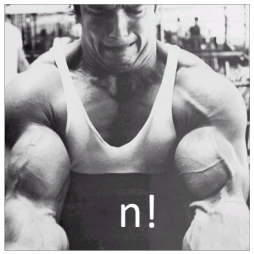
Lecture 15

CSE 331

Feb 28, 2020

Main Steps in Algorithm Design



Problem Statement

Problem Definition

Algorithm

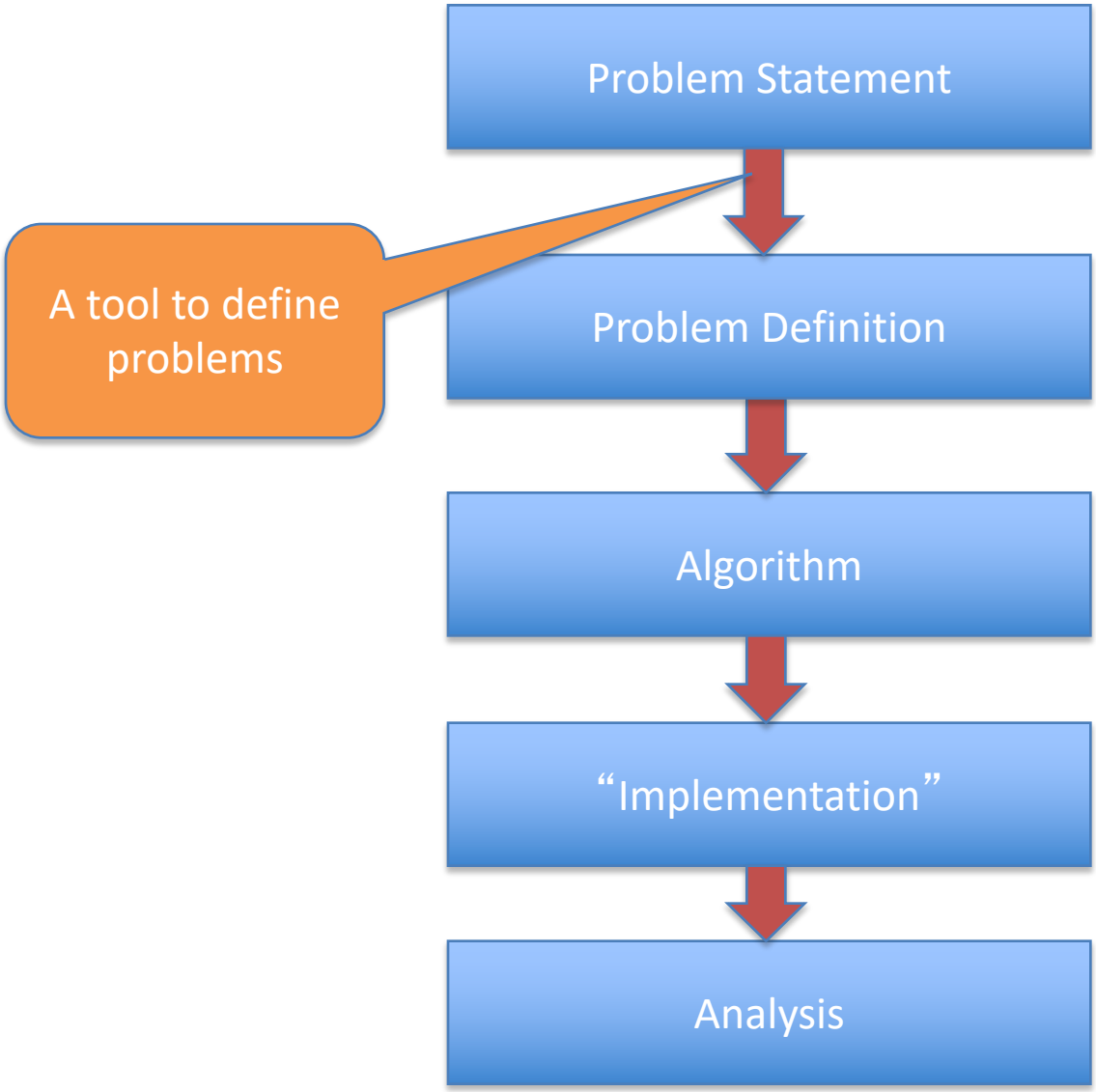
"Implementation"

