CSE 4/587

Data Intensive Computing

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Classifiers: Naive Bayes and Logistic Regression

Classification

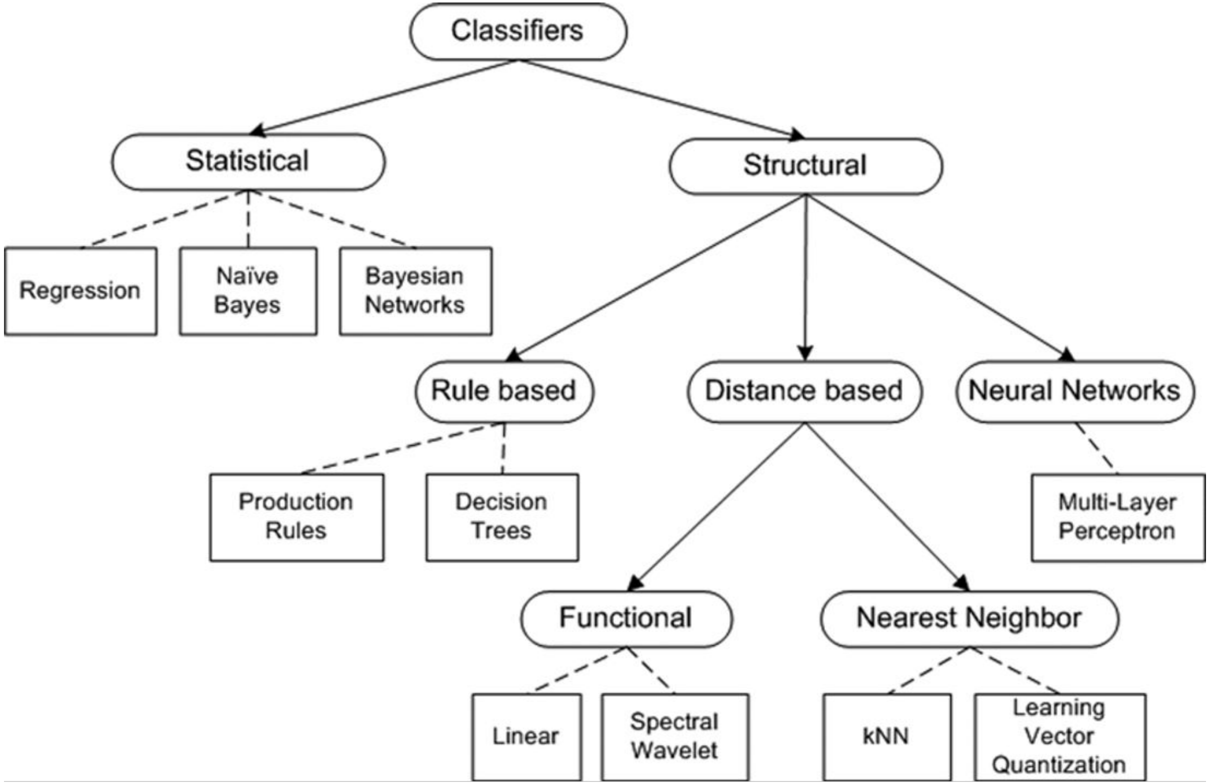
- Classification involves taking a set of unlabeled data points and labeling them in some fashion
- Why?
  - To learn from the classification/data
  - To discover patterns
  - Automate some process, ie handwriting recognition

Classification of Classification Algorithms

Classification algorithms can be divided into two broad categories:

- Statistical algorithms
  - Regression
  - Probability based classification: Bayes
- Structural algorithms
  - Rule-based algorithms: if-else, decision trees
  - Distance-based algorithm: similarity, nearest neighbor
  - Neural networks

