

# Lecture 24

CSE 331

April 1, 2020

# Rankings



# How close are two rankings?

Google   [Advanced Search](#)

Web [Show options...](#) Results 1 - 10 of about 23,700,000 for compare rankings. (0.30 seconds)

**Comparison Reviews** Sponsored Link  
[Angleslist.com](#) Your neighbors' ratings on local service companies. Award-winning.

**Ranking - Wikipedia, the free encyclopedia**  
In competition **ranking**, items that **compare** equal receive the same **ranking** number, and then a gap is left in the **ranking** numbers. The number of **ranking** ...  
[en.wikipedia.org/wiki/Ranking](#) - [Cached](#) - [Similar](#) - [Print](#) - [Share](#)

**A Comparison of Ranking Methods for Classification Algorithm Selection**  
is based on Spearman's rank correlation coefficient. To **compare ranking** methods, a combination of Friedman's test and Dunn's multiple com- ...  
[www.springerlink.com/index/mh87p44744x7Q258.pdf](#) - [Similar](#) - [Print](#) - [Share](#)  
by PB Brazdil - 2000 - [Cited by 46](#) - [Related articles](#)

**College Football Ranking Comparison**  
College Football **Ranking** Comparison. Sunday, November 8, 2009 (127 **Rankings**). | Last Week | Archived | FBS | FCS | Summary | CSV Data | ...  
[www.masseyratings.com/cf/compare.htm](#) - [Cached](#) - [Similar](#) - [Print](#) - [Share](#)

**College Basketball Ranking Comparison**  
Monday, April 6, 2009 (36 **Rankings**) ..... Cor 923 810 923 898 819 868 888 777 839 899 880  
243 246 245 248 241 251 247 249 248 244 **Ranking** Violation % 254 ...  
[www.masseyratings.com/cb/compare.htm](#) - [Cached](#) - [Print](#) - [Share](#)

**FIFA.com - Compare Teams**  
With FIFA.com you can **compare** the progress of up to four teams in the FIFA/Coca-Cola World **Ranking**. See how teams have risen and fallen since August 1993 ...  
[www.fifa.com/.../ranking/compare/compareteams.html](#) - [Cached](#) - [Similar](#) - [Print](#) - [Share](#)

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GetEducated.com's directory of online degrees provides **rankings**, ratings, and comparison tools to help you choose the best online degrees from the best ...  
[www.geteducated.com/](#) - [Cached](#) - [Similar](#) - [Print](#) - [Share](#)

**Compare your website rankings in Google Caffeine « Advanced Web ...**  
Aug 18, 2009 ... To help you **compare** your website **rankings** from Google with the **rankings** from Google Caffeine we have created a quick step-by-step tutorial

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bing

**ALL RESULTS** ALL RESULTS 1-10 of 8,810,000 results · [Advanced](#)

**FIFA.com - Compare Teams**  
With FIFA.com you can **compare** the progress of up to four teams in the FIFA/Coca-Cola World **Ranking**. See how teams have risen and fallen since August 1993 and pinpoint their ...  
[www.fifa.com/worldfootball/ranking/compare/compareteams.html](#) · [Cached page](#)

**How to Compare Graduate School Rankings | eHow.com**  
Choosing a graduate school can be a stressful and confusing process for any student. By comparing graduate schools and their **rankings** a student can choose which one fits her ...  
By eHow Education Editor · Difficulty: Moderate · 0 posts  
[www.ehow.com/how\\_2051516\\_compare-graduate-school-rankings.html](#) · [Cached page](#)

**Ranking - Wikipedia, the free encyclopedia**  
Items that **compare** equal receive the same **ranking** number, which is the mean of what they would have under ordinal **rankings**. Equivalently, the **ranking** number of 1 plus the number of ...  
[en.wikipedia.org/wiki/Ranking](#) · [Enhanced view](#)

**Ranking Compare**  
Ask Google Yahoo Search | Web browser tool – Shows search results for a given keyword or phrase on the top three engines, Google, Yahoo and Msn.  
[tools.essential-seo.com/ranking-compare](#) · [Cached page](#)

**SchoolDigger.com - School Rankings, Reviews and More - Public and ...**  
Find the best elementary, middle, and high schools. Search for schools near any address, **compare** test scores, sort by school **ranking**, class sizes, and more using SchoolDigger.  
[www.schooldigger.com](#) · [Cached page](#)