Data Structures

Analysis

Correctness+Runtime Analysis

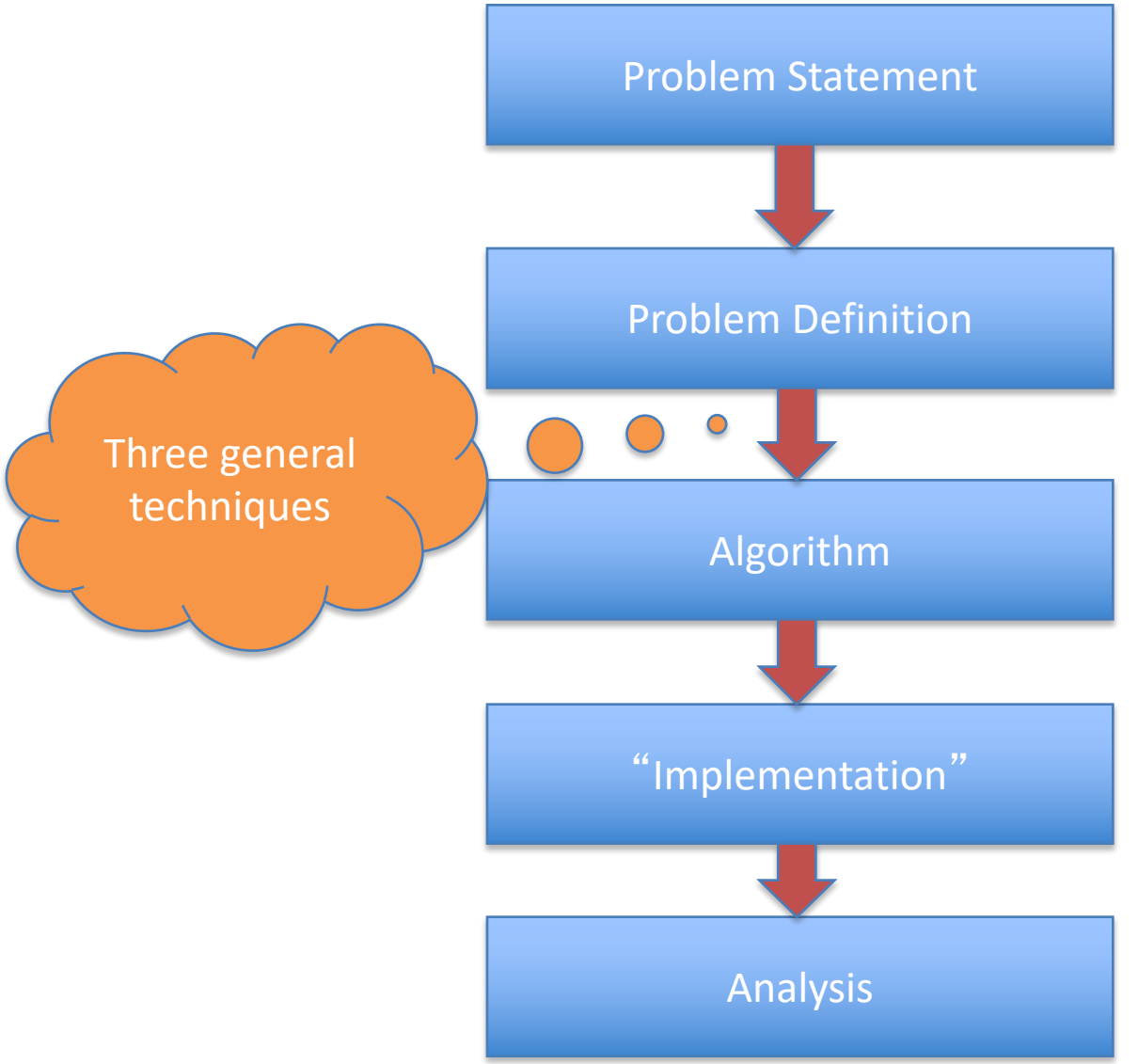
Where do graphs fit in?