Classification of Classification Algorithms



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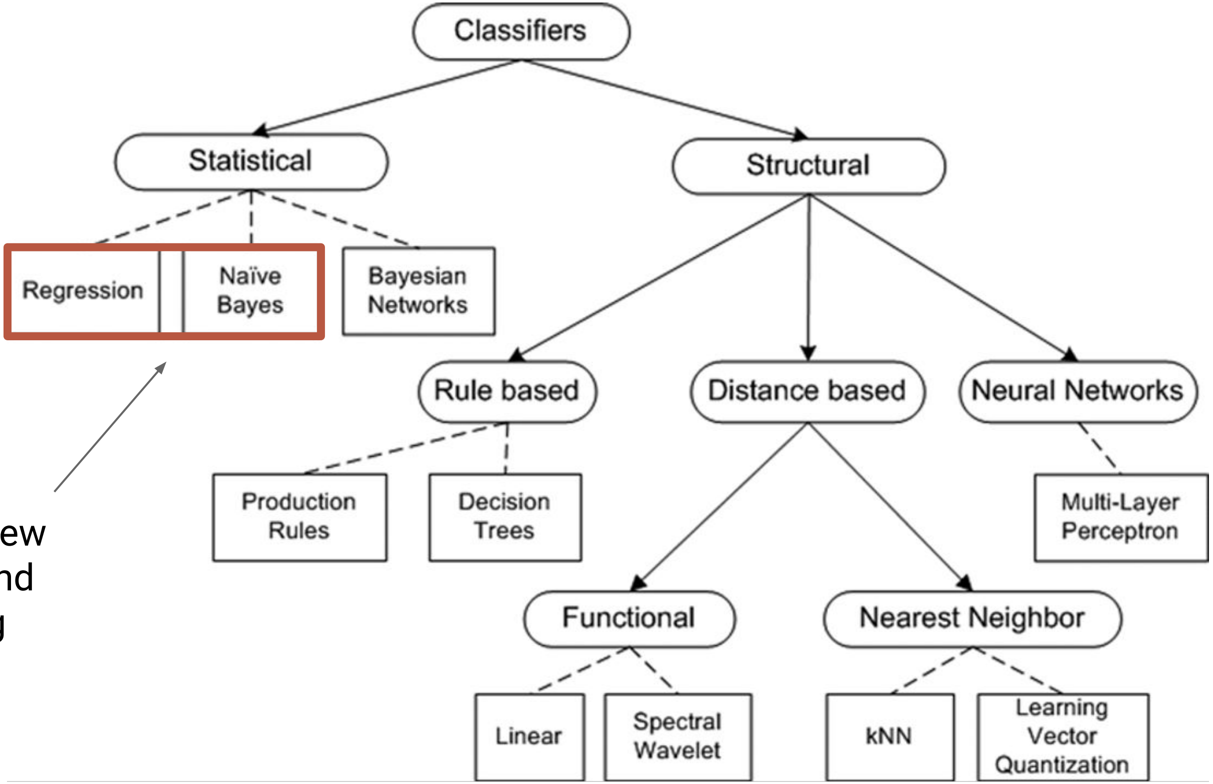
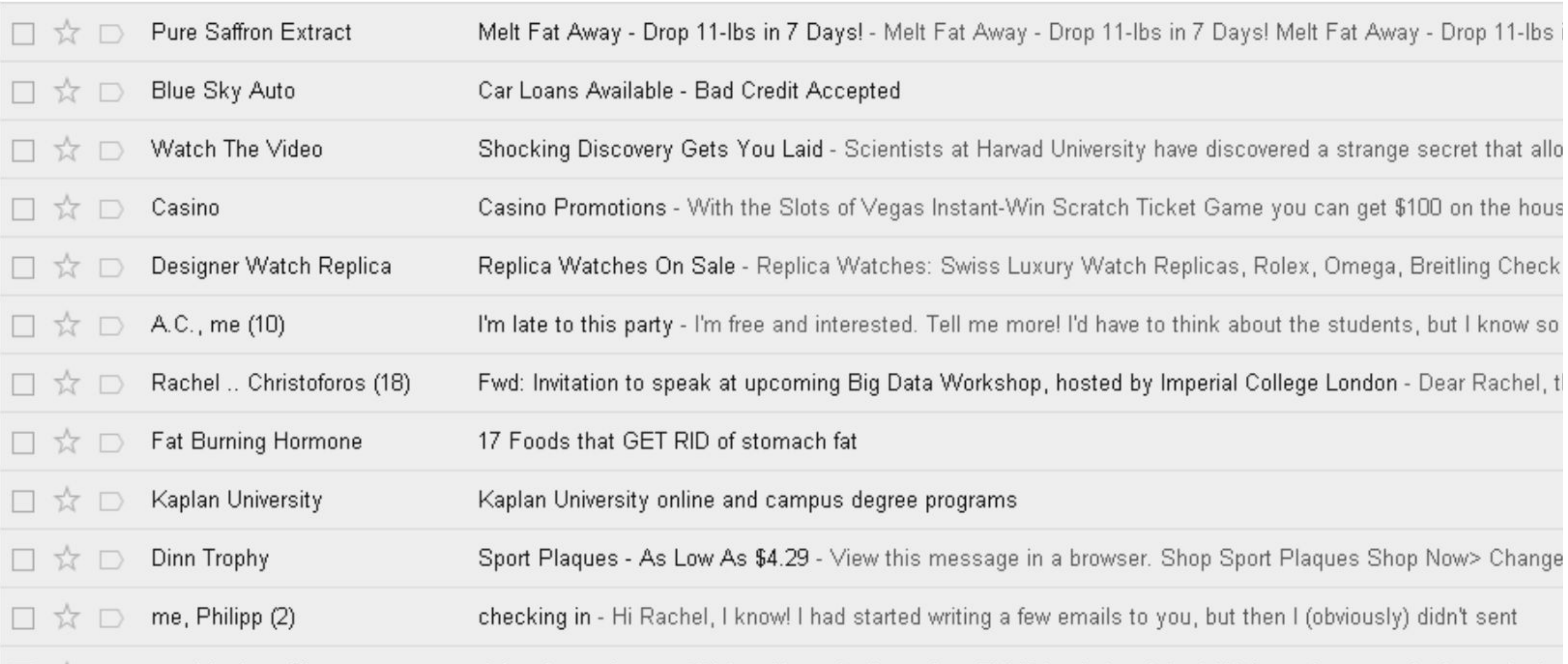


