Lecture 25

CSE 331

Apr 3, 2020

Logistics

- Homework 6 is out today
- Deadline for regrading requests is Mon (Apr 6)
- Mid-semester temp grades will be out on Tue (Apr 7)
  - Based on 4 hws, midterms, quiz 1 (nothing dropped)
- Video Project (remember?)
  - Due April 20
  - See mini project website for details

- New S/U policy: It's about you, not me ☺
  - I'll assign letter grades as usual;
  - YOU choose to convert your letter grade to S/U
    - Chance to prevent any possible damage to your GPA
  - C and above: S
  - C- and below: U

- **Most importantly: Take care of yourself!**
  - **I mean mentally!**
  - **Go easy on yourself!**

Divide and Conquer

Divide up the problem into at least two sub-problems

Recursively solve the sub-problems

“Patch up” the solutions to the sub-problems for the final solution

Improvements on a smaller scale

Greedy algorithms: exponential → poly time

(Typical) Divide and Conquer: $O(n^2)$ → asymptotically smaller running time

Multiplying two numbers

Given two numbers a and b in binary

$a=(a_{n-1},..,a_0)$ and $b = (b_{n-1},...,b_0)$

Compute $c = a \times b$

Elementary school algorithm is $O(n^2)$

The current algorithm scheme

Shift by O(n) bits

Adding O(n) bit numbers

Mult over n bits

$$a \bullet b = a^1b^1 \bullet 2^{2[n/2]} + (a^1b^0 + a^0b^1) \bullet 2^{[n/2]} + a^0b^0$$

Multiplication over n/2 bit inputs

$T(n) \leq 4T(n/2) + cn$

$T(1) \leq c$

$T(n)$ is $O(n^2)$

The key identity

$$a^1b^0+a^0b^1 = (a^1+a^0)(b^1+b^0) - a^1b^1 - a^0b^0$$

The final algorithm

Input: $a = (a_{n-1},..,a_0)$ and $b = (b_{n-1},...,b_0)$

Mult (a, b)

If $n = 1$ return $a_0 b_0$

$a^1 = a_{n-1},...,a_{[n/2]}$ and $a^0 = a_{[n/2]-1},..., a_0$

Compute b^1 and b^0 from b

$x = a^1 + a^0$ and $y = b^1 + b^0$

Let $p =$ Mult (x, y), $D =$ Mult (a^1, b^1), $E =$ Mult (a^0, b^0)

$F = p - D - E$

return $D \bullet 2^{2[n/2]} + F \bullet 2^{[n/2]} + E$

$T(1) \leq c$

$T(n) \leq 3T(n/2) + cn$

$O(n^{\log_2 3}) = O(n^{1.59})$

run time

All green operations are $O(n)$ time

$$a \bullet b = a^1b^1 \bullet 2^{2[n/2]} + ((a^1+a^0)(b^1+b^0) - a^1b^1 - a^0b^0) \bullet 2^{[n/2]} + a^0b^0$$