Questions?

- Scala?
- PA1?
- Asymptotic Notation?
- Runtime Analysis?
Logarithms Cheat Sheet

- Let $a, b, c, n > 0$
- **Exponent Rule:** $\log(n^a) = a \log(n)$
- **Product Rule:** $\log(an) = \log(a) + \log(n)$
- **Division Rule:** $\log\left(\frac{n}{a}\right) = \log(n) - \log(a)$
- **Change of Base from $b$ to $c$:** $\log_b(n) = \frac{\log_c(n)}{\log_c(b)}$
- **Log/Exponent are Inverses:** $b^{\log_b(n)} = \log_b(b^n) = n$
1. \( \sum_{i=j}^{k} c = (k - j + 1)c \)

2. \( \sum_{i=j}^{k} cf(i) = c \sum_{i=j}^{k} f(i) \)

3. \( \sum_{i=j}^{k} (f(i) + g(i)) = (\sum_{i=j}^{k} f(i)) + (\sum_{i=j}^{k} g(i)) \)

4. \( \sum_{i=j}^{k} f(i) = (\sum_{i=\ell}^{k} f(i) - (\sum_{i=\ell}^{j-1} f(i))) \) (for any \( \ell < j \))

5. \( \sum_{i=j}^{k} f(i) = f(j) + f(j + 1) + \ldots + f(k - 1) + f(k) \)

6. \( \sum_{i=j}^{k} f(i) = f(j) + \ldots + f(\ell - 1) + (\sum_{i=\ell}^{k} f(i)) \) (for any \( j < \ell \leq k \))

7. \( \sum_{i=j}^{k} f(i) = (\sum_{i=j}^{\ell} f(i)) + f(\ell + 1) + \ldots + f(k) \) (for any \( j \leq \ell < k \))

8. \( \sum_{i=1}^{k} i = \frac{k(k+1)}{2} \)

9. \( \sum_{i=0}^{k} 2^i = 2^{k+1} - 1 \)
Inequalities Cheat Sheet

- $x \geq y$ is true if $x/a \geq x/y$ (for any $a > 0$)
- $x \geq y$ is true if $xa \geq xy$ (for any $a > 0$)
- $x + a \geq y + b$ is true if $x \geq y$ and $a \geq b$ (for any $a, b$)
- $x \geq y$ is true if $x \geq a$ and $a \geq y$ (for any $a$)
Examples

Show that there is some $c$ for which...

$$12 \log(10 \times 2^n) \leq c \ n$$

... for all $n$ greater than some $n_0$
Examples

Show that there is some $c$ for which...

$$n^2 + n \log(n) \leq c \ 2^n$$

... for all $n$ greater than some $n_0$
Examples

Let $f(x)$ be a function defined as follows:

- If $x$ is odd, $f(x) = 10x$
- If $x$ is even, $f(x) = 100x^2$

Show that there is some $c_{\text{low}}$ and $c_{\text{high}}$ for which...

$$f(n) \geq c_{\text{low}} n \quad \text{and} \quad f(n) \leq c_{\text{high}} n^2$$

... for all $n$ greater than some $n_0$
Let $f(x)$ be a function defined as follows:

- If $x$ is odd, $f(x) = 10x$
- If $x$ is even, $f(x) = 100x^2$

Is there a $c_{\text{low}}$ and $c_{\text{high}}$ for which...

$f(n) \geq c_{\text{low}} n^2$ and $f(n) \leq c_{\text{high}} n$

... for all $n$ greater than some $n_0$
Examples

For each of the functions to the left:

1. Compute the closed form of the summation (remove all summation terms)
2. Provide a big-Θ bound for the function.
3. Arrange them in order of complexity class.

\[ f_1(n) = \sum_{i=1}^{5n} (n^2 \cdot 2^i) \]
\[ f_2(n) = \sum_{i=\frac{3}{2}n}^{n} i \]
\[ f_3(n) = \sum_{i=1}^{n} \sum_{j=i}^{n} 2 \]
\[ f_4(n) = \sum_{i=1}^{\log(n)} (5 \cdot 2^i + n) \]
\[ f_5(n) = \sum_{i=5}^{\log(n) + 5} (3 \cdot i \cdot \log(n)) \]
\[ f_6(n) = \sum_{i=2}^{4} \log(i) \]