CSE-250 Recitation

Sept 13-Sept 19: Inequalities, Logarithms, and Bounds

Questions?

- Scala?
- PA1?
- Asymptotic Notation?
- Runtime Analysis?

Logarithms Cheat Sheet

- Let $a, b, c, n > 0$
- **Exponent Rule:** $\log(n^a) = a\log(n)$
- **Product Rule:** $\log(an) = \log(a) + \log(n)$
- **Division Rule:** $\log\left(\frac{n}{a}\right) = \log(n) - \log(a)$
- **Change of Base from b to c:** $\log_b(n) = \frac{\log_c(n)}{\log_c(b)}$
- **Log/Exponent are Inverses:** $b^{\log_b(n)} = \log_b(b^n) = n$

Summation Cheat Sheet

1. $\sum_{i=j}^{k} c = (k - j + 1)c$
2. $\sum_{i=j}^{k}(cf(i)) = c \sum_{i=j}^{k} f(i)$
3. $\sum_{i=j}^{k}(f(i) + g(i)) = \left(\sum_{i=j}^{k} f(i)\right) + \left(\sum_{i=j}^{k} g(i)\right)$
4. $\sum_{i=j}^{k}(f(i)) = \left(\sum_{i=\ell}^{k}(f(i))\right) - \left(\sum_{i=\ell}^{j-1}(f(i))\right)$ (for any $\ell < j$)
5. $\sum_{i=j}^{k} f(i) = f(j) + f(j+1) + \ldots + f(k-1) + f(k)$
6. $\sum_{i=j}^{k} f(i) = f(j) + \ldots + f(\ell - 1) + \left(\sum_{i=\ell}^{k} f(i)\right)$ (for any $j < \ell \leq k$)
7. $\sum_{i=j}^{k} f(i) = \left(\sum_{i=j}^{\ell} f(i)\right) + f(\ell + 1) + \ldots + f(k)$ (for any $j \leq \ell < k$)
8. $\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$
9. $\sum_{i=0}^{k} 2^i = 2^{k+1} - 1$

Inequalities Cheat Sheet

$x \ge y$ is true if $x/a \ge x/y$ (for any $a > 0$)

$x \ge y$ is true if $xa \ge xy$ (for any $a > 0$)

$x + a \ge y + b$ is true if $x \ge y$ and $a \ge b$ (for any a, b)

$x \ge y$ is true if $x \ge a$ and $a \ge y$ (for any a)

Examples

Show that there is some **c** for which…

$12 \log(10 \times 2^n) \le c\, n$

… for all **n** greater than some n_0

Examples

Show that there is some **c** for which…

$$n^2 + n \log(n) \leq c\, 2^n$$

… for all **n** greater than some n_0

Examples

Let **f(x)** be a function defined as follows:

- If **x** is odd, $f(x) = 10x$
- If **x** is even, $f(x) = 100x^2$

Show that there is some c_{low} and c_{high} for which…

$f(n) \geq c_{low}\, n$ and $f(n) \leq c_{high}\, n^2$

… for all **n** greater than some n_0

Examples

Let **f(x)** be a function defined as follows:

- If **x** is odd, $\mathbf{f(x) = 10x}$
- If **x** is even, $\mathbf{f(x) = 100x^2}$

Is there a $\mathbf{c_{low}}$ and $\mathbf{c_{high}}$ for which…

$\mathbf{f(n) \geq c_{low}\, n^2}$ and $\mathbf{f(n) \leq c_{high}\, n}$

… for all **n** greater than some $\mathbf{n_0}$

Examples

$$f_1(n) = \sum_{i=1}^{5n} (n^2 2^i)$$

$$f_2(n) = \sum_{i=\frac{3}{4}n}^{n} i$$

$$f_3(n) = \sum_{i=1}^{n} \sum_{j=i}^{n} 2$$

$$f_4(n) = \sum_{i=1}^{\log(n)} (5 \cdot 2^i + n)$$

$$f_5(n) = \sum_{i=5}^{\log(n)+5} (3 \cdot i \cdot \log(n))$$

$$f_6(n) = \sum_{i=2}^{4} \log(i)$$

For each of the functions to the left:

1. Compute the closed form of the summation (remove all summation terms)
2. Provide a big-Ө bound for the function.
3. Arrange them in order of complexity class.