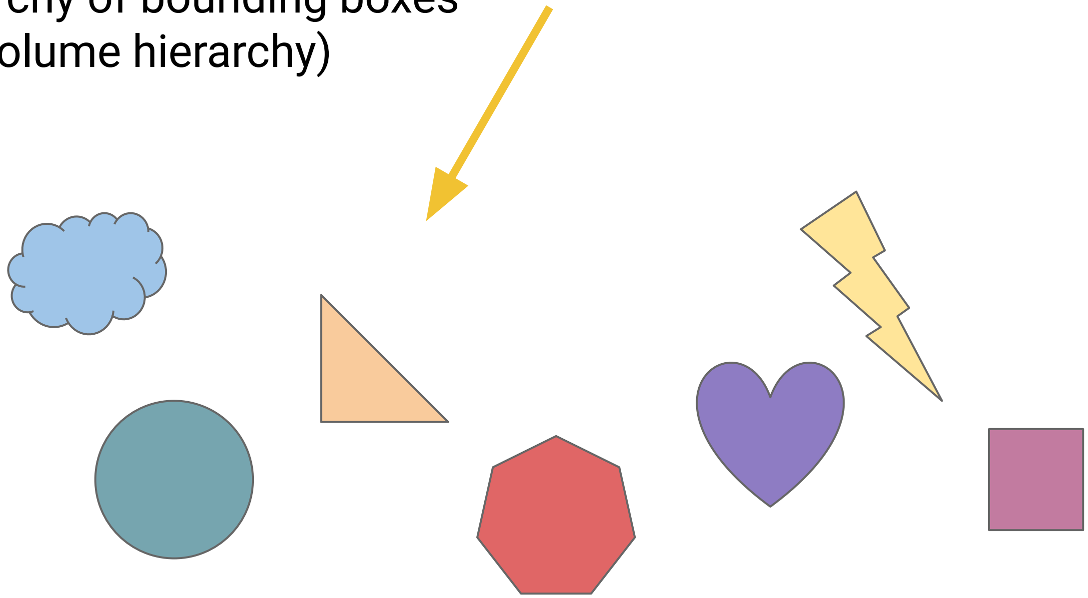
Other Problems: Ray/Path Tracing

Which object does this ray of light hit?
Do we have to check every single object?
How can we organize these objects?



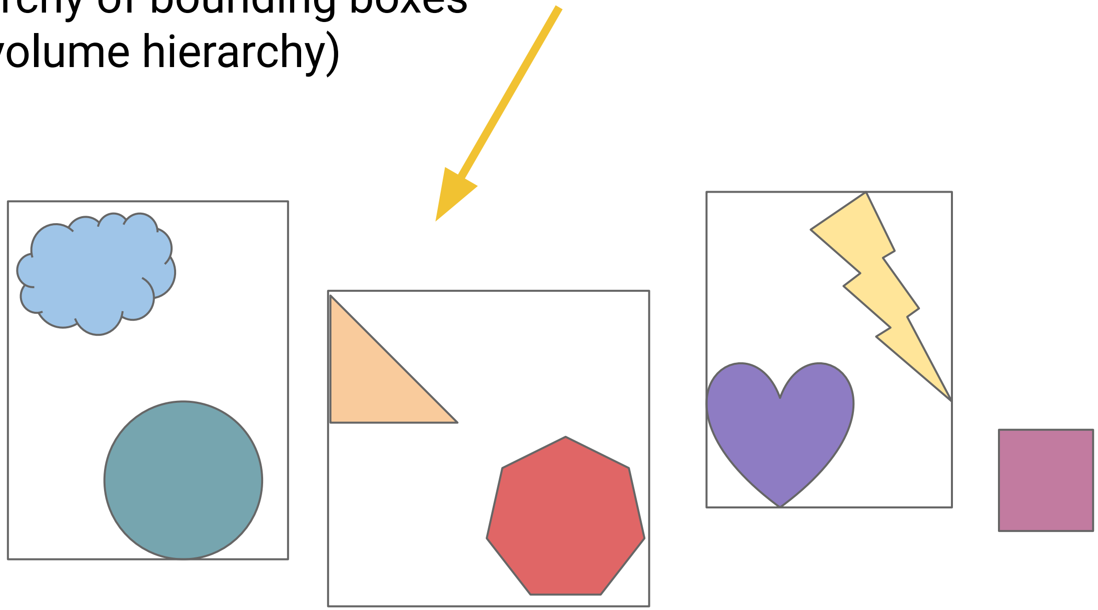
Other Problems: Ray/Path Tracing

Idea: Build a hierarchy of bounding boxes
(BVH - Bounding volume hierarchy)