Data Structures

Correctness+Runtime Analysis

Rest of the course*



Data Structures

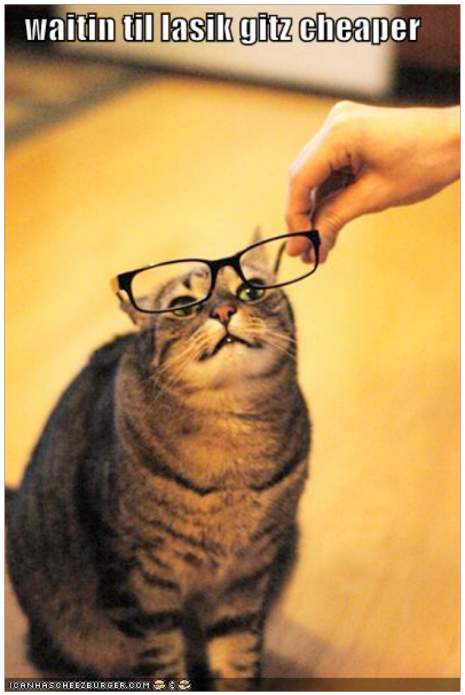
Correctness+Runtime Analysis

Greedy algorithms

Build the final solution piece by piece

Being short sighted on each piece

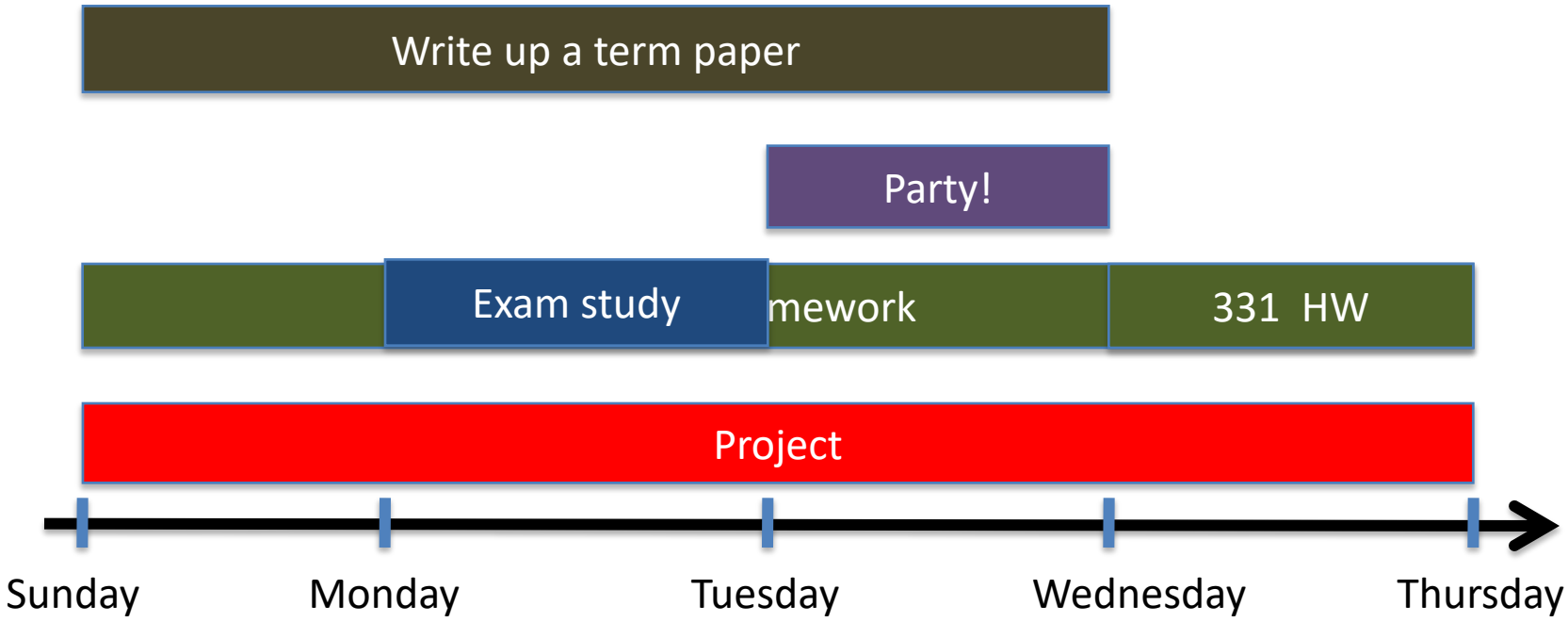
Never undo a decision



Know when you see it

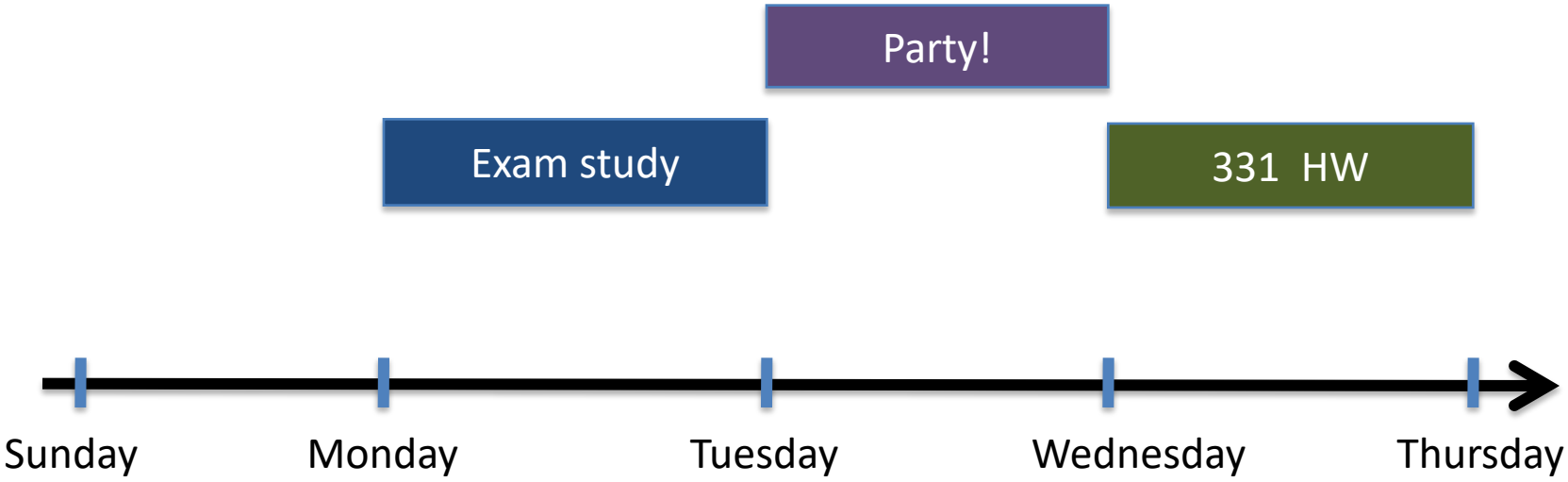
End of Semester blues

Can only do one thing at any day: what is the maximum number of tasks that you can do?



The optimal solution

Can only do one thing at any day: what is the maximum number of tasks that you can do?



Interval Scheduling Problem

Input: n intervals $[s(i), f(i))$ for $1 \le i \le n$

$\{ s(i), \ldots, f(i)-1 \}$

Output: A *schedule* S of the n intervals

No two intervals in S conflict

|S| is maximized

Algorithm with examples

Interval Scheduling via examples

In which we derive an algorithm that solves the Interval Scheduling problem via a sequence of examples.

The problem

In these notes we will solve the following problem:

Interval Scheduling Problem

Input: An input of n intervals $[s(i), f(i))$, or in other words, $\{s(i), \ldots, f(i) - 1\}$ for $1 \leq i \leq n$ where i represents the intervals, $s(i)$ represents the start time, and $f(i)$ represents the finish time.

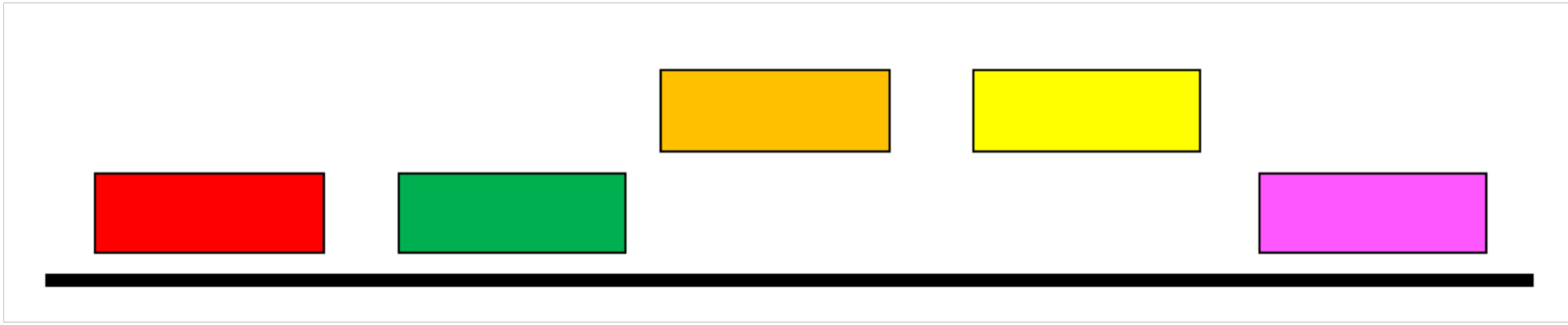
Output: A schedule S of n intervals where no two intervals in S conflict, and the total number of intervals in S is maximized.

Sample Input and Output

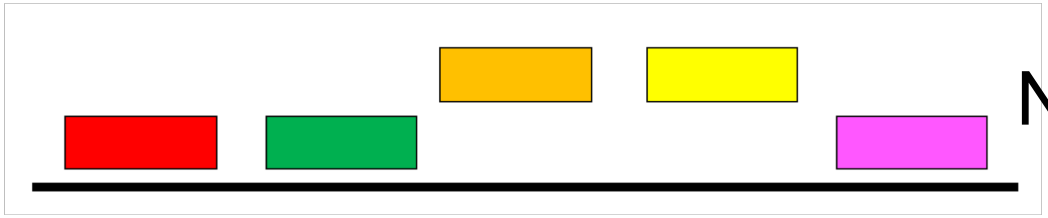
Input:

Example 1

No intervals overlap



Algorithm?



