CSE 250 Recitation

Mar 06 - Mar 10: Probability, Stacks and Queues

Random Variables

A random variable X is the value of some unknown outcome.

- E.g., Roll 🎲: X is 1 with probability ⅙, 2 with probability ⅙, 3 with…
- Random variables are usually capital letters
- We write P[X = i] to say the probability that random variable X has value i

Probability Rules

Probabilities are all between 0 and 1 (0% chance to 100% chance)

- $P[X = i]$ is a number between 0 and 1

The sum of probabilities for all possible outcomes is 1.0

- $\Sigma_i P[X = i] = 1$

The probability that something does not happen is 1 - the probability it does

- $P[X \neq i] = (1 - P[X = i])$

Expectation Rules

- $E[X] = P_1X_1 + P_2X_2 + P_3X_3 + P_3X_3 + \ldots + P_nX_n$

- $E[X + Y] = E[X] + E[Y]$ (always)
- $E[XY] = E[X]E[Y]$ (if X, Y are independent)

Expectations

Suppose you roll a 6-sided die 10 times (With rolls X_1, X_2, … X_{10})

What is the expected sum of the rolls?

What is the expected product of the rolls?

How does this change for, say, n rolls?

What if X_i is an i-sided die?

Probability Example

Let's say you draw a card from a standard deck of cards, and if that card is a diamond, then you win $50, if it is a heart you win $10, otherwise you win 0.

Let X be the random variable representing the amount of money you win.

What is P[X = 50]? P[X = 10]? P[X = 0]? P[X = 15]?

What is E[X]?

Relating this back to code

When we make a random decision in our algorithm that affects the runtime of our code, then we can consider the expected value of the runtime.

Here the outcome is a specific runtime

And the probabilities are the probability that your code chooses a given outcome

Each outcome occurs with probability 1/n

$$\begin{cases} T(0) + T(n-1) + \Theta(n) & \text{if } X = 1 \\ T(1) + T(n-2) + \Theta(n) & \text{if } X = 2 \\ T(2) + T(n-3) + \Theta(n) & \text{if } X = 3 \\ .. & \\ T(n-2) + T(1) + \Theta(n) & \text{if } X = n-1 \\ T(n-1) + T(0) + \Theta(n) & \text{if } X = n \end{cases}$$