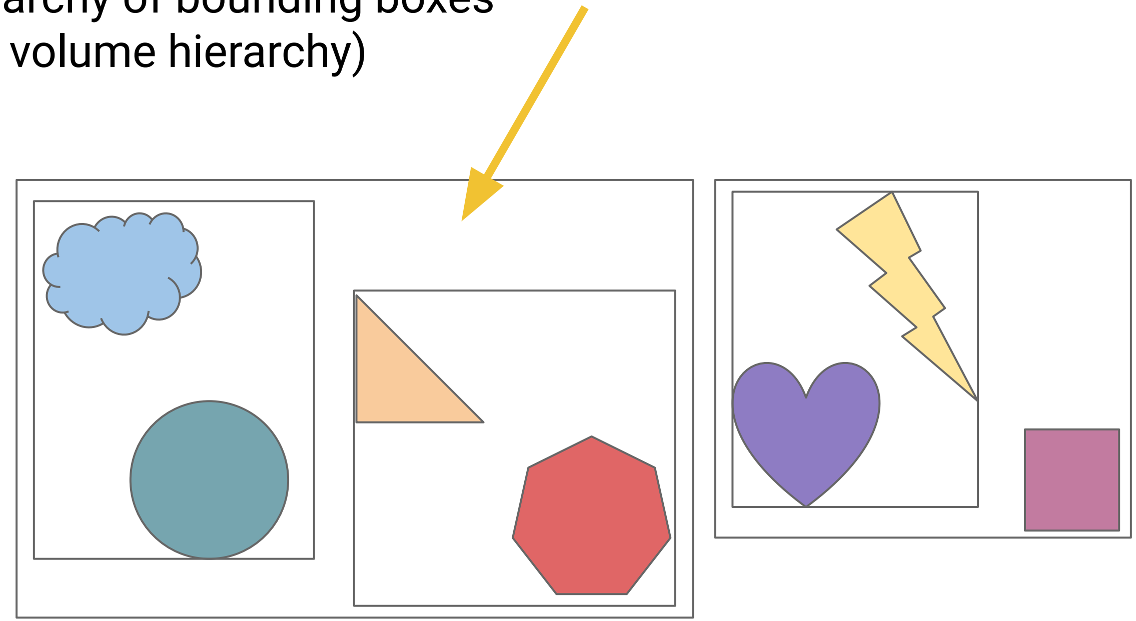
Other Problems: Ray/Path Tracing

Idea: Build a hierarchy of bounding boxes
(BVH - Bounding volume hierarchy)



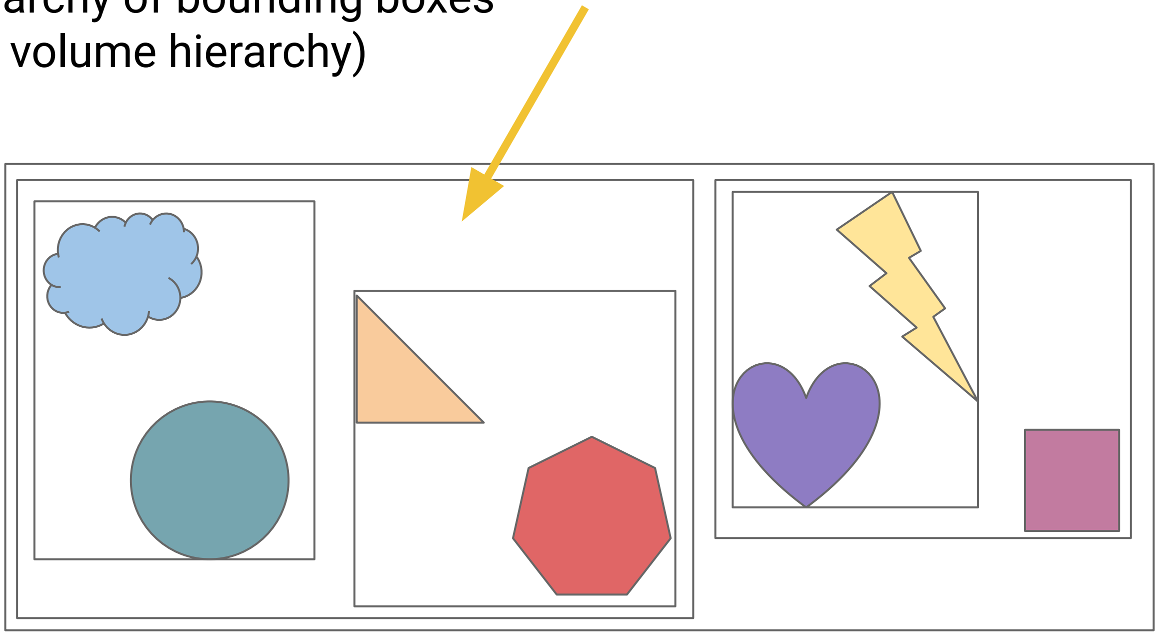
Other Problems: Ray/Path Tracing

Idea: Build a hierarchy of bounding boxes
(BVH - Bounding volume hierarchy)