No intervals overlap

R: set of requests

Set S to be the empty set

While R is not empty

Choose i in R

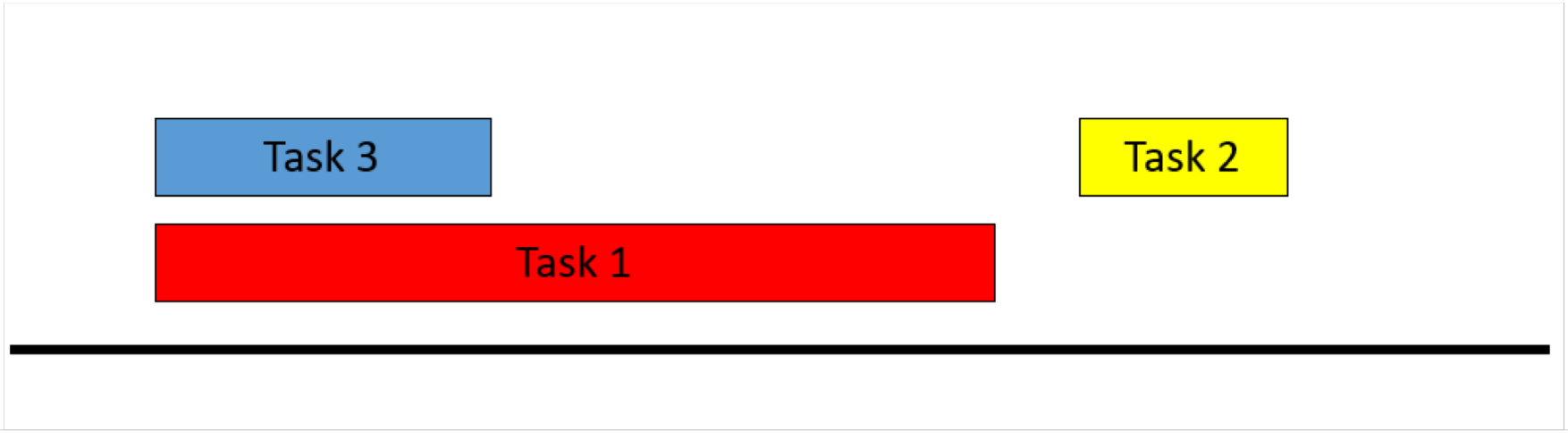
Add i to S

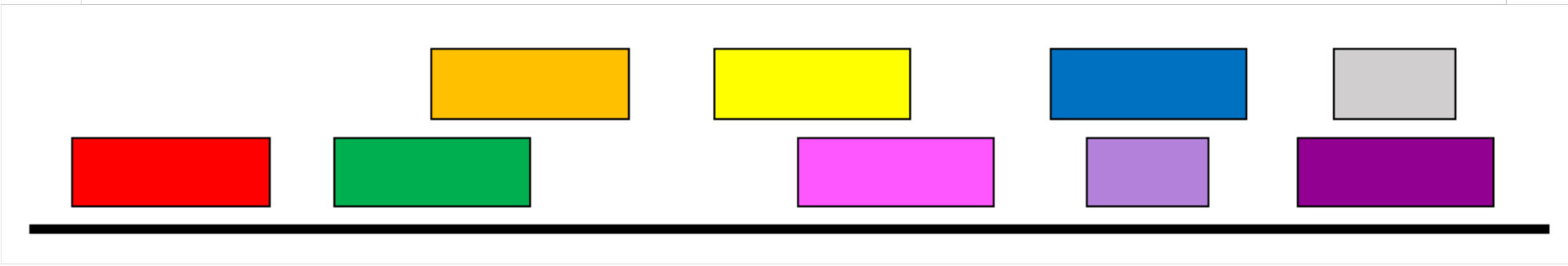
Remove i from R

Return $S^* = S$

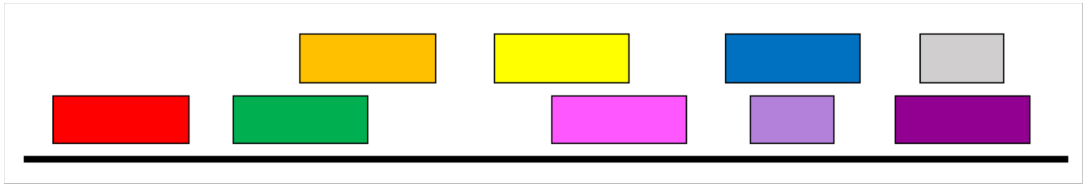
Example 2

At most one overlap





Algorithm?