Today we'll review Naive Bayes and introduce log regression

Motivating Example: Spam Classification



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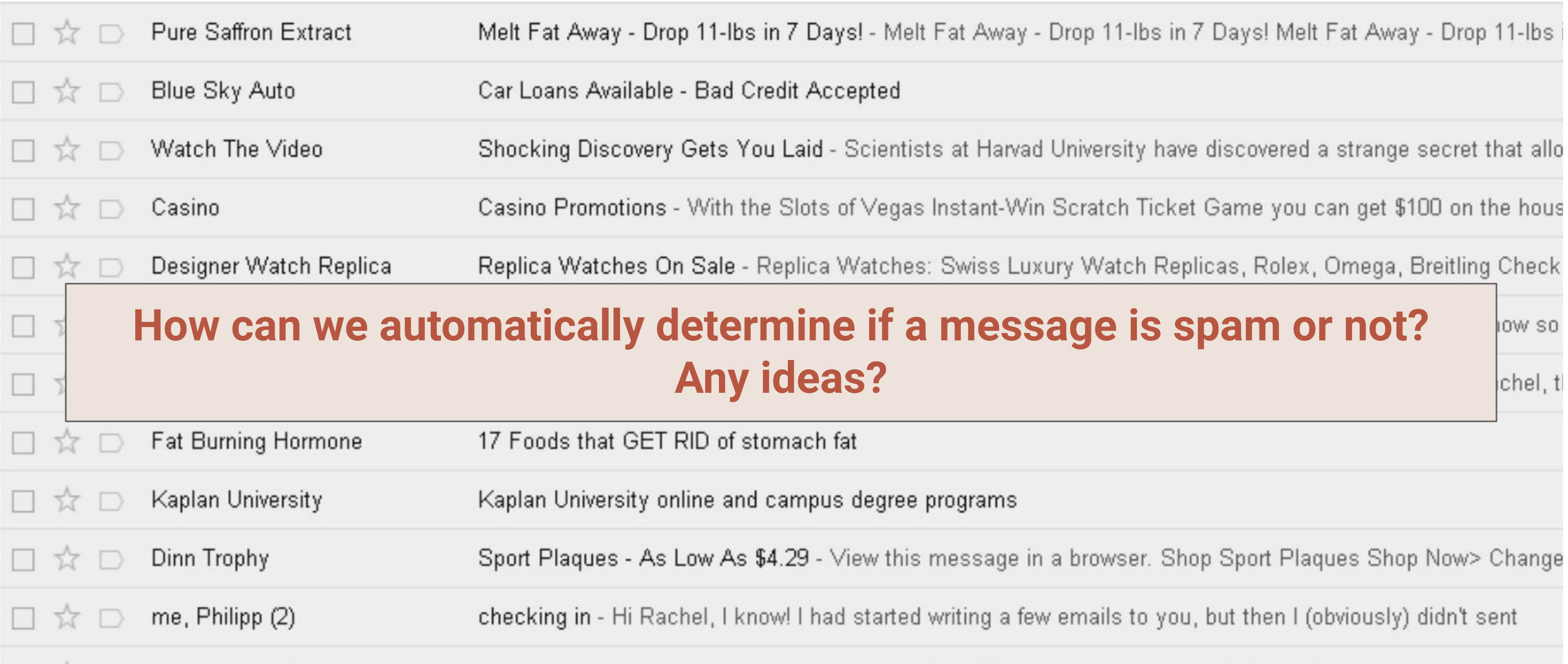