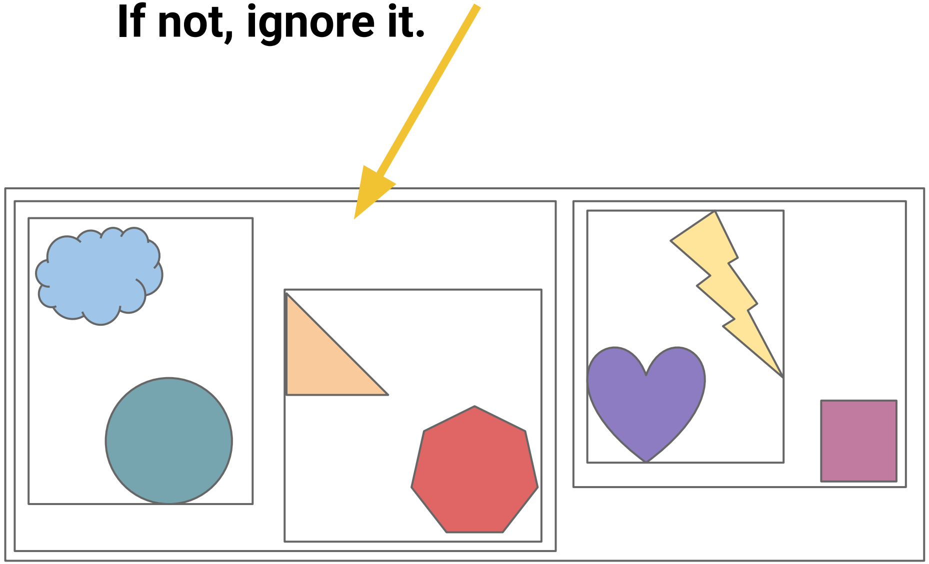
Other Problems: Ray/Path Tracing

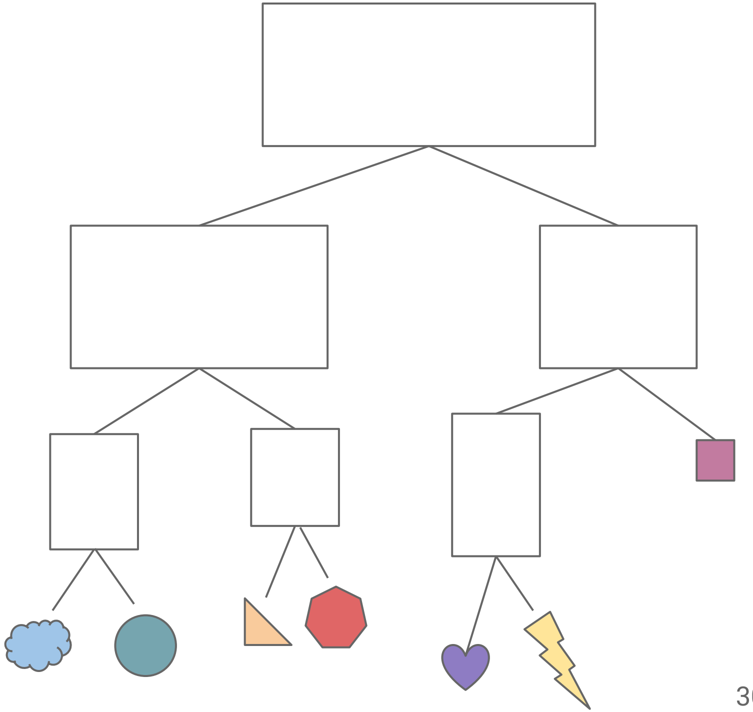
Idea: Build a hierarchy of bounding boxes
(BVH - Bounding volume hierarchy)



Other Problems: Ray/Path Tracing

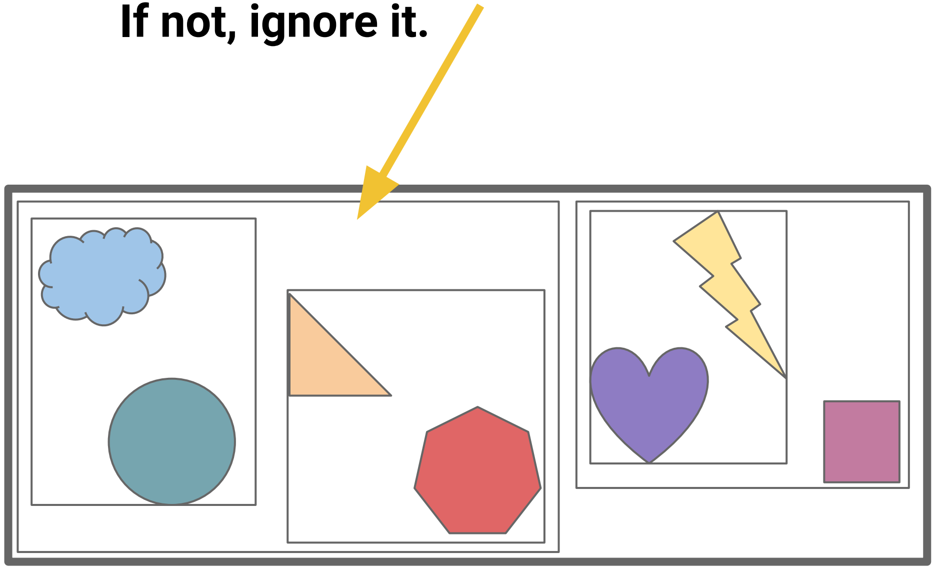
These bounding boxes form a tree…
We can check if the ray intersects a bounding box.
  If it does, explore its children.
  If not, ignore it.