At most one overlap

R: set of requests

Set S to be the empty set

While R is not empty

Choose i in R

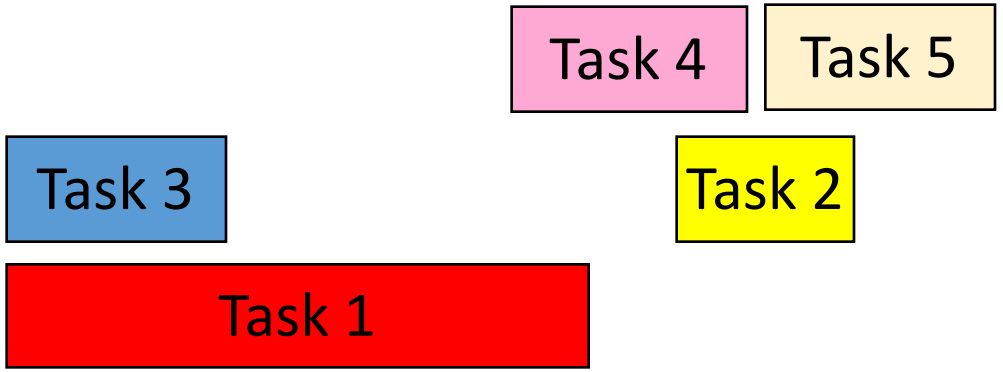
Add i to S

Remove all tasks that conflict with i from R

Return $S^* = S$

Example 3

More than one conflict



Set S to be the empty set

While R is not empty

Choose i in R

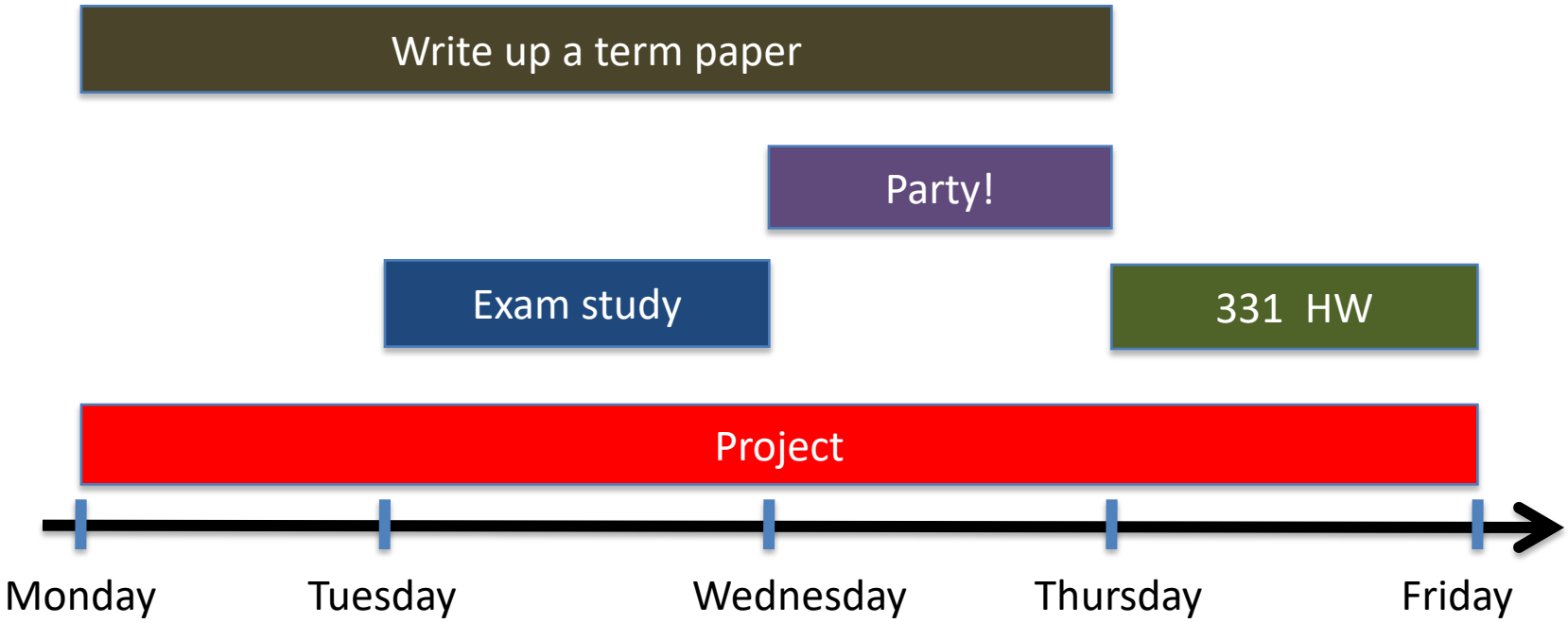
Add i to S

Remove all tasks that conflict with i from R

Return $S^* = S$

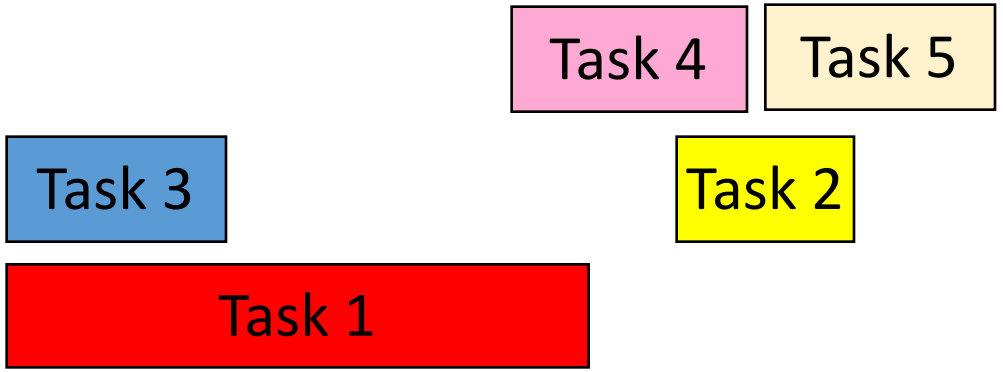
Greedily solve your blues!

Arrange tasks in some order and iteratively pick non-overlapping tasks



Making it more formal

More than one conflict



Set S to be the empty set

While R is not empty

Choose i in R that minimizes v(i)

Add i to S

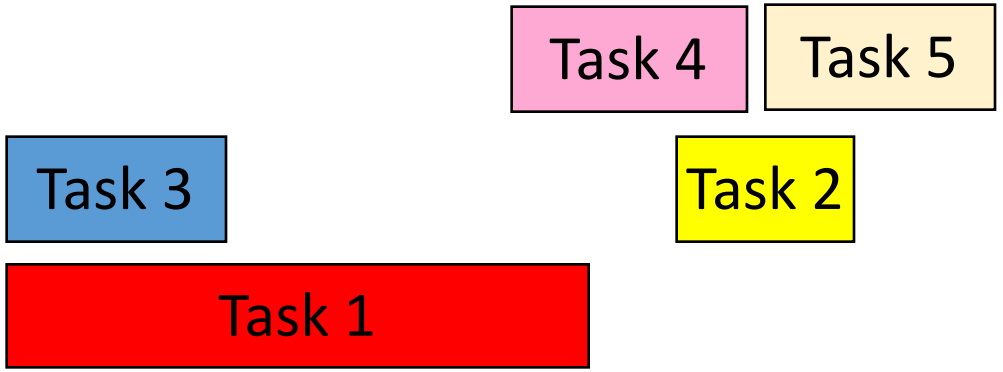
Remove all tasks that conflict with i from R

Return $S^* = S$

Associate a value v(i) with task i

What is a good choice for v(i)?