How can we automatically determine if a message is spam or not? Any ideas?

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How do we know right away that this email is spam?

Idea: The use of certain words, ie lottery, can indicate an email is spam.

Naive Bayes

Basic Idea: Make a probabilistic model – have many ***simple rules***, and aggregate those rules together to provide a probability.

Bayes Law and Probability Theory

Basic Law: $P(H \mid E) = P(E \mid H) * P(H) / P(E)$

Bayes Law - Example

Suppose you know that I work 5 days out of the week.

Also suppose you know that on work days, I never wear flip flops, and on non-work days I wear flip flops 70% of the time.

Given this information, if you see me on a random day of the week wearing shoes, what is the probability that I had work that day?

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Bayes Law - Example

Basic principle: P(*H* | *E*) = P(*E* | *H*) * P(*H*) / P(*E*)

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- What is the evidence, E, that we observed?

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Therefore, if you see me in shoes, there is an 88% I went to work today

Bayes Law - Spam Classification

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Let's start one word at a time:

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Bayes Law - Spam Classification

Given Bayes Law, how can we start classifying emails as spam?

Let's start one word at a time:

Probability that the given word appears in an email

$$P(spam|word) = P(word|spam) * P(spam) / P(word)$$

Probability that an email is spam if it contains a given word

Probability that the given word appears in an email known to be spam

Probability that an email is spam

Bayes Law - Spam Classification

We've now boiled our classification problem down to a counting problem:

Given a set of emails that have been classified as spam or not spam (ham):

1. Count number of spam vs ham emails to compute **P(*spam*)**
2. Count number of times the given word, ie lottery, appears in emails to compute **P(*word*)**
3. Count number of times the given word appears in spam emails to compute **P(*word*|*spam*)**

Enron Email Example - DDS Chapter 4

- **Input:** Enron data set containing employee emails
- A small subset chosen for EDA
- 1500 spam, 3672 ham
- Test word is “meeting”
- Running a simple shell script reveals that there are 16 spam emails containing “meeting” and 153 ham emails containing "meeting"
- **Output:** What is the probability that an email containing "meeting" is spam? What is your intuition? Now prove it using Bayes Law…

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P(*meeting*) = (16+153) / (1500+3672) = 0.0326

P(*spam|meeting*) = **P(*meeting|spam*)*P(*spam*)/P(*meeting*)** = 0.094 (9.4%)

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Let's say we have ***i*** words. Let ***x*** be a vector of size ***i***,

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Let's say we have ***i*** words. Let ***x*** be a vector of size ***i***,

where $\boldsymbol{x_j}$ **= 1** if the $\boldsymbol{j^{th}}$ word is present in an email, **0** otherwise.