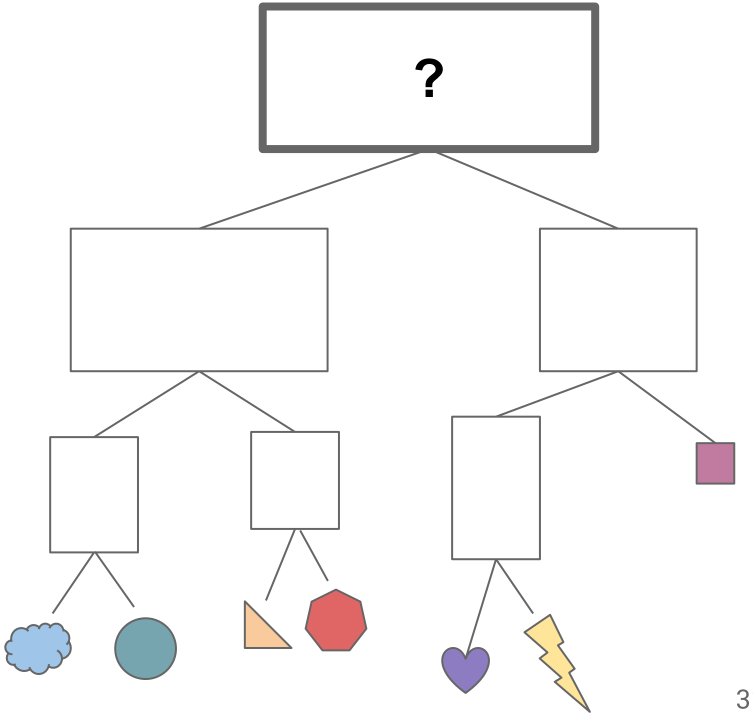




Other Problems: Ray/Path Tracing

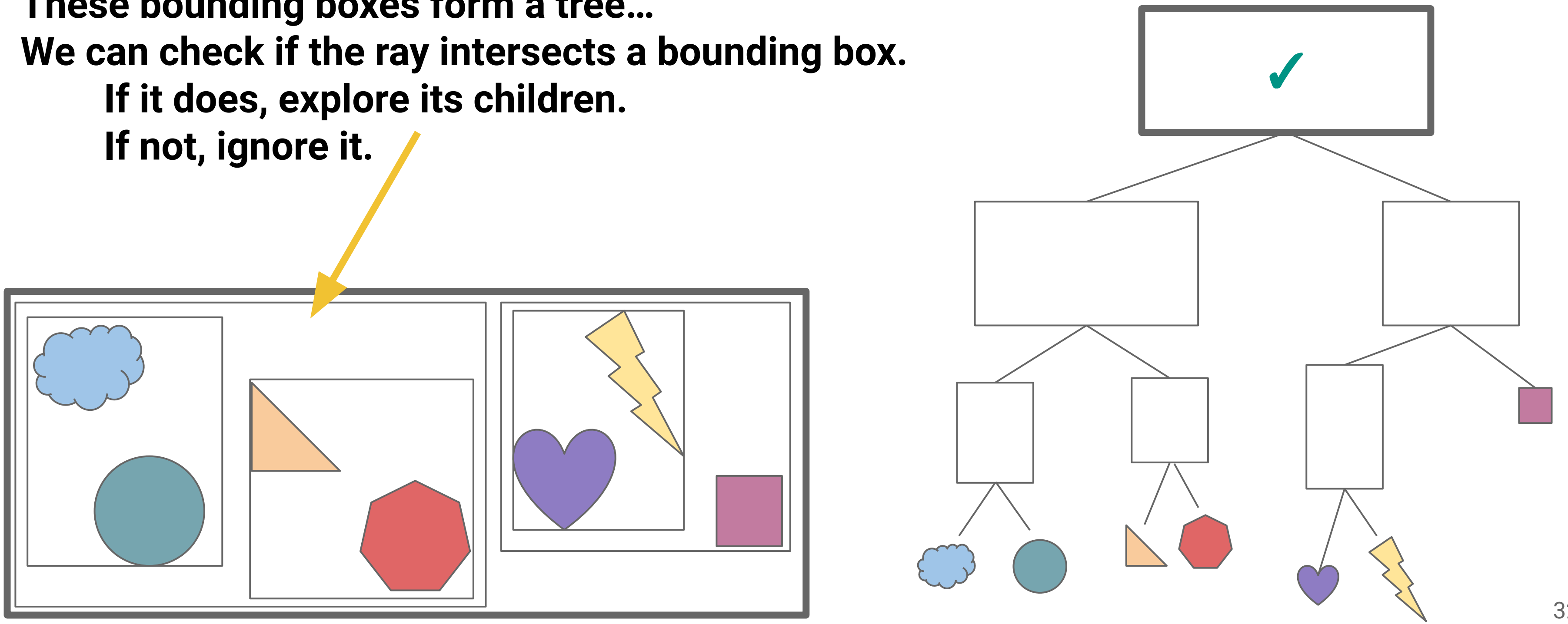
These bounding boxes form a tree…
We can check if the ray intersects a bounding box.
If it does, explore its children.
If not, ignore it.





