# Lecture 18

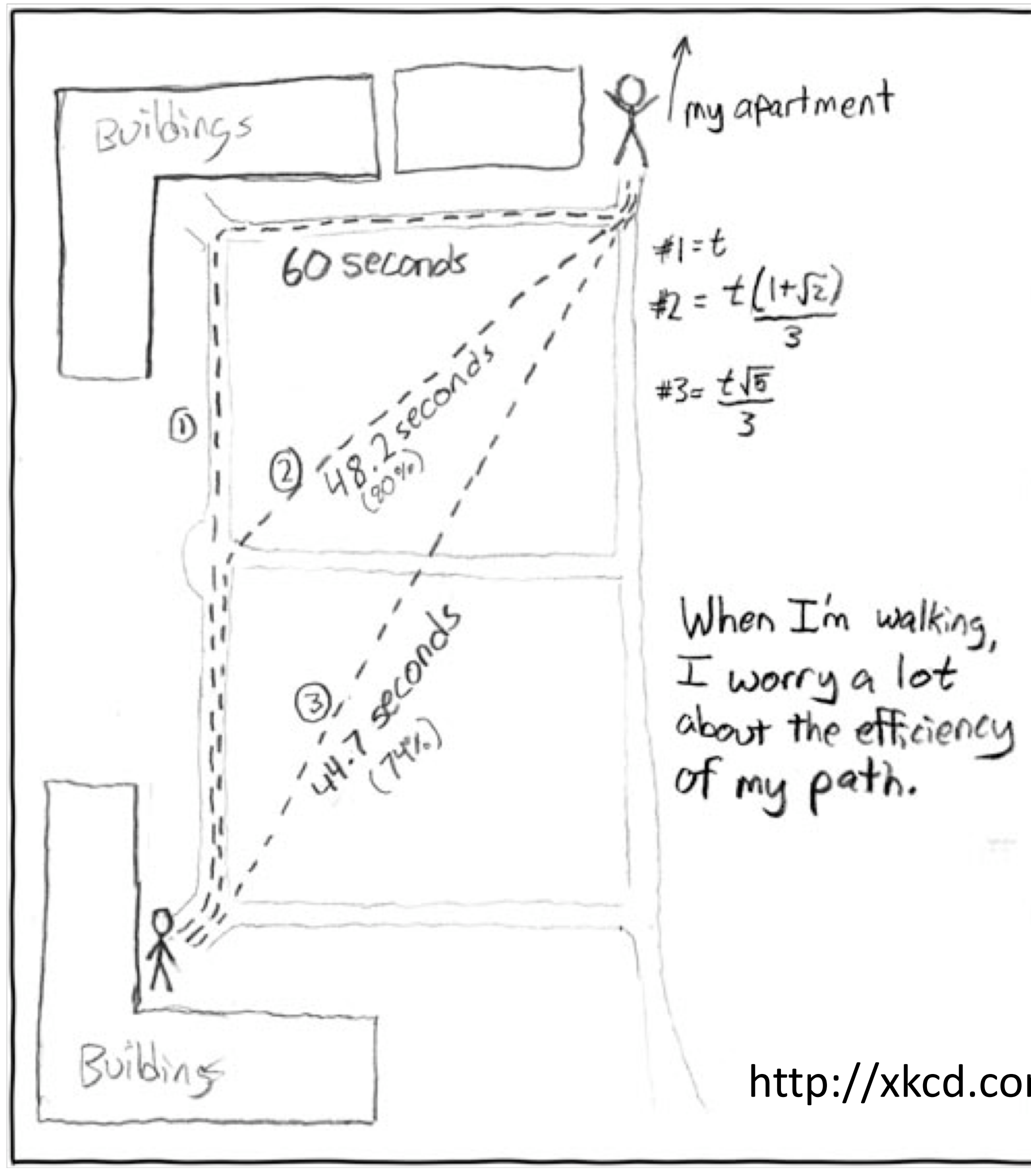
CSE 331

Mar 6, 2020

Quiz starts at **2:00pm** and ends at **2:10pm**

Lecture starts at 2:15pm

# Shortest Path Problem



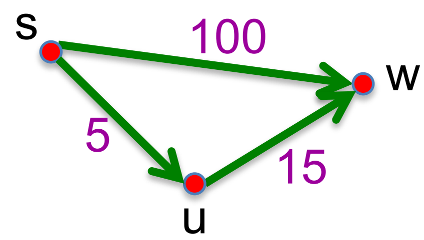

http://xkcd.com/85/

# Another more important application

Is BGP a known acronym for you?