**College and university rankings - Wikipedia, the free encyclopedia**  
The College and university **rankings** are a lists of universities and liberal arts colleges in higher education, an order determined by any combination of factors.  
[International ...](#) · [Regional and national ...](#) · [Criticism \(North America\)](#)  
[en.wikipedia.org/wiki/College\\_and\\_university\\_rankings](#) · [Enhanced view](#)

**www.hospitalcompare.hhs.gov**  
[www.hospitalcompare.hhs.gov/Hospital/Search/SearchCriteria.asp?version=default&browser=IE...](#)

# Rest of today's agenda

Formal problem: Counting inversions


Divide and Conquer algorithm

# Solve a harder problem

Input:  $a_1, \dots, a_n$

Output: LIST of all inversions

```
L =  $\phi$ 
for i in 1 to n-1
  for j in i+1 to n
    if  $a_i > a_j$ 
      add (i,j) to L
return L
```



Optimal for  
the listing  
problem

# Example 1: All inversions-- $(2i-1, 2i)$

2	1	3	4	6	5	7	8
---	---	---	---	---	---	---	---

Only check  $(i, i+1)$  pairs

Q1: Solve listing problem in  $O(n)$  time?

Q2: Recursive divide and conquer algorithm to count the number of inversions?

CountInv ( $a, n$ )

if  $n = 1$  return 0

if  $n = 2$  return  $a_1 > a_2$

$a_L = a_1, \dots, a_{\lfloor n/2 \rfloor}$

$a_R = a_{\lfloor n/2 \rfloor + 1}, \dots, a_n$

return CountInv( $a_L, \lfloor n/2 \rfloor$ ) + CountInv( $a_R, n - \lfloor n/2 \rfloor$ )

# This can be horribly wrong...

CountInv (a,n)

if  $n = 1$  return 0

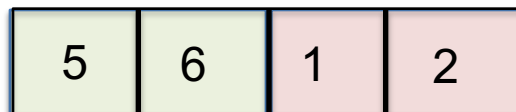
if  $n = 2$  return  $a_1 > a_2$

$a_L = a_1, \dots, a_{\lfloor n/2 \rfloor}$

$a_R = a_{\lfloor n/2 \rfloor + 1}, \dots, a_n$

return CountInv( $a_L, \lfloor n/2 \rfloor$ ) + CountInv( $a_R, n - \lfloor n/2 \rfloor$ )

Example where instance has non-zero (can be  $\Omega(n^2)$ ) inversions and algo returns 0?



All 4 "crossing" pairs are inversions

# Bad case: “crossing inversions”

CountInv (a,n)

if  $n = 1$  return 0

if  $n = 2$  return  $a_1 > a_2$

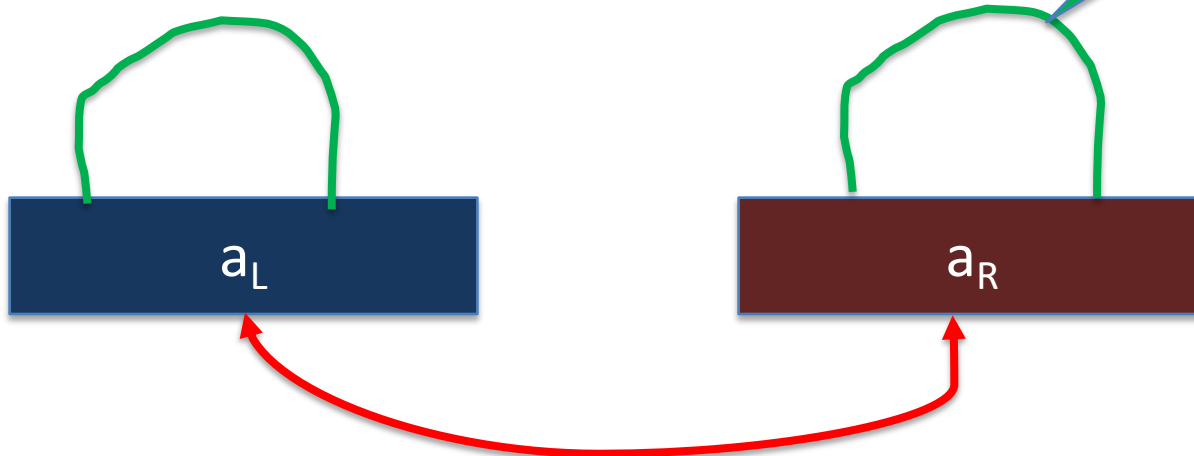
$a_L = a_1, \dots, a_{\lfloor n/2 \rfloor}$

$a_R = a_{\lfloor n/2 \rfloor + 1}, \dots, a_n$

return CountInv( $a_L, \lfloor n/2 \rfloor$ ) + CountInv( $a_R, n - \lfloor n/2 \rfloor$ )

Yes!