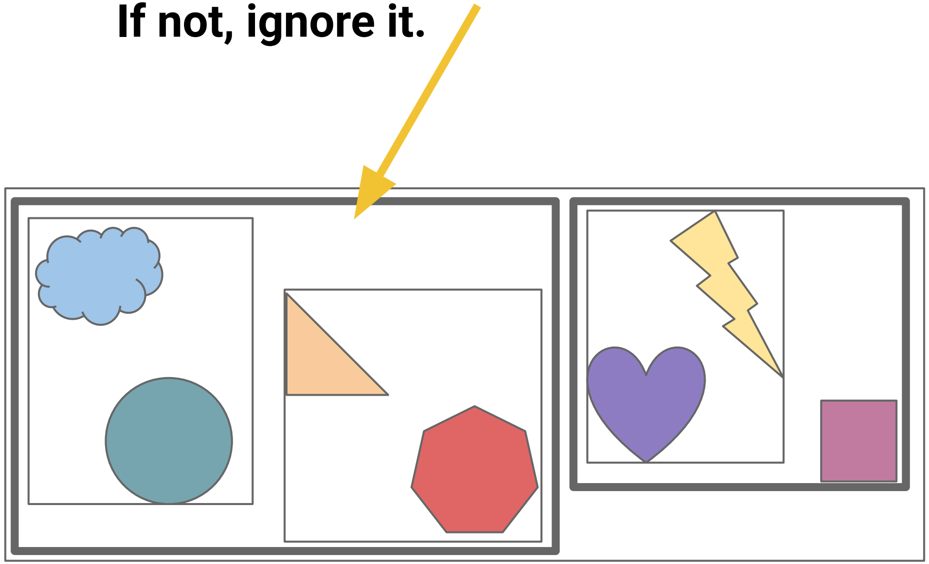
Other Problems: Ray/Path Tracing

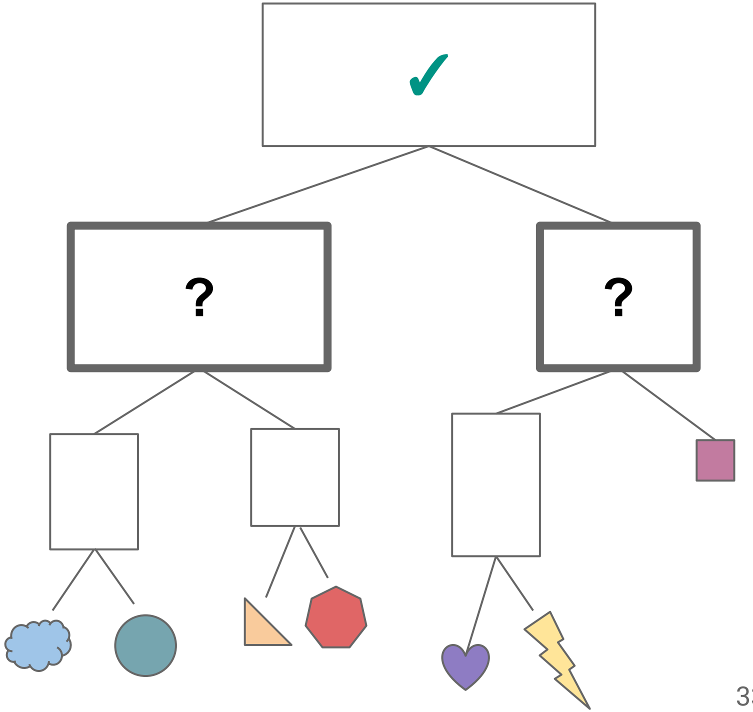
These bounding boxes form a tree…
We can check if the ray intersects a bounding box.
If it does, explore its children.
If not, ignore it.