Routing uses shortest path algorithm

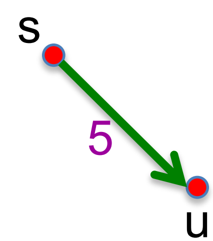
# Shortest Path problem

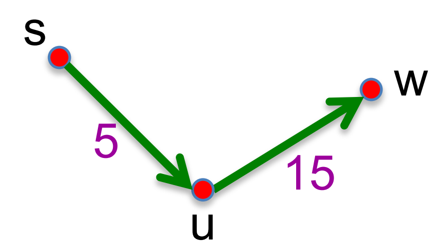



**Input:** *Directed* graph $G=(V,E)$

Edge lengths, $l_e$ for $e$ in $E$

"start" vertex $s$ in $V$



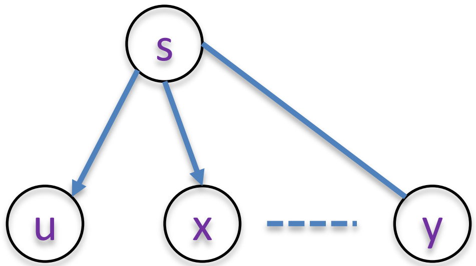

**Output:** Length of shortest paths from $s$ to all nodes in $V$

# Dijkstra's shortest path algorithm

# Towards Dijkstra's algo: part 1

Determine d(t) one by one

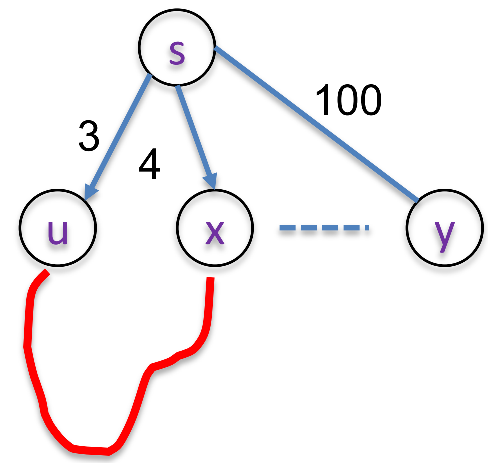
$\boxed{d(s) = 0}$



# Towards Dijkstra's algo: part 2

Determine $d(t)$ one by one

Let $u$ be a neighbor of $s$ with smallest $l_{(s,u)}$




$$d(u) = l_{(s,u)}$$

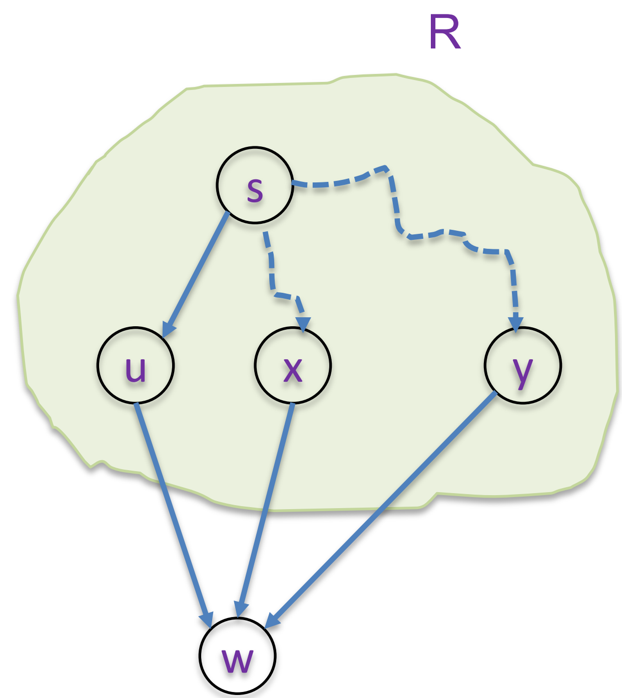
Length of is $\geq 0$

Not making any claim