More than one conflict



Set S to be the empty set

While R is not empty

Choose i in R that minimizes v(i)

Add i to S

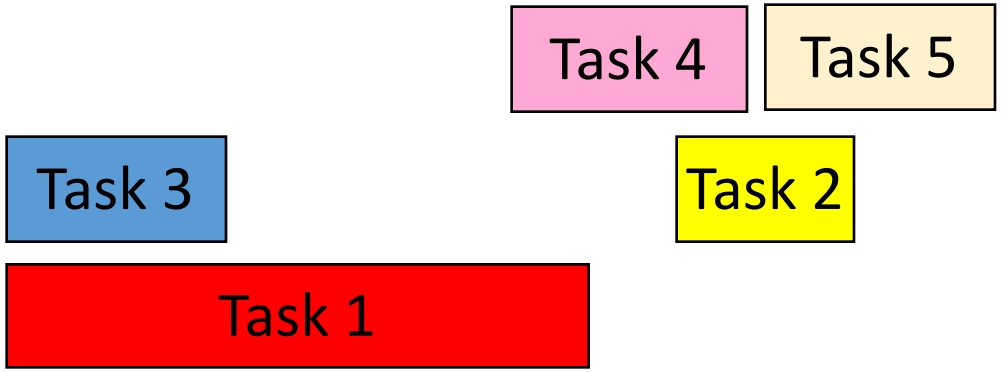
Remove all tasks that conflict with i from R

Return $S^* = S$

Associate a value v(i) with task i

$v(i) = f(i) - s(i)$

Smallest duration first



Set S to be the empty set

While R is not empty

Choose i in R that minimizes f(i) – s(i)

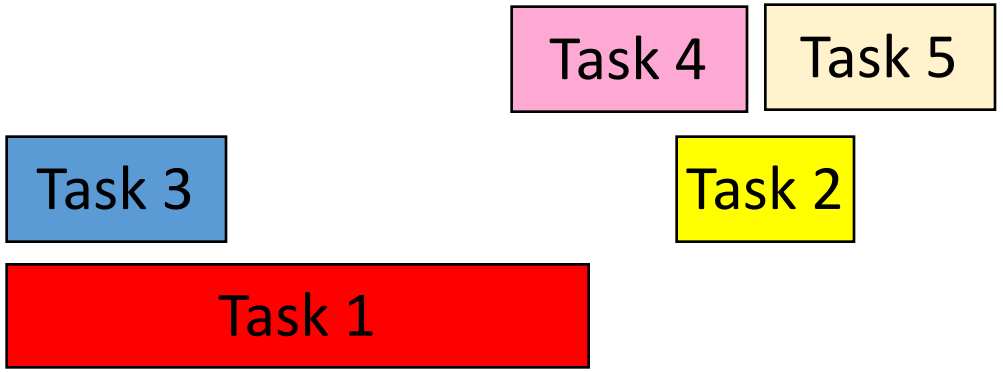
Add i to S

Remove all tasks that conflict with i from R

Return $S^* = S$

v(i) = s(i)

Earliest time first?



Set S to be the empty set

While R is not empty

Choose i in R that minimizes s(i)

Add i to S

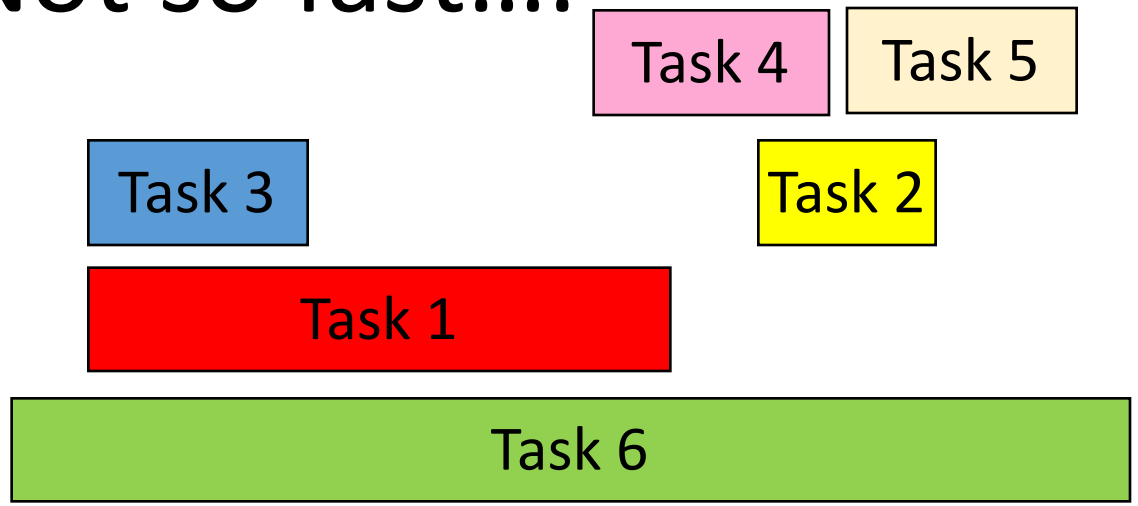
Remove all tasks that conflict with i from R

Return $S^* = S$

So are we done?

Not so fast….

Earliest time first?



Set S to be the empty set

While R is not empty

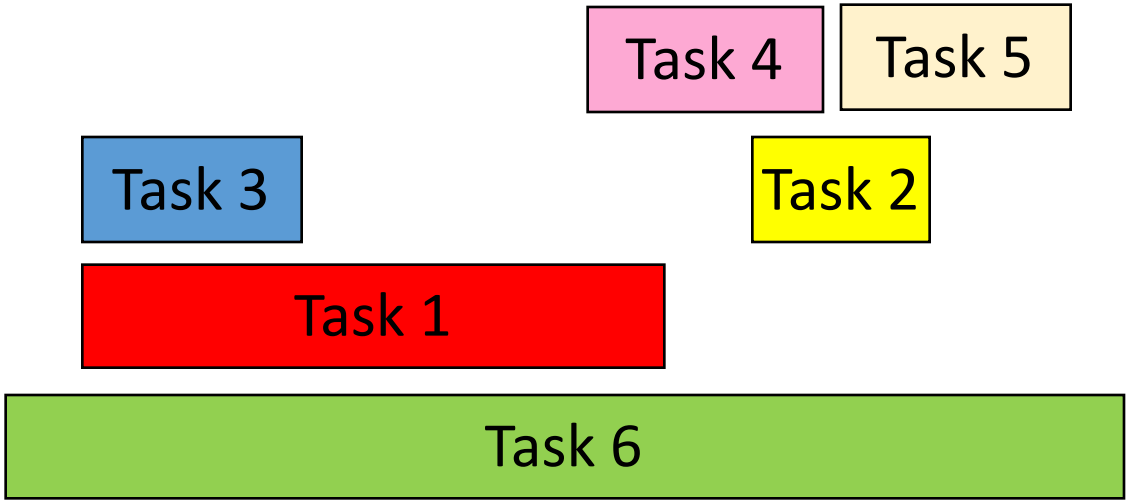
Choose i in R that minimizes s(i)