Other Problems: Ray/Path Tracing

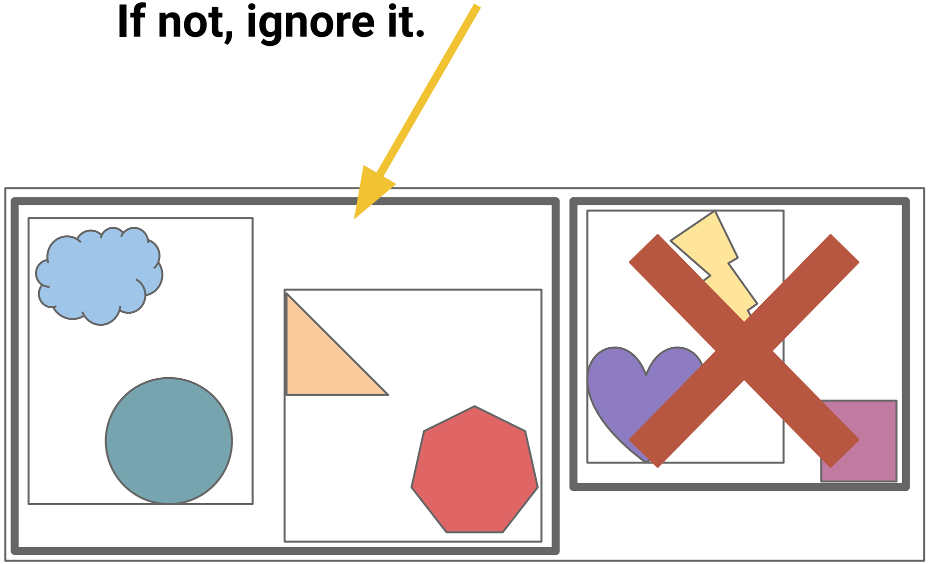
These bounding boxes form a tree…
We can check if the ray intersects a bounding box.
If it does, explore its children.
If not, ignore it.

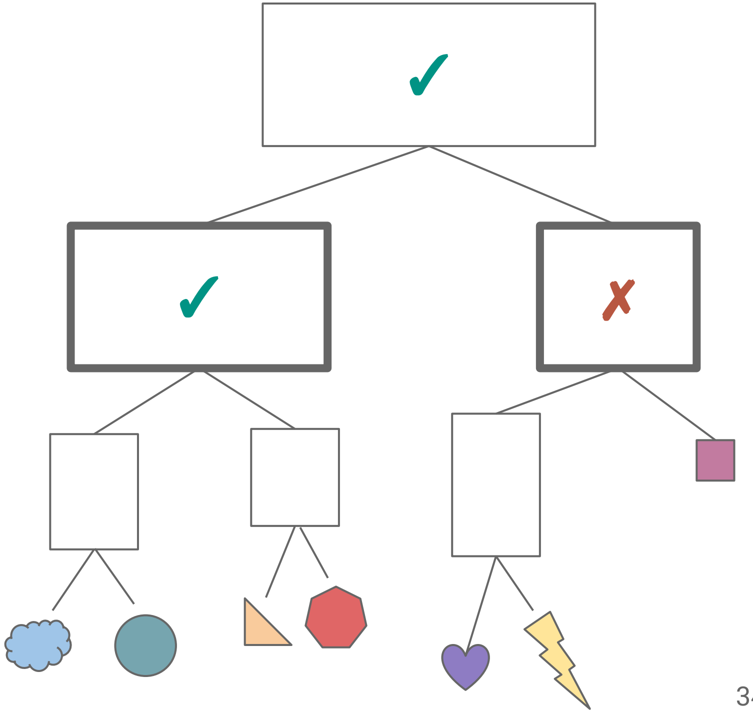




Other Problems: Ray/Path Tracing

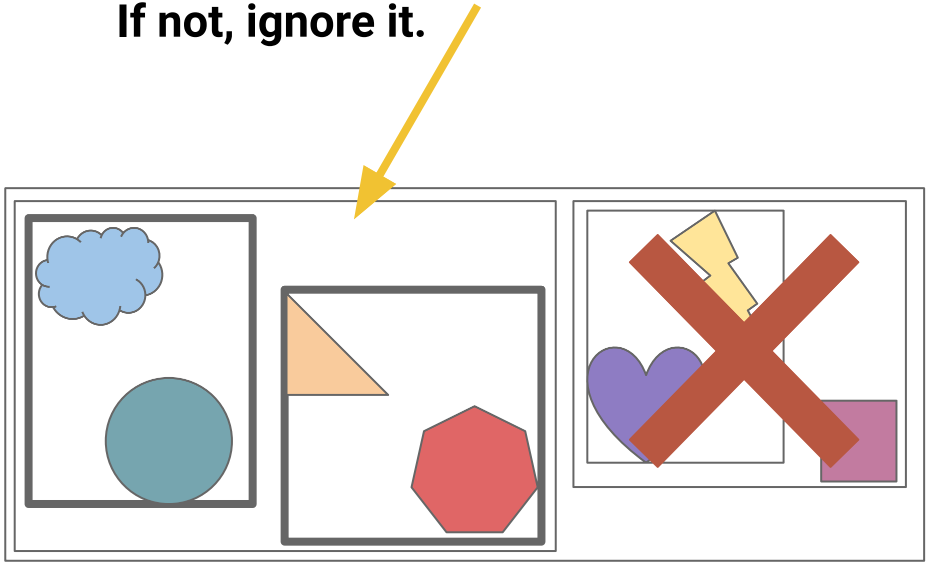
These bounding boxes form a tree…
We can check if the ray intersects a bounding box.
If it does, explore its children.
If not, ignore it.

