Lecture 5

CSE 331

Feb 5, 2020

Stable Matching

A perfect matching with no instability

What is instability?

Given

- 2n preference lists; L_m for each man m, L_w for each woman m
- a perfect matching S

A pair (m, w) $\notin$ S is instability, if

- m is ranked before w's husband in L_w

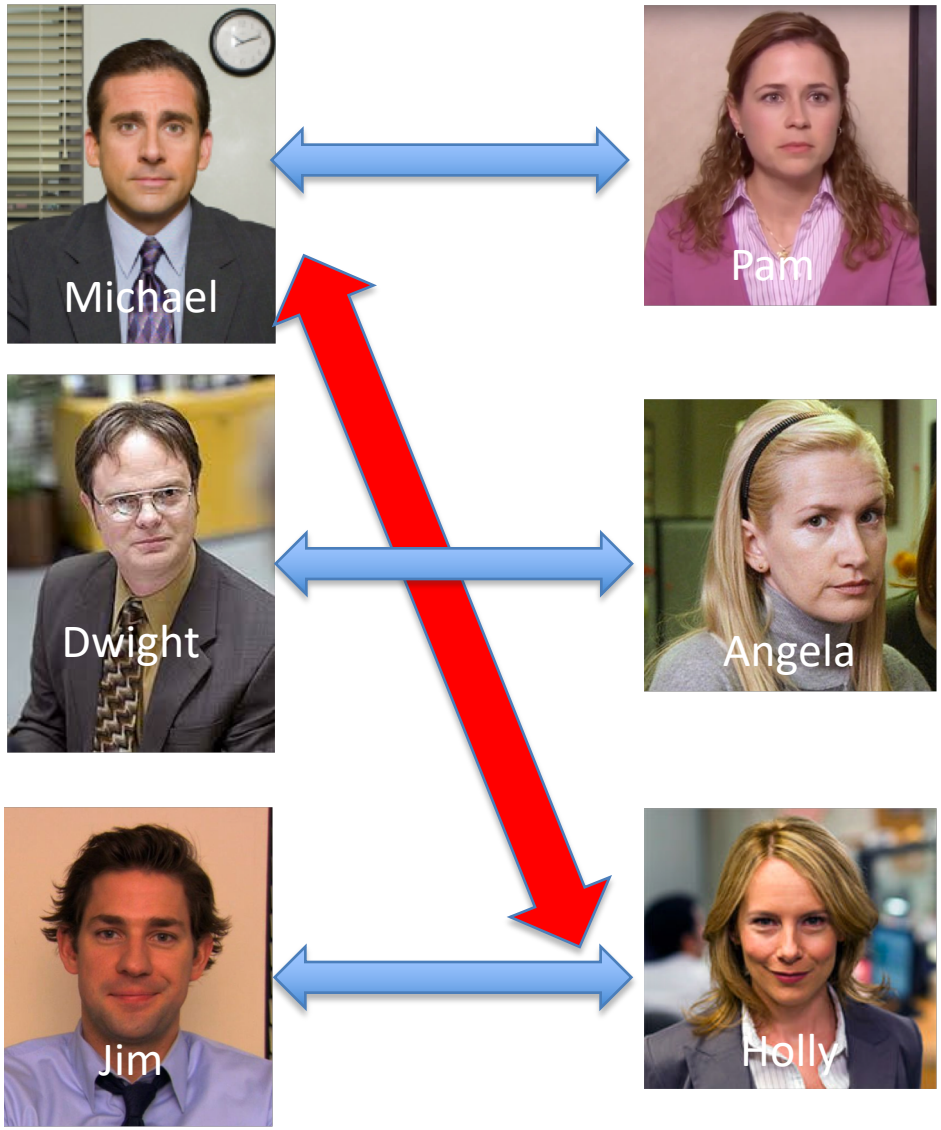
AND

- w is ranked before m's wife in L_m

I.e., if a man AND a woman would be both happier in a new marriage

Preferences

Instability



A stable marriage

Even though BBT and JA are not very happy

Brad Pitt (BP)

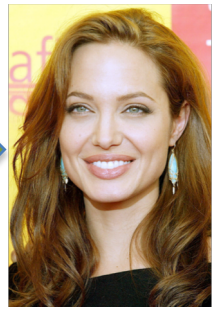
Angelina Jolie (AJ)

Billy Bob Thornton (BBT)

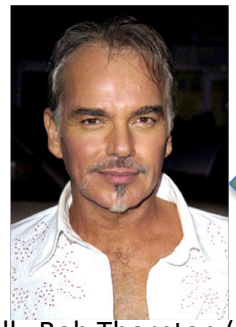
Jennifer Aniston (JA)

Two stable marriages possible!