Now how do we compute P(*x*|*spam*)?

Once we do this, we can apply Bayes Law to find P(*spam*|*x*)

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$\boldsymbol{1-\theta_{jc}}$ if the $\boldsymbol{j^{th}}$ word is not in the email

$\boldsymbol{\theta_{jc}}$ if the $\boldsymbol{j^{th}}$ word is in the email

Example

"meeting": 1% chance of being in a spam email

"money": 10% chance of being in a spam email

"viagra": 4% chance of being in a spam email

"enron": 0% chance of being in a spam email

What is the probability that a spam email contains "meeting" and "money"?

(but not "viagra" or "enron")

Example

$x = [1,1,0,0]$ $\theta_{1c} = 0.01$ $\theta_{2c} = 0.10$ $\theta_{3c} = 0.04$ $\theta_{4c} = 0.0$

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$$p(x|\mathrm{c}) = \theta_{1c}\theta_{2c}(1 - \theta_{3c})(1 - \theta_{4c})$$

$$p(x|\mathrm{c}) = 0.01 * 0.1 * 0.96 * 1.0 = 0.00096$$

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$$p(x|\mathrm{c}) = \theta_{1c}\theta_{2c}(1 - \theta_{3c})(1 - \theta_{4c})$$

$$p(x|\mathrm{c}) = 0.01 * 0.1 * 0.96 * 1.0 = 0.00096$$

There is a 0.09% chance that this exact vector *x* appears in a spam email

Cleaning it up...

- Multiplying many small probabilities can result in numerical issues
- A common method for avoiding this is to take the log of both side

$$log(p(x|c)) = \sum_j x_j log(\theta_j/(1-\theta_j)) + \sum_j log(1-\theta_j)$$

Cleaning it up...

Many of these terms don't depend on the email and can be precomputed

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Call this $\boldsymbol{w_0}$

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$$log(p(x|c)) = \sum_j x_j w_j + w_0$$

The Final Formula

Now given $p(x|spam)$ we can use Baye's Law we can compute $p(spam|x)$:

$$p(spam|x) = p(x|spam) * p(spam) \,/\, p(x)$$

The Final Formula

Now given $p(x|spam)$ we can use Baye's Law we can compute $p(spam|x)$:

$$p(spam|x) = p(x|spam) * p(spam) / p(x)$$

These other two terms are pretty straightforward to compute, and $p(spam)$ is independent of the input email

Naive Bayes

A few notes:

- Occurrences of words are considered independent events
  - Don't care how many times a word appears
  - Don't care about combinations of words
  - This is why it's called "naive"

Extending our Model: Laplace Smoothing

From the previous formula, $\boldsymbol{\theta_{jc}}$ is just a ratio of counts: $\boldsymbol{n_{jc} / n_j}$

Where $\boldsymbol{n_{jc}}$ is the number of times the word appears in a spam email

and $\boldsymbol{n_j}$ is the number of times the word appears in any email

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and $\boldsymbol{n_j}$ is the number of times the word appears in any email

This is just an estimate based on our dataset…what if $\boldsymbol{\theta_{jc}}$ = 1 (or 0)?

Extending our Model: Laplace Smoothing

Laplace Smoothing is a technique to avoid these extreme probabilities

Introduce parameters α, β to our computation of θ_{jc}

$$\theta_{jc} = \frac{n_{jc} + \alpha}{n_j + \beta}$$

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More data allows you to relax the values of $\boldsymbol{\alpha}, \boldsymbol{\beta}$

Extending our Model: Multiple Classes

What if we want more than two classes?

Example from DDS: Classifying NYTimes articles based on section

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Example from DDS: Classifying NYTimes articles based on section

Idea: For a given article, compute the probabilities for each class (section), and then classify the article as the one with the highest probability

More on Classifiers

Example Questions and Answers

● "Will someone click on this ad?"	0 or 1 (no or yes)
● "What number is this (image recognition)?"	0, 1, 2, 3, etc
● "What is this news article about?"	"Sports"
● "Is this spam?"	0 or 1
● "Is this pill good for headaches?"	0 or 1

Answering these questions can be done with classifiers