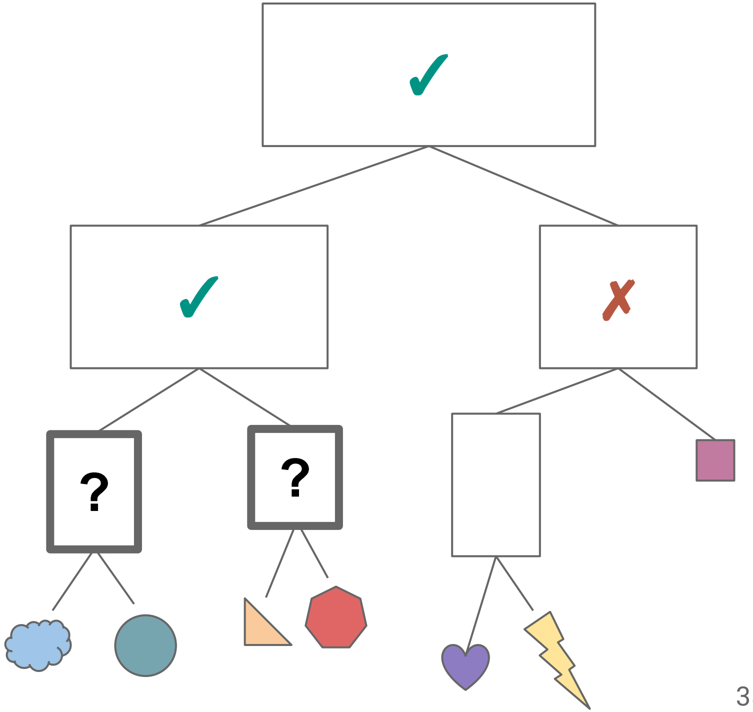




Other Problems: Ray/Path Tracing

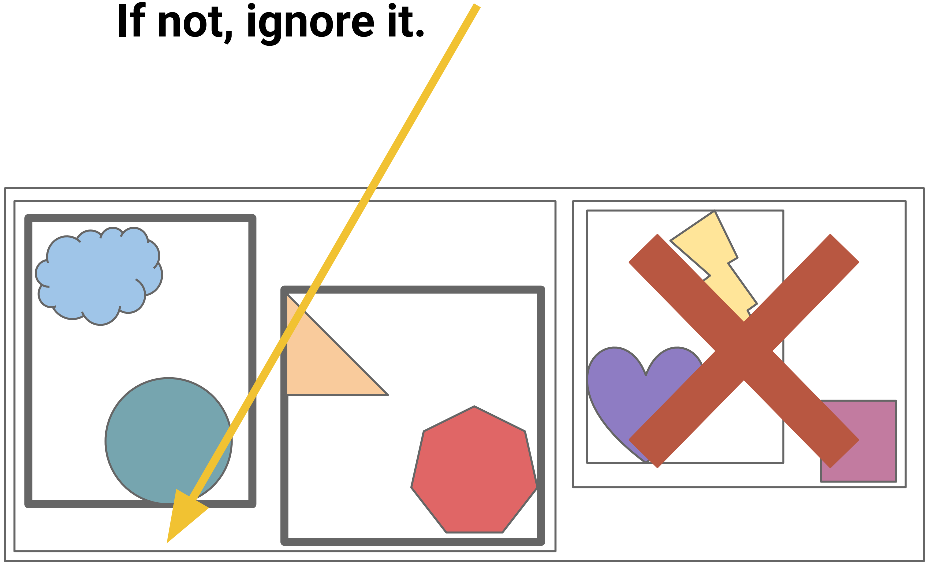
These bounding boxes form a tree…
We can check if the ray intersects a bounding box.
If it does, explore its children.
If not, ignore it.