Add i to S

Remove all tasks that conflict with i from R

Return $S^* = S$

Pick job with minimum conflicts



Set S to be the empty set

While R is not empty

Choose i in R that has smallest number of conflicts

Add i to S

Remove all tasks that conflict with i from R

Return $S^* = S$

So are we done?

Nope (but harder to show)

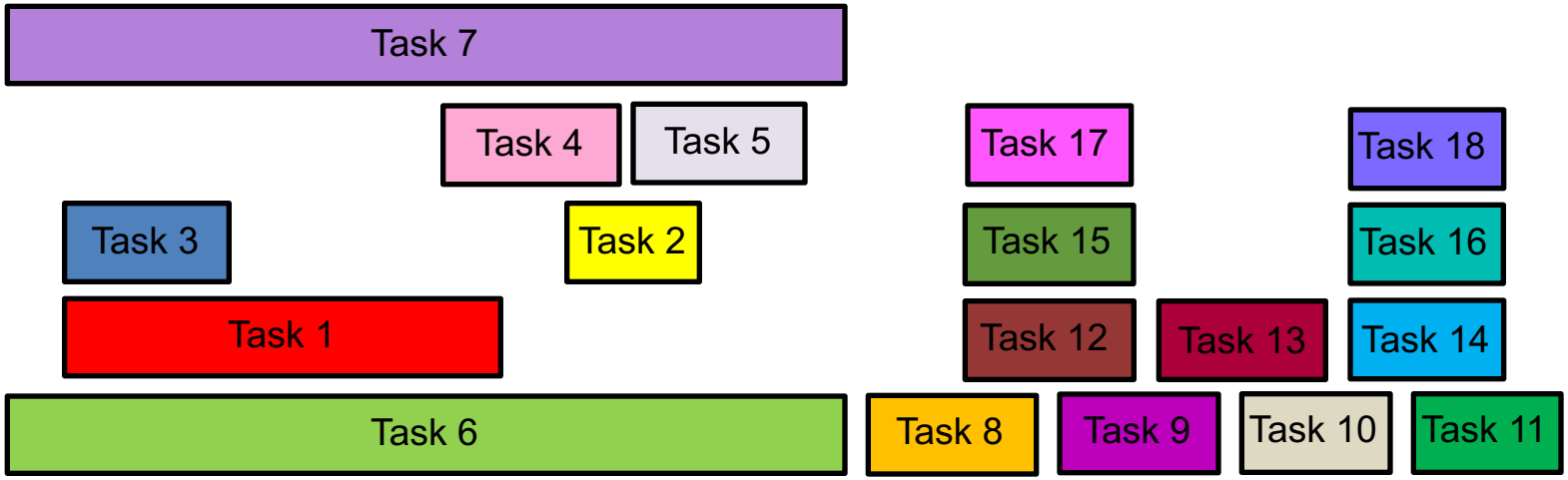
Set S to be the empty set

While R is not empty

Choose i in R that has smallest number of conflicts
Add i to S

Remove all tasks that conflict with i from R

Return $S^* = S$



Set S to be the empty set

While R is not empty

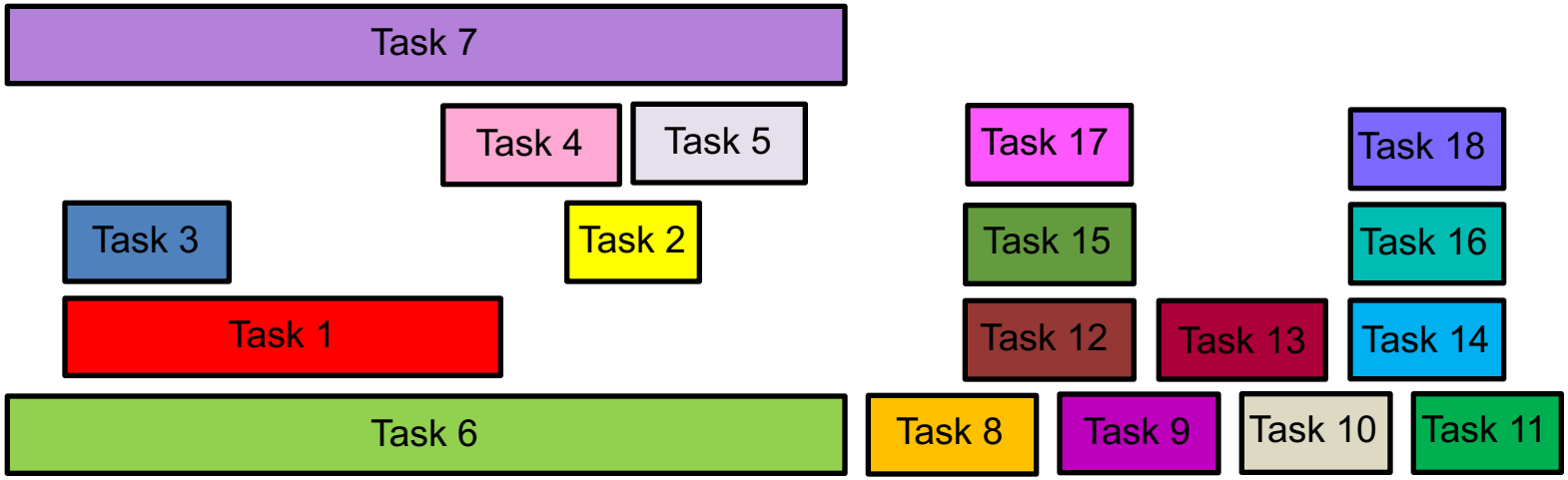
Choose i in R that has smallest number of conflicts

Add i to S

Remove all tasks that conflict with i from R

Return $S^* = S$

Algorithm?