Are  $a_L$   
and  $a_R$   
sorted?





# Example 2: Solving the bad case



$a_L$



$a_R$

$a_L$  is sorted

First element in  $a_L$  is larger than first/only element in  $a_R$

$O(1)$  algorithm to count number of inversions?

return size of  $a_L$

# Example 3: Solving the bad case

1
---

$a_L$

5	6	.....
---	---	-------

$a_R$

$a_R$  is sorted

First/only element is  $a_L$  is smaller than first element in  $a_R$

$O(1)$  algorithm to count number of inversions?

return 0

# Solving the bad case

First element of  $a_L$  is larger than first element of  $a_R$



$a_L$



$a_R$

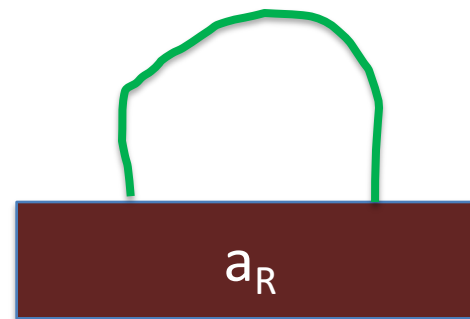
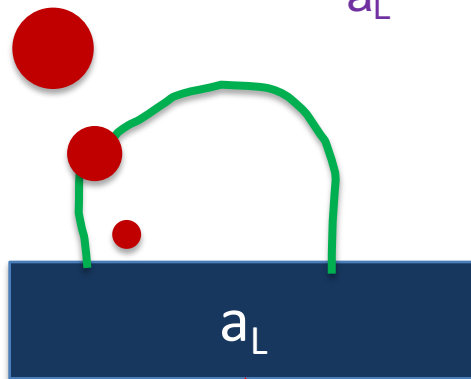
First element of  $a_L$  is smaller than first element of  $a_R$



$a_L$



$a_R$



Try to  
modify  
the  
MERGE  
algorithm



# Divide and Conquer

Divide up the problem into at least two sub-problems

Solve all sub-problems: Mergesort

Recursively solve the sub-problems

Solve stronger sub-problems: Inversions

“Patch up” the solutions to the sub-problems for the final solution

# Mergesort-Count algorithm

Input:  $a_1, a_2, \dots, a_n$

Output: Numbers in sorted order+ #inversion

MergeSortCount(  $a, n$  )

If  $n = 1$  return ( 0 ,  $a_1$  )

If  $n = 2$  return (  $a_1 > a_2$ ,  $\min(a_1, a_2)$ ;  $\max(a_1, a_2)$  )

$a_L = a_1, \dots, a_{n/2}$       $a_R = a_{n/2+1}, \dots, a_n$

( $c_L, a_L$ ) = MergeSortCount( $a_L, n/2$ )

( $c_R, a_R$ ) = MergeSortCount( $a_R, n/2$ )

( $c, a$ ) = MERGE-COUNT( $a_L, a_R$ )

return ( $c+c_L+c_R, a$ )

$$T(2) = c$$

$$T(n) = 2T(n/2) + cn$$

$O(n \log n)$  time

$O(n)$

Counts #crossing-inversions+  
MERGE

# MERGE-COUNT( $a_L, a_R$ )

$a_L = l_1, \dots, l_{n'}$

$a_R = r_1, \dots, r_m$

```
c = 0
```

```
i, j = 1
```

```
while i ≤ n' and j ≤ m
```

```
    if  $l_i < r_j$ 
```

```
        i ++
```

```
        add  $l_i$  to output
```

```
    else
```

```
        add  $r_j$  to output
```

```
        j ++
```

```
        c += n' - i + 1
```

```
Output any remaining items
```

```
return c
```



$a_L$



$a_R$



$a_L$



$a_R$