Mar 2

Greedy algorithm

0. $R = [n]$

1. $S = \emptyset$

2. while $R \neq \emptyset$

   (2.1) Let i be the smallest index in R

   (2.2) Add i to S

   (2.3.) Remove i from R

   (2.4) Delete all j from R that conflicts with i

3. Return $S^* = S$

Thm 1: S^* is an optimal solution.

↳ $\forall$ inputs, among all possible valid schedules for given input, S^* has the maximum number of jobs.

Ex 1. Algo terminates.

Ex 2. S^* is a valid schedule.

Pf. of correctness of greedy alg
→ Greedy stays ahead (next)
→ Exchange argument (min. max latenc[y] sec 4.2)

Let O be an optimal solution

Ex 3: Convince yourself that such an O exists.

Idea: $S^* = \mathcal{O}$

$\longmapsto \mathcal{O}$

$\longmapsto S^*$

problem: can have >1 optimal solutions

Idea: $|S^*| = |\mathcal{O}|$

Notation: $S^* = \{i_1, \dots, i_k\}$ $f(i_1) \le f(i_2) \le \dots \le f(i_k)$

$\mathcal{O} = \{j_1, \dots, j_m\}$ $f(j_1) \le f(j_2) \le \dots \le f(j_m)$

$\boxed{\text{Thm 1: } k = m}$

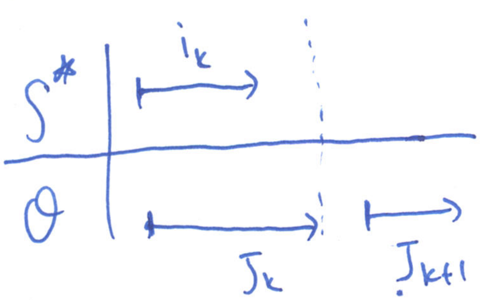
Claim 1: $k \le m$ (as $\mathcal{O}$ is optimal)

Lemma 1: ("Greedy stays ahead") $\forall\ 1 \le \ell \le k$

$$f(i_\ell) \le f(j_\ell)$$

(Assume Lemma 1 is true)

Pf (idea) of Thm 1: By contradiction: $k \ne m \underset{\text{by claim 1}}{\Longrightarrow} k < m \Leftrightarrow m \ge k+1$



By Lemma 1, $f(i_k) \le f(j_k)$

$\rightarrow$ Consider the situation after i_k is added to S

Ex $\nearrow$ $\Rightarrow j_{k+1} \in R$ (as j_{k+1} doesn't conflict with any i_ℓ $\ell \le k$)

$\Rightarrow R$ is non-empty $\Rightarrow$ Greedy alg. did not terminate $\Rightarrow$ contradict.

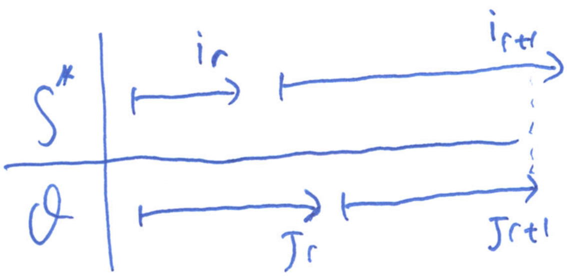
Pf (idea) Lemma 1: By induction on ℓ

Base case: $\ell = 1$. $f(i_1) \leq f(J_1) \rightarrow$ By alg. definition, $f(i_1)$ is the smallest finish tim

I.H: $f(i_\ell) \leq f(J_\ell) \quad \forall\ 1 \leq \ell \leq r \quad (r \geq 1)$

I.S: show $f(i_{r+1}) \leq f(J_{r+1})$

for the sake of contradiction $f(i_{r+1}) > f(J_{r+1})$



(.) Consider the situation after i_r is added to S by alg.

$\Rightarrow i_{r+1} \in R \Rightarrow i_{r+1}$ cannot be picked by greedy

$\Rightarrow J_{r+1} \in R$ (as $f(J_{r+1}) < f(i_{r+1})$).

$\Downarrow$

contradiction ∎