Set S to be the empty set

While R is not empty

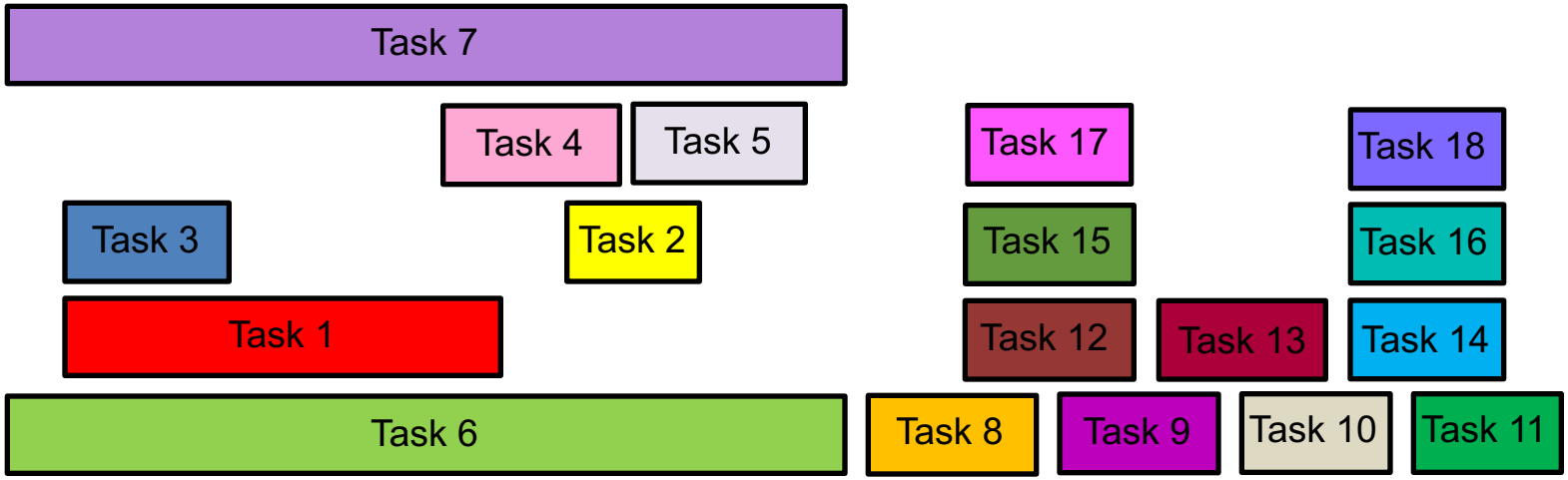
Choose i in R that minimizes v(i)

Add i to S

Remove all tasks that conflict with i from R

Return $S^* = S$

Earliest finish time first



Set S to be the empty set

While R is not empty

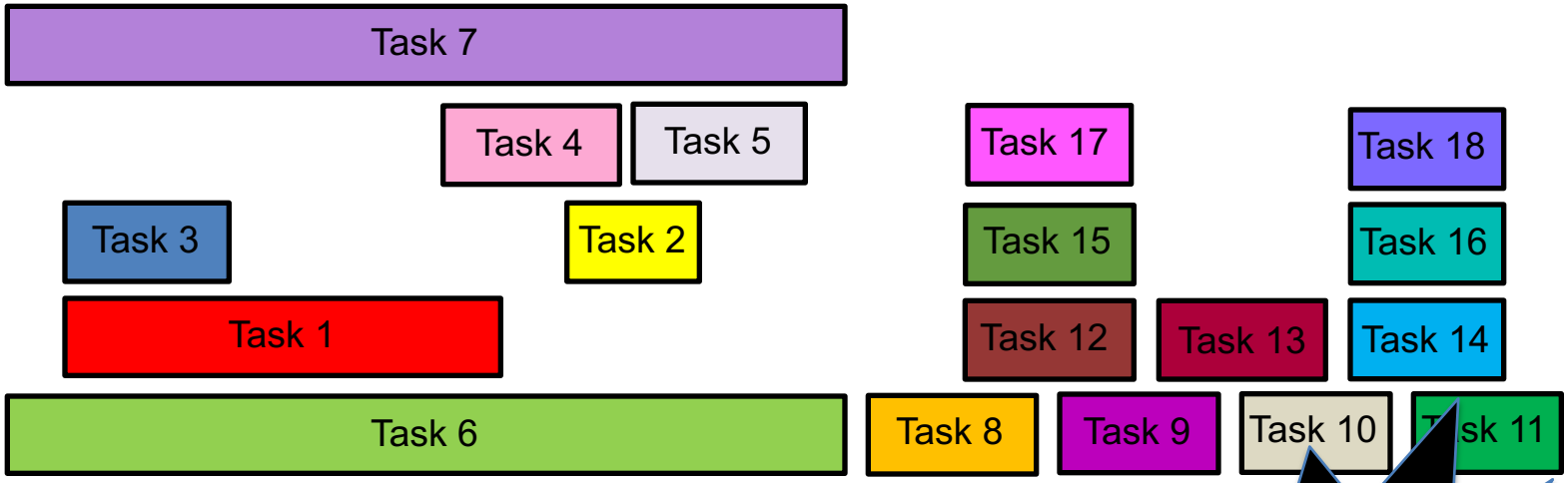
Choose i in R that minimizes f(i)

Add i to S

Remove all tasks that conflict with i from R

Return $S^* = S$

Find a counter-example?



Set S to be the empty set

While R is not empty

Choose i in R that minimizes f(i)

Add i to S

Remove all tasks that conflict with i from R

Return $S^* = S$

It works!

Questions?

Today’s agenda

Prove the correctness of the algorithm

Final Algorithm

R: set of requests

Set S to be the empty set

While R is not empty

    Choose i in R with the earliest finish time

    Add i to S

    Remove all requests that conflict with i from R

Return $S^* = S$