Brad Pitt (BP)



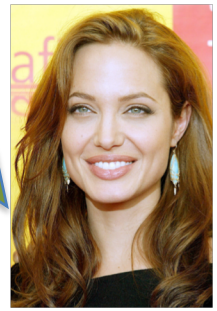
Angelina Jolie (AJ)



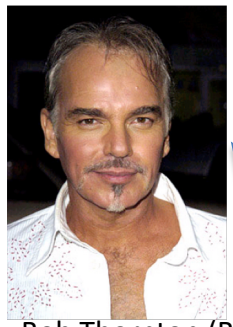
Billy Bob Thornton (BBT)

Jennifer Aniston (JA)

Brad Pitt (BP)



Angelina Jolie (AJ)



Billy Bob Thornton (BBT)

Jennifer Aniston (JA)

Stable Marriage problem

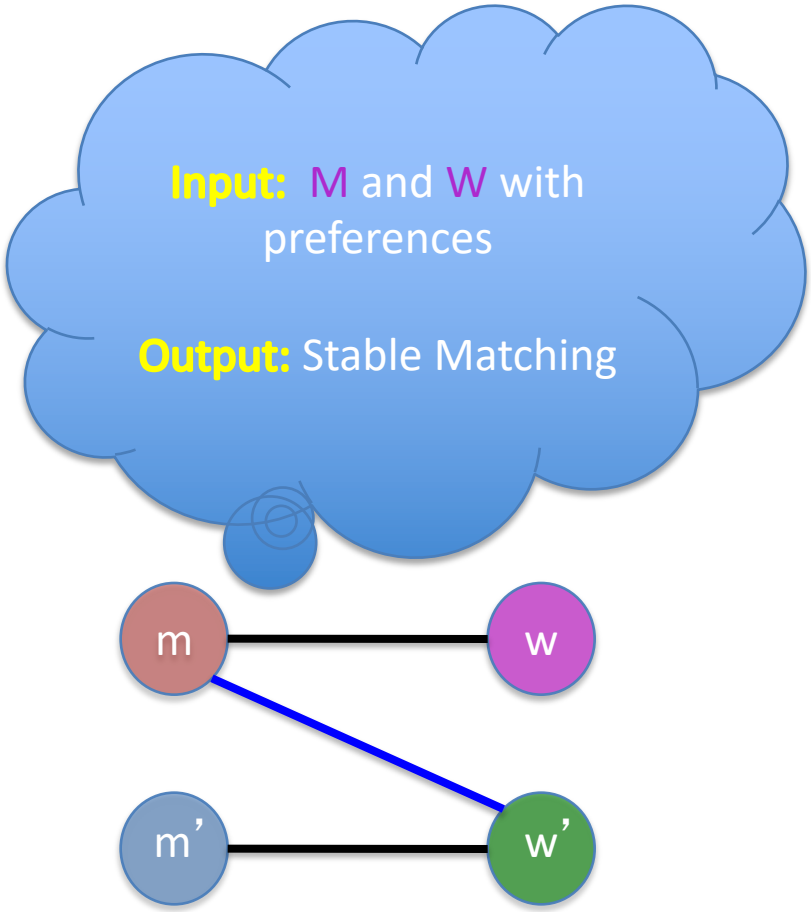
Set of men M and women W

Preferences (ranking of potential spouses)

Matching (no polyandry/gamy in M X W)

Perfect Matching (everyone gets married)

Instability



Stable matching = perfect matching + no instability

Questions/Comments?

Two Questions

Does a stable marriage always exist?