on other vertices

# Towards Dijkstra's algo: part 3

Determine $d(t)$ one by one

Assume we know $d(v)$ for every $v$ in $R$




Compute an upper bound $d'(w)$ for every $w$ not in $R$

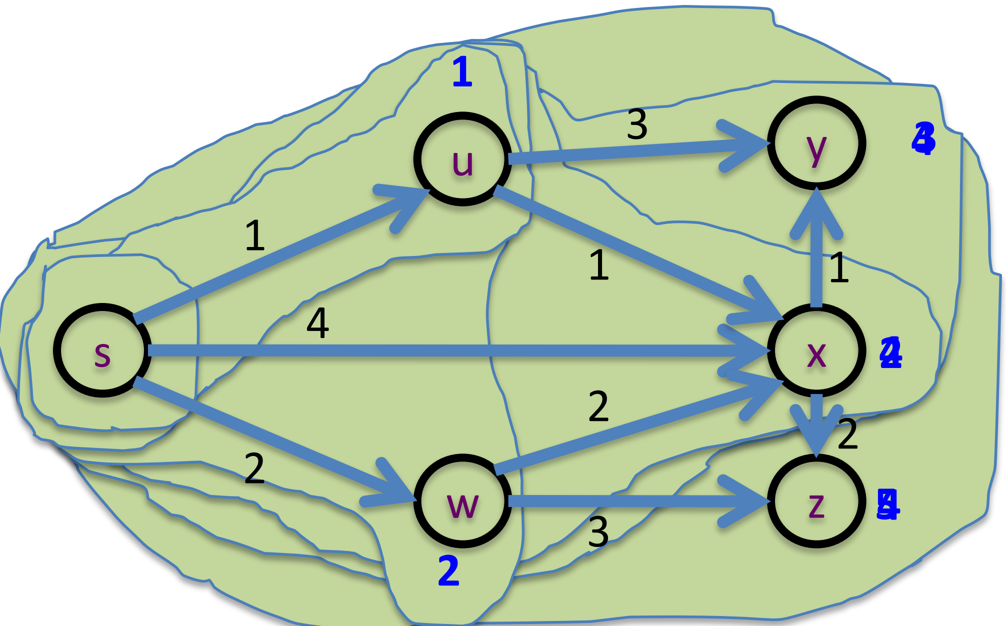
$$d(w) \leq d(u) + l_{(u,w)}$$

$$d(w) \leq d(x) + l_{(x,w)}$$

$$d(w) \leq d(y) + l_{(y,w)}$$

$$d'(w) = \min_{e=(u,w) \text{ in } E,\ u \text{ in } R} d(u)+l_e$$

# Dijkstra's shortest path algorithm




$d'(w) = \min_{e=(u,w) \text{ in } E,\ u \text{ in } R} d(u)+l_e$

d(s) = 0 d(u) = 1

d(w) = 2 d(x) = 2

d(y) = 3 d(z) = 4

Input: Directed G=(V,E), $l_e \geq 0$, s in V

R = {s}, d(s) =0

While there is a x not in R with (u,x) in E, u in R

Pick w among all x with smallest d'(w) value

Add w to R

d(w) = d'(w)

Shortest paths