Lecture 6

CSE 331

Feb 7, 2020

If you need it, ask for help. Take advantage of OHs (No one came on Wed!)

Read recitation notes

Recitation 2

Recitation notes for the week of **February 3, 2020**.

Overview

- Recitation 1 Review and HW0 Answers
- Stable Matching Background
- Proof by Induction
- Proof by Counterexample
- Proof by Contradiction

Recitation 1 Review and HW0 Answers

Before we begin, go back and review the content from recitation 1, particularly the Reduction section where we go over Geometric Search.

In addition, the HW0 answer key has been posted here. Going forward, the answer keys will not be given out online, but instead will be released at the end of Friday's lecture.

⊘ Common Mistakes

The common mistakes we saw in submissions were either:

- Not using anything specific to the problem at hand (proving $n!$, but not how it relates to the problem).
- Simply restating the problem statement (while important to the proof, you need to show how the problem set-up proves a certain statement).

Stable Matching Review

The problem

Input:

- Set of n men $M = \{m_1, m_2, \ldots, m_n\}$
- Set of n women $W = \{w_1, w_2, \ldots, w_n\}$
- For every $m \in M$, L_m a total ranking of all women
- For every $w \in W$, L_w a total ranking of all men

Reading Assignments and Preparation Videos

Date	Topic	Notes
Mon, Jan 27	Introduction S20 F19 F18	(**HW 0 out**) Week 1 recitation notes
Wed, Jan 29	Main Steps in Algorithm Design S20 F19 F18	
Fri, Jan 31	Stable Matching Problem S20 F19 F18 x^2	[KT, Sec 1.1]
Mon, Feb 3	Perfect Matchings S20 F19 F18 x^2	[KT, Sec 1.1] (**HW 0 in**) Week 2 recitation notes
Wed, Feb 5	Stable Matching Problem S20 F19 F18 x^2	[KT, Sec 1.1]
Fri, Feb 7	Gale Shapley algorithm F19 F18 x^2	[KT, Sec 1.1] *Reading Assignment:* Pigeonhole principle *Reading Assignment:* Asymptotic notation care package
Mon, Feb 10	Gale Shapley algorithm outputs a stable matching F19 F18 x^2	[KT, Sec 1.1] (**HW 1 out**)
Wed, Feb 12	Efficient algorithms and asymptotic analysis F19 F18 x^2	[KT, Sec 1.1] *Reading Assignment:* Worst-case runtime analysis notes *Reading Assignment:* [KT, Sec 1.1, 2.1, 2.2, 2.4]

Sign-up for mini projects

Deadline: Friday, Feb 28, 11:00am

Actions

Signup for Mini Video project

Folks,

Please check the video project page: https://cse.buffalo.edu/~erdem/cse331/spring20/mini-project/index.html. Go over the details and make sure you understand what's expected.

Then, form groups of exactly 3 (three) by signing up here: https://docs.google.com/forms/d/e/1FAIpQLSctYIXnwY_riSI38-x226TstaZ2wwXIvC64-B-NWP1ISvedbw/viewform (link is available in the project page too). There are 153 of you, so don't worry about being left out in the remainder (I told you! :)).

You can use Piazza to find teammates. Then you can decide on the topic.

The deadline for team formation and algorithm/case study selection is **February 28, 11am ET.**

project

edit · good note | 0 Updated Just now by A. Erdem Sariyuce

followup discussions *for lingering questions and comments*

Start a new followup discussion

Compose a new followup discussion

Questions/Comments?

