## Lecture 19

CSE 331
Mar 9, 2020

## Mid-terms this week

## note @297 (오

## The midterm post

First, midterm-I is on Wednesday, Mar 11 and midterm-II is on Friday, Mar 13 during the usual class timings (i.e. 2:00-2:50pm in Knox 109). Belon

## Logistics:

- The exam will be closed book and closed notes. However, you can bring in one $8.5^{\prime \prime} \mathrm{X} 11^{1 "}$ (letter-sized) review sheet. (If you prefer you can br as long as it is one sheet (you can use both sides). It can be hand-written or typed up doesn't matter-- however, you are not allowed to bring i but can concentrate on the main ideas in the material we have covered. The exam (as you can probably understand from the sample midterm


## Midterm material:

- The questions will be from all the material in Section 1.1, Chapters 2 and 3 and Sections $4.1,4.2$ (which will be a reading assignment) and 4.4 week. This also includes the support page material, reading assignments and recitation notes. Also, see the next couple of points.
- Most of the questions (over both the midterm exams) will be from Chapters 1,3 and 4. (You could have guessed this from the time we spend 2 in some sense is very basic: you can be asked to analyze the run time of an algorithm related to stuff from Chapter 1,3 or 4.)
- Most of the questions will be from the sections we covered in class. This, however, does not mean that there can't be any question from the s pages. (Also see next point.)
- We did not cover certain sections in the book, e.g., we did not cover Sec 3.4 and 3.5 at all in class. However, Sec 3.4 and 3.5 are very nice ap on BFS/DFS algorithms.
- You guys will not have a HW problem on Sec 3.6 or Section 4 before the midterm exams. So in the worst-case, I'll only ask $T / F$ question(s) frc be trifled with.)


## For preparation:

- Work through the sample midterm exams (@233). Do not use the sample midterm to deduce anything about the relative coverage of different exams will be harder than the sample midterm exams. The actual midterms will follow the exact same format for the sample midterms: i.e. the
- I encouraae vou to not look at the solutions to the samble midterms before vou have sDent some aualitv time bv vourself on the midterm aues


## Instructor OHs: 4-6 today, no on wed

## Quiz 1: T/F dilemma!

(a) (Part 1) Argue why the following statement is TRUE.

If $f(n)=c \cdot g(n)$, then $2^{f(n)}=\left(2^{g(n)}\right)^{c}$ for every real number $c$.
(Part 2) Is the following statement true or false? Also remember to briefly JUSTIFY your answer.
If $f(n)$ is $\Theta(g(n))$, then $2^{f(n)}$ is $\Theta\left(2^{g(n)}\right)$.

True False (Please CIRCLE your answer)

Those are not okay; you'll get zero!
(a) (Part 1) Argue why the following statement is TRUE.

If $f(n)=c \cdot g(n)$, then $2^{f(n)}=\left(2^{g(n)}\right)^{c}$ for every real number $c$. It is talse.

True
False
(Please CIRCLE your answer)

## True False (Please CIRCLE your answer)



Also; it makes no sense to leave it unanswered; just choose one: you have nothing to lose

## Dijkstra's shortest path algorithm



Input: Directed $G=(V, E), I_{e} \geq 0$, s in $V$
$R=\{s\}, d(s)=0$
While there is a $x$ not in $R$ with $(u, x)$ in $E, u$ in $R$
Pick $w$ among all $x$ with smallest $d^{\prime}(w)$ value Add w to R $d(w)=d^{\prime}(w)$

$$
d^{\prime}(w)=\min _{e=(u, w) \text { in } E, u \text { in } R} d(u)+l_{e}
$$

$$
\begin{array}{ll}
d(s)=0 & d(u)=1 \\
d(w)=2 & d(x)=2 \\
d(y)=3 & d(z)=4
\end{array}
$$

Shortest paths


## Couple of remarks

The Dijkstra's algo does not explicitly compute the shortest paths

Can maintain "shortest path tree" separately

Dijkstra's algorithm does not work with negative weights

Left as an exercise

## Rest of Today's agenda

Prove the correctness of Dijkstra's Algorithm

Runtime analysis of Dijkstra's Algorithm

## Dijkstra's shortest path algorithm

$P_{u}$ shortest s-u path in "Dijkstra tree"

$$
d^{\prime}(v)=\min _{e=(u, v) \text { in } E, u \text { in } R} d(u)+I_{e}
$$

Input: Directed $G=(\mathrm{V}, \mathrm{E}), \mathrm{I}_{\mathrm{e}} \geq 0$, s in V

```
R={s},d(s)=0
While there is a x not in R with ( }u,x\mathrm{ ) in E, u in R
    Pick w among all }x\mathrm{ with smallest d'(w) value
    Add w to R
    d(w) = d'(w)
```

Lemma 1: At end of each iteration, if $u$ in $R$, then $P_{u}$ is a shortest s-u path

Lemma 2: If $u$ is connected to $s$, then $u$ in $R$ at the end

## Dijkstra's shortest path algorithm

$$
d^{\prime}(v)=\min _{e=(u, v) \text { in } E, u \text { in } R} d(u)+l_{e}
$$

Input: Directed $G=(\mathrm{V}, \mathrm{E}), \mathrm{I}_{\mathrm{e}} \geq 0$, s in V

$$
R=\{s\}, d(s)=0
$$

While there is a $x$ not in $R$ with $(u, x)$ in $E, u$ in $R$

At most n iterations

Pick $w$ among all $x$ with smallest $d^{\prime}(w)$ value Add w to R $d(w)=d^{\prime}(w)$


$$
\begin{aligned}
& \Sigma_{\mathrm{x} \in \mathrm{~V}} \mathrm{O}\left(\mathrm{In}_{\mathrm{x}}+1\right) \\
& =O(m+n) \text { time }
\end{aligned}
$$

$O((m+n) n)$ time bound is trivial
$\mathrm{O}((\mathrm{m}+\mathrm{n}) \log \mathrm{n})$ time implementation with priority Q

## Reading Assignment

Sec 4.4 of [KT]

## Mid-term topics done!

Anything till now is fair game for the mid-terms

