Feb 17

$G = (V, E)$ $\rightarrow$ set of edges

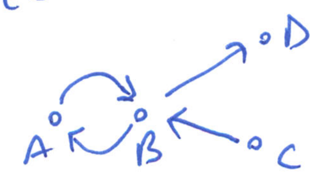
$\downarrow$ set of vertices/nodes

$E \subseteq V \times V$

<u>Default</u>: $n = |V|$; $m = |E|$

<u>Def:</u> G is undirected $\Leftrightarrow \forall u \neq w \in V, (u,w) \in E \Leftrightarrow (w,u) \in E$

o.w. G is directed



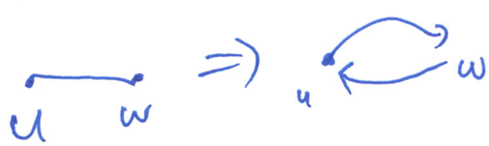
(.) Airline map (u)

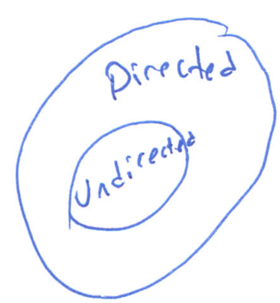
(.) Wikipedia page

Feb 19 <u>Default:</u> G is undirected.

<u>Claim</u>: Every undirected graph is also directed.

<u>Pf. idea:</u>





Def: A path in $G=(V,E)$ is sequence of vertices $u_1, u_2, \ldots, u_k$ s.t. $\forall i \in [k-1]$, $(u_i, u_{i+1}) \in E$

Notes: (i) u_i need not be distinct (ii) holds for directed graphs

D, C, B, A ✓
A, B, C, D ✓
A, B, C, B ✓
A, C, D ✗

D, C, B, A ✗
A, B, C, D ~~✗~~ ✓
A, B, C, B ✗
A, C, D ✗

Def: A <u>simple path</u> does NOT have any repeated vertices.

By default: All paths are <u>simple</u>.

Def: length of a path = # edges in the path

$len(A,B,C,D) = 3$

Q: What is max length of a (simple) path? of n nodes

A: n-1. Ex (Hint: use PHP)

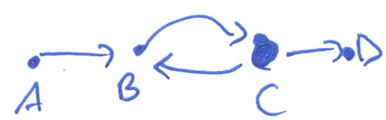
Def: A cycle $u_1, u_2, \ldots, u_k$ is a path s.t.

(1) $u_1, \ldots, u_{k-1}$ are distinct

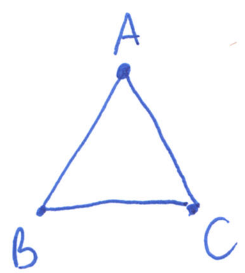
(2) $u_1 = u_k$

(3) • G is undirected : $k \geq 4$

• G is directed : $k \geq 3$

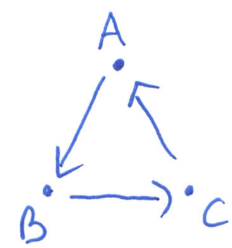


B, C, B ✓



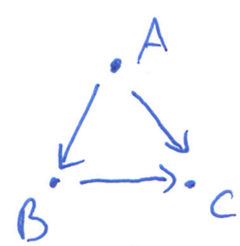
A, C, B, A ✓

A, B, C, A ✓



A, C, B, A ✗

A, B, C, A ✓



A, C, B, A ✗

A, B, C, A ✗

↑ Directed Acyclic Graph

Def u & w are connected if $\exists$ u-w path
(undirected G)

u & w are strongly connected if $\exists$ u-w path, w-u path

Def A (directed) graph is (strongly) connected if $\forall$ $u \neq w$, u & w are (strongly) connected.