Feb 21

Prop: Let T be a BFS tree for $G=(V,E)$

If $(u,w) \in E$ s.t. $u \in L_i$ and $w \in L_J$

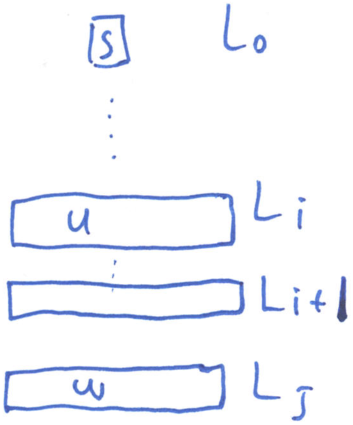
$\Rightarrow |i-J| \leq 1 \iff i \in \{J-1, J, J+1\}$

Pf idea: Pf by contradiction.

w.l.o.g. assume $i \leq J$ [o.w. switch i & J]

for contradiction assume $|i-J| > 1 \Rightarrow J > i+1$

$\iff J \geq i+2$



Consider situation when BFS is creating L_{i+1}

$\Rightarrow u \in L_i$, $w \notin L_0, \ldots L_{i-1}$

$\Rightarrow (u,w) \in E$

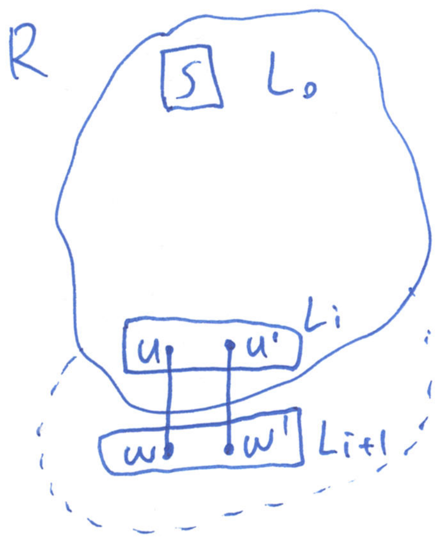
$\Rightarrow$ By BFS defn $w \in L_{i+1}$

$\Rightarrow$ contradicts $w \in L_J$ for $J \geq i+2$ ■

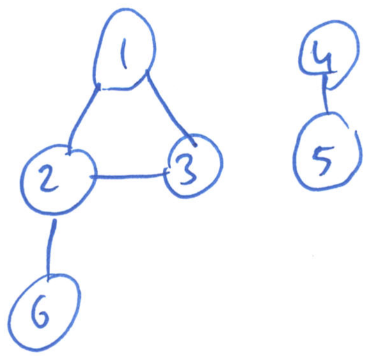
Explore(s)

0. $R = \{s\}$
1. While $\exists\, (u,w) \in E$ s.t. $u \in R$ and $v \notin R$
   Add w to R
2. Output $R^* = R$

Lem 0: Explore always terminates } Ex



Def: Set of all vertices connected to s is called its connected component $CC(s)$



$CC(2) = \{1, 2, 3, 6\}$

$CC(5) = \{4, 5\}$

Theorem: For all G, start vertices s, $R^* = CC(s)$

General trick: to show $A = B \iff A \subseteq B$ and $B \subseteq A$

Lemma 1: $R^* \subseteq CC(s)$

Lemma 2: $CC(s) \subseteq R^*$

Lemma 1 + Lemma 2 $\Rightarrow$ Thm

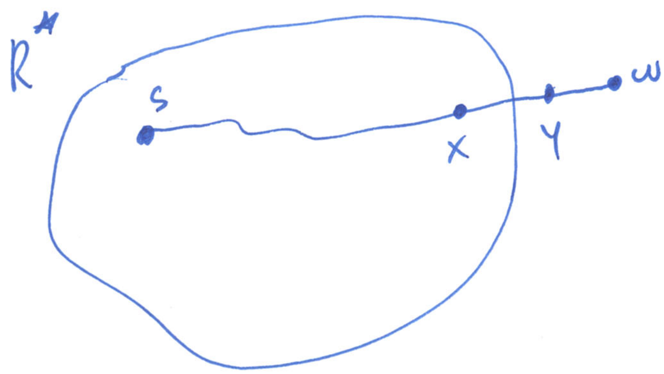
Thm $\Rightarrow$ BFS is correct

Ex: By induction

Pf idea of Lemma 2: Pf. by contradiction.

Assume $CC(s) \not\subseteq R^*$ $\Rightarrow$ $\exists\ w \in CC(s)$ BUT $w \notin R^*$

$\Leftrightarrow$ $\exists$ s-w path p in G but $w \notin R^*$
[$w \in CC(s)$]



Since p starts inside of R^* but ends up outside of R^*,

$\Rightarrow \exists\ (x,y) \in P$ s.t. $x \in R^*$, $y \notin R^*$

$\Rightarrow$ y should have been added to R by Explore

$\Rightarrow$ Algo has not terminated

$\Rightarrow$ contradicts with the existence of R^* ∎