Feb 26

BFS(G, s) // G is in adj. list format

$O(n)$ $\begin{cases} 0.\ CC[s] = T \text{ and } CC[u] = F \ \forall u \neq s \in V \\ 1.\ i = 0 \\ 2.\ L_0 = \{s\} \end{cases}$

3\. While $L_i \neq \emptyset$ } T_1: # times this loop is run

3.1. $L_{i+1} = \emptyset \rightarrow O(1)$

3.2. For all $u \in L_i$ ~~T~~

for all $(u,w) \in E \rightarrow T_{12}$: # times alg. gets here

T_{123}: # times alg gets here $\leftarrow$ if $CC[w] == F$ $\left.\begin{matrix} \\ CC[w] = T \\ \text{Add } w \text{ to } L_{i+1} \end{matrix}\right\} O(1)$

$[T_1 \leq T_{123}]$

3.3 $i{+}{+} \rightarrow O(1)$

$O(n) \leftarrow$ 4. Return CC

[pass by value]

Total runtime $= O(n) + \underset{\substack{\uparrow\uparrow \\ 3.1\ 3.3}}{T_1} \cdot O(1) + T_{123} \cdot O(1) + O(n)$

$\leq O(n) + T_{123} \cdot O(1) + T_{123} \cdot O(1) = O(n) + O(T_{123})$

Goal: Bound total

Analysis 1: $T_{123} = O(n^3) \Rightarrow$ overall $O(n) + O(n^3) = O(n^3)$

Analysis 2: $T_{123} \leq n^2$ (obs: every vertex u appears in ≤ 1 L_i)

Claim: $T_{12} \leq n$ ($\Leftarrow$ $T_{123} \leq n \cdot T_{12} = n \cdot n = n^2$)