Stable Marriage problem

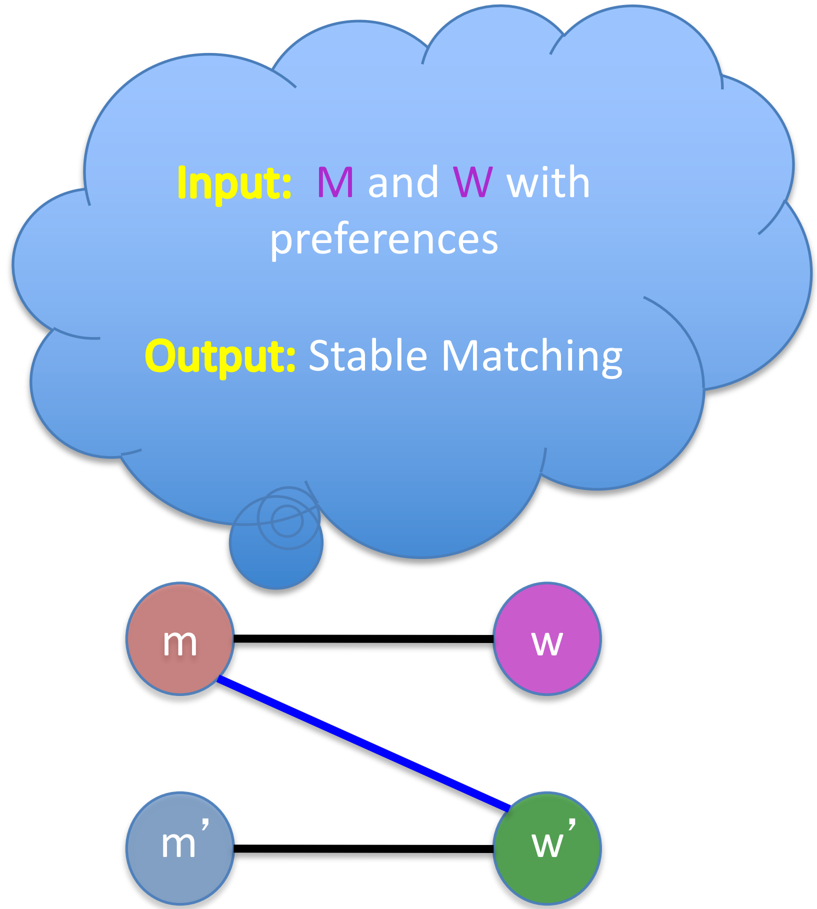
Set of men M and women W

Preferences (ranking of potential spouses)

Matching (no polyandry/gamy in M X W)

Perfect Matching (everyone gets married)

Instablity



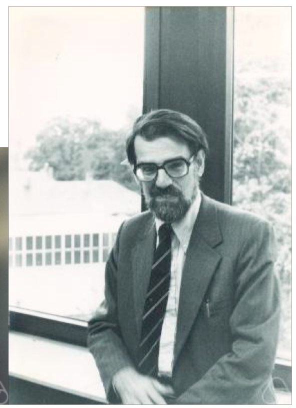
Stable matching = perfect matching+ no instablity

Remember Two Questions

Does a stable marriage always exist?

If one exists, how quickly can we compute one?

Moral of the story…



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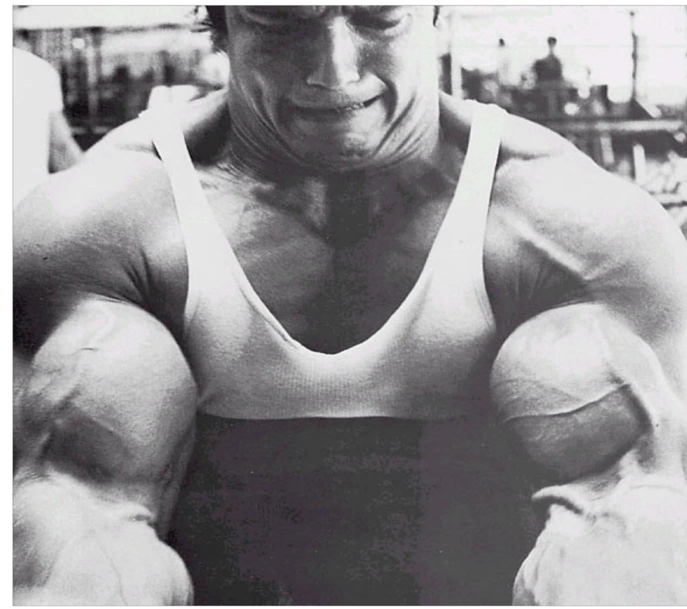


Rest of today's agenda

GS algorithm

Run of GS algorithm on an instance

Prove correctness of the GS algorithm

Gale-Shapley Algorithm (cont.)

Gale-Shapley Algorithm

Intially all men and women are free

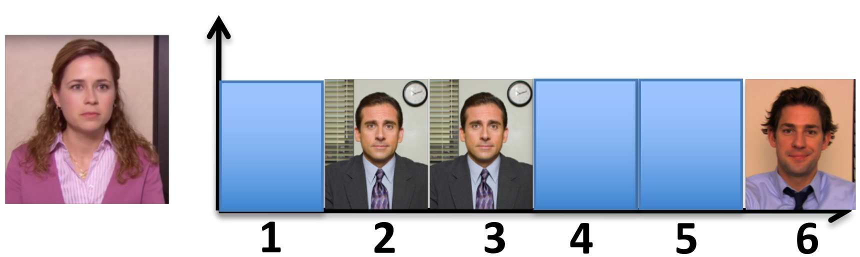
While there exists a free woman who can propose

> Let w be such a woman and m be the best man she has not proposed to
>
> w proposes to m
>
> If m is free
>
> (m,w) get engaged
>
> Else (m,w') are engaged
>
> If m prefers w' to w
>
> w remains free
>
> Else
>
> (m,w) get engaged and w' is free