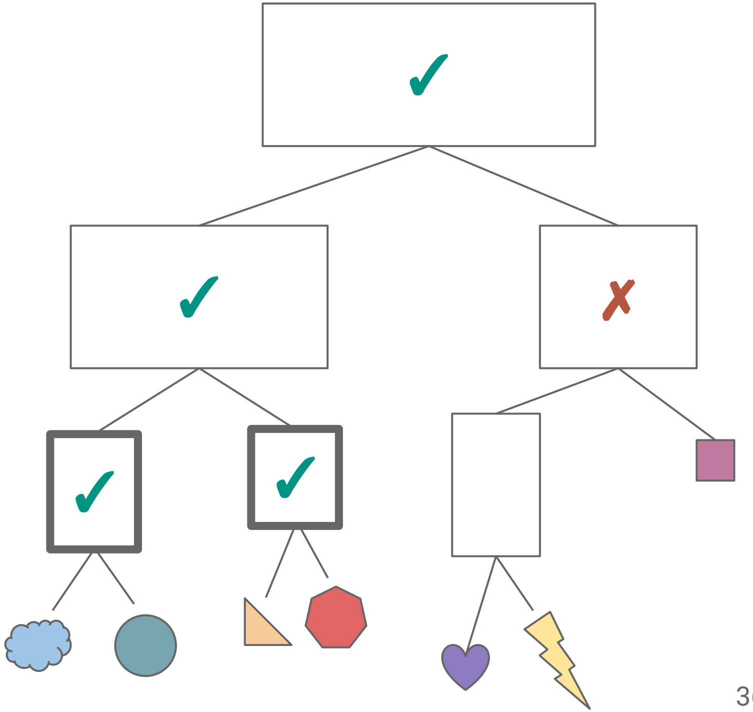




Other Problems: Ray/Path Tracing

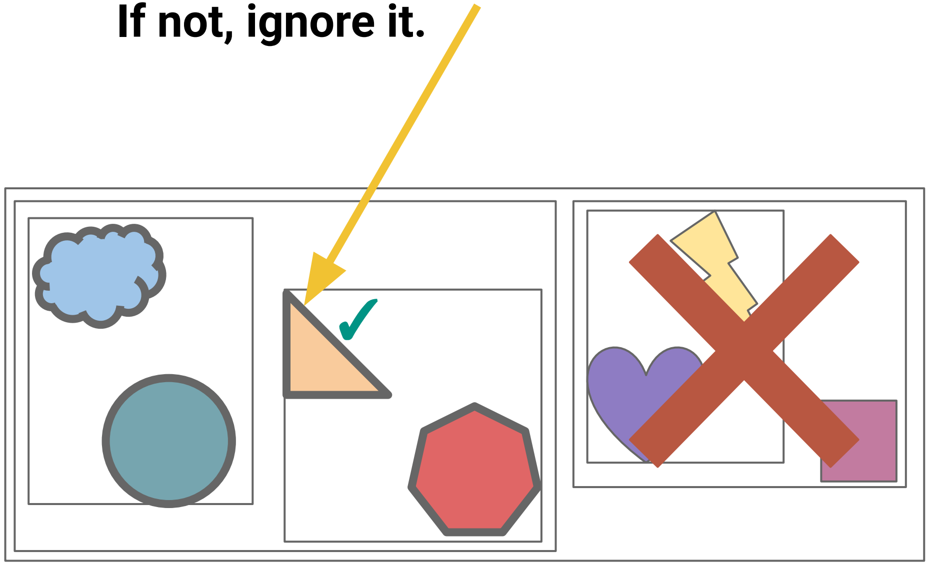
These bounding boxes form a tree…
We can check if the ray intersects a bounding box.
- **If it does, explore its children.**
- **If not, ignore it.**

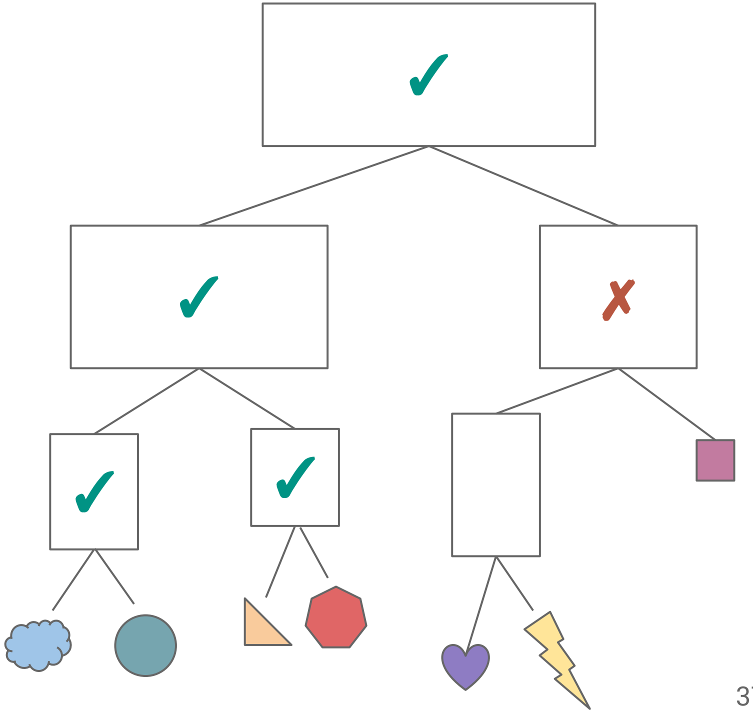




Other Problems: Ray/Path Tracing

These bounding boxes form a tree…
We can check if the ray intersects a bounding box.
- **If it does, explore its children.**
- **If not, ignore it.**





Other Problems: Ray/Path Tracing

- By using a ***bounding-volume hierarchy***, we can avoid checking all ***n*** objects for collisions
  - When we are projecting millions+ rays of light, this is a huge savings
- In practice, we hope to end up with a runtime of **~*O*(*m* log *n*)** where ***m*** is the number of rays and ***n*** is the number of objects
  - This depends on how effectively we can build our BVH
- In both ray tracing and Barnes-Hut, the exact structure of the hierarchy will vary based on the specific data we are using

High-Level Summary

- We've seen both trees and hash tables as effective ways to organize our data if we know we are going to be searching it often
- **HashTables** can be great for exact lookups
  - Think PA4: you may want to lookup a person with an exact (birthday, zipcode) pair, and HashTable lets you do that very fast
- **Trees** and tree like structures work very well for "fuzzier" searches
  - What is "close" to this point? What object might this projectile hit? etc
  - The input to your search is not necessarily an exact element in your tree, but the tree organizes the data in a way that directs your search