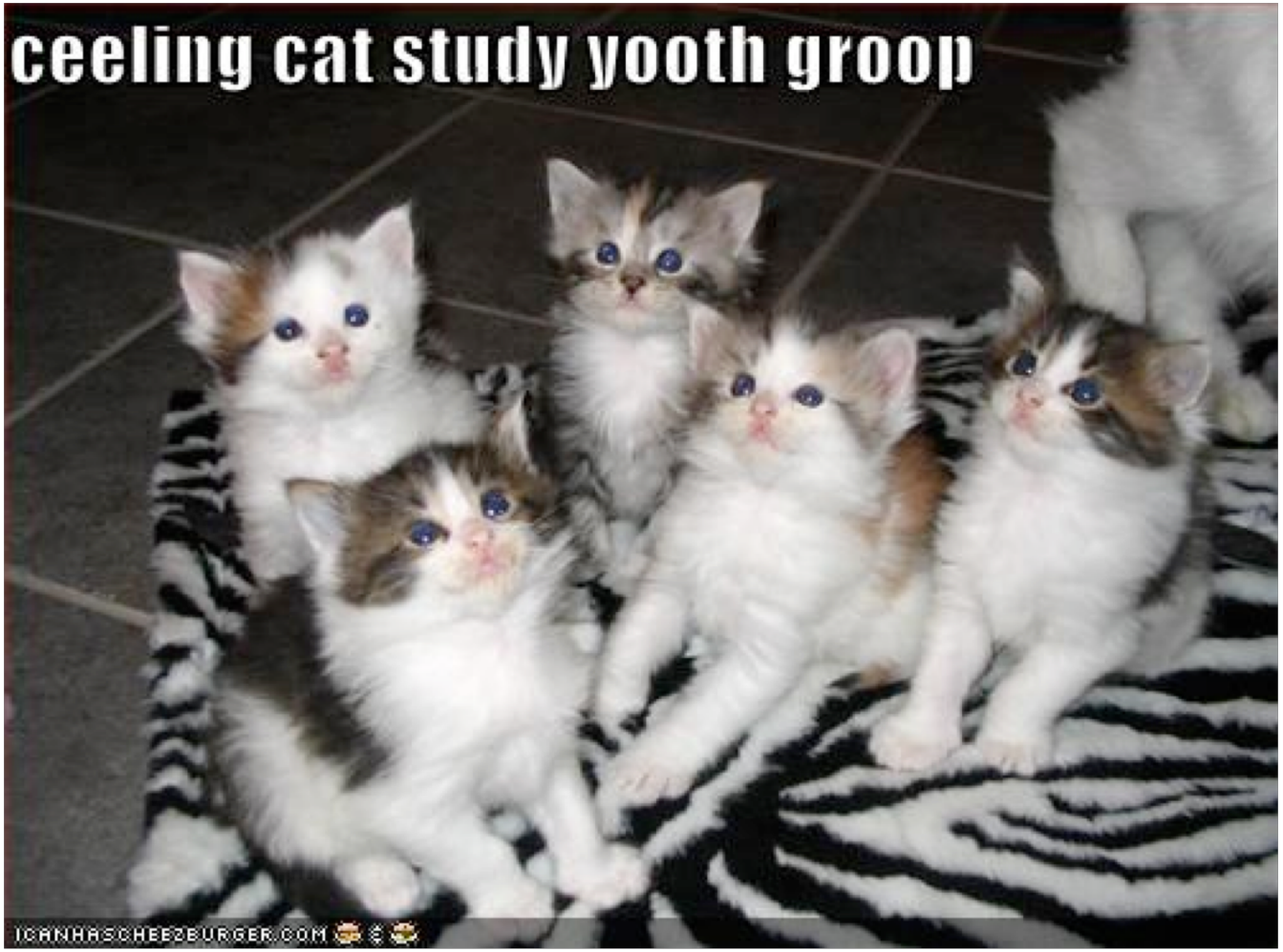
If one exists, how quickly can we compute one?

Today's lecture

Naïve algorithm

Gale-Shapley algorithm for Stable Marriage problem

Discuss: Naïve algorithm!



The naïve algorithm

Incremental algorithm to produce all n! perfect matchings?

Go through all possible perfect matchings S

If S is a stable matching

then Stop

Else move to the next perfect matching

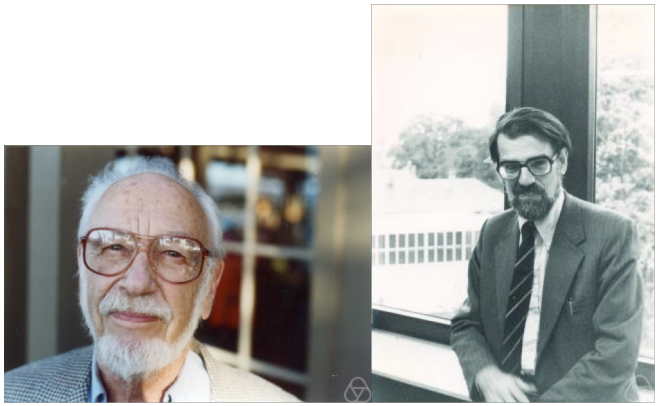
Gale-Shapley Algorithm

David Gale

Lloyd Shapley

$O(n^3)$ algorithm

Moral of the story…



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Questions/Comments?

Rest of today's agenda

GS algorithm

Run of GS algorithm on an instance

Prove correctness of the GS algorithm