Output the engaged pairs as the final output

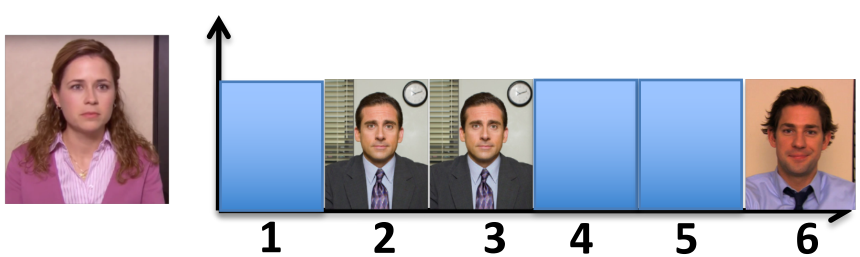
Preferences

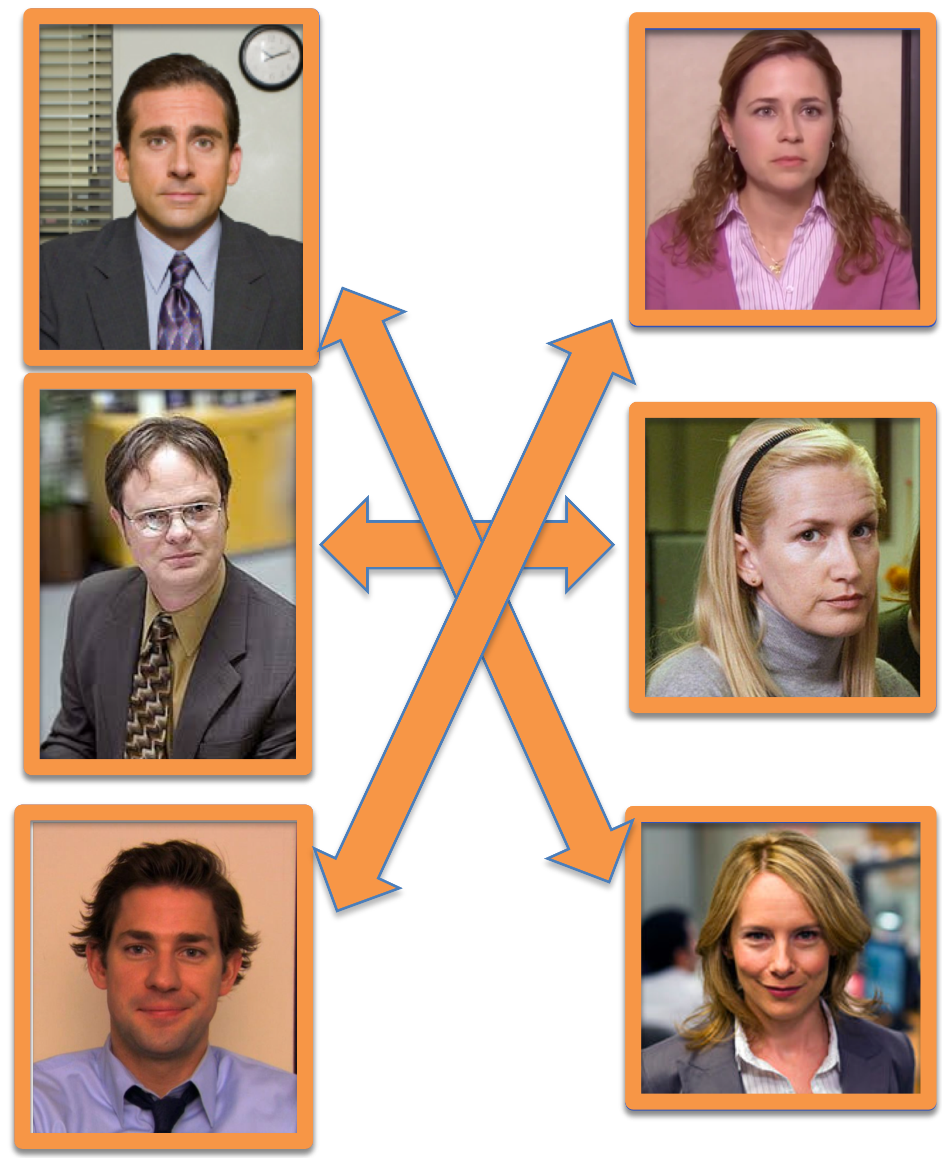
GS algorithm: The Office Edition



Any other stable matching in this example?

No!





Observation 1

Intially all men and women are free

While there exists a free woman who can propose

> Let w be such a woman and m be the best man she has not proposed to
>
> w proposes to m
>
> If m is free
>
> (m,w) get engaged
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> If m prefers w' to w
>
> w remains free
>
> Else
>
> (m,w) get engaged and w' is free

> Once a man gets engaged, he remains engaged (to "better" women)

Output the engaged pairs as the final output

Observation 2

Intially all men and women are free

While there exists a free woman who can propose

Let w be such a woman and m be the best man she has not proposed to

w proposes to m

If m is free

(m,w) get engaged

Else (m,w') are engaged

If m prefers w' to w

w remains free

Else

(m,w) get engaged and w' is free

If w proposes to m after m', then she prefers m' to m

Output the set S of engaged pairs as the final output

How many iterations? n^2

Intially all men and women are free

While there exists a free woman who can propose

Let w be such a woman and m be the best man she has not proposed to

w proposes to m

If m is free

(m,w) get engaged

Else (m,w') are engaged

If m prefers w' to w

w remains free

Else

(m,w) get engaged and w' is free

Output the set S of engaged pairs as the final output