Apr 8

KICKASS PROPERTY LEMMA

$S = \{(x,y) \in P \mid |x - x^*| \leq \delta\}$

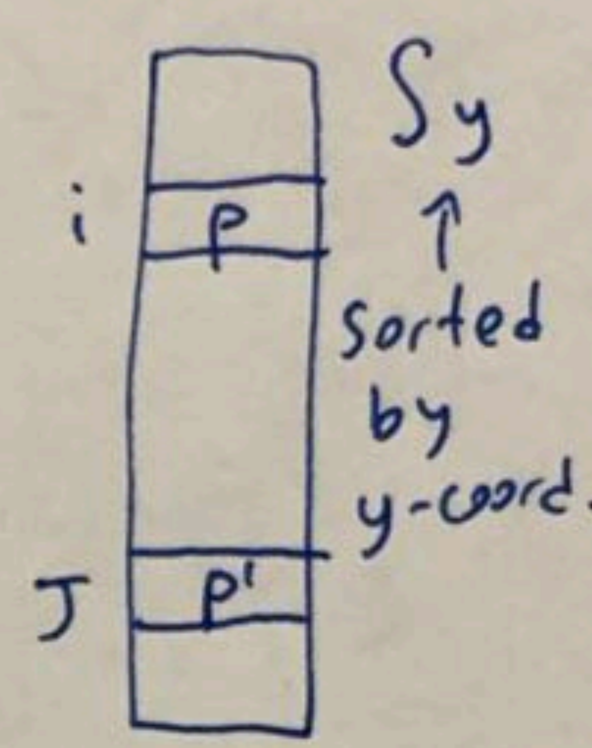
For every $p \neq p' \in S$ s.t.

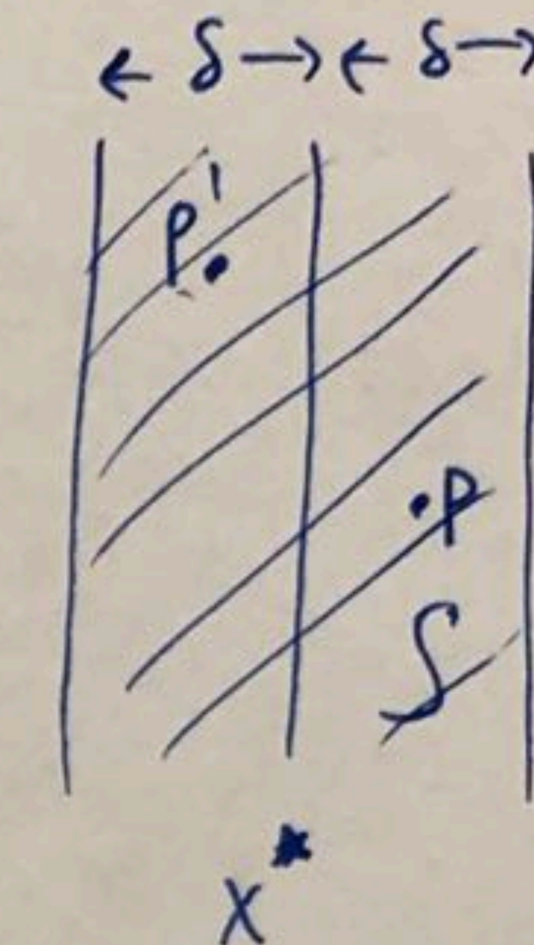
$$d(p,p') < \delta$$

s.t. $S_y[i] = p$

$S_y[j] = p'$

then $|i - j| \leq 15$





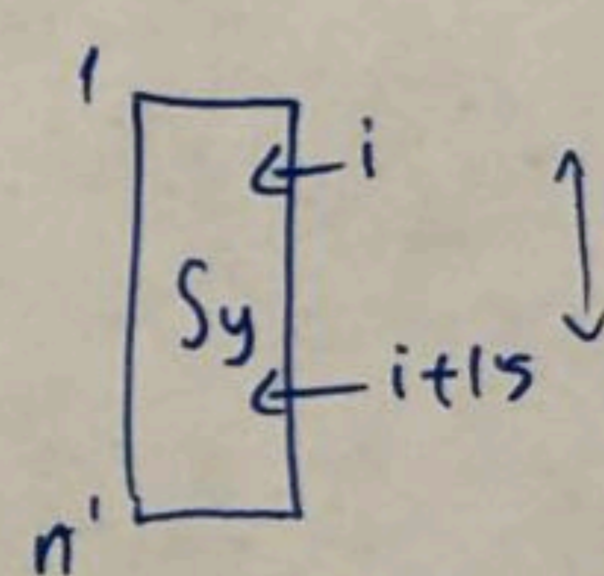
Note: (1) "15" can be made to "9" (can be as small as "7")

for $i = 1 \ldots n' - 1$

Let (p_i, p_i') among

$(S_y[i], S_y[i+1]), (S_y[i], S_y[i+2])$

$\ldots\ldots (S_y[i], S_y[\underbrace{i+15}_{\min(i+15,\, n')}])$ with the smallest distance



Let (p, p') be the closest pair of points among

$(p_1, p_1'), (p_2, p_2') \ldots (p_{n'}, p_{n'}')$

If $d(p,p') < \delta$

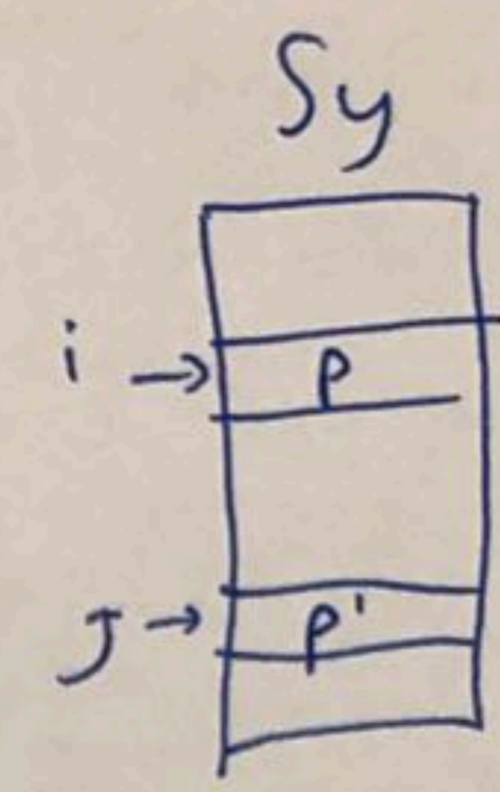
return (p,p')

else

return NULL

Pf (idea) of Kickass Property Lemma:

For contradiction assume $|i - j| \geq 16$



$d(p, p') < \delta \ (*)$

$d(p, p') \geq \frac{3\delta}{2} > \delta$

$\Rightarrow$ contradicts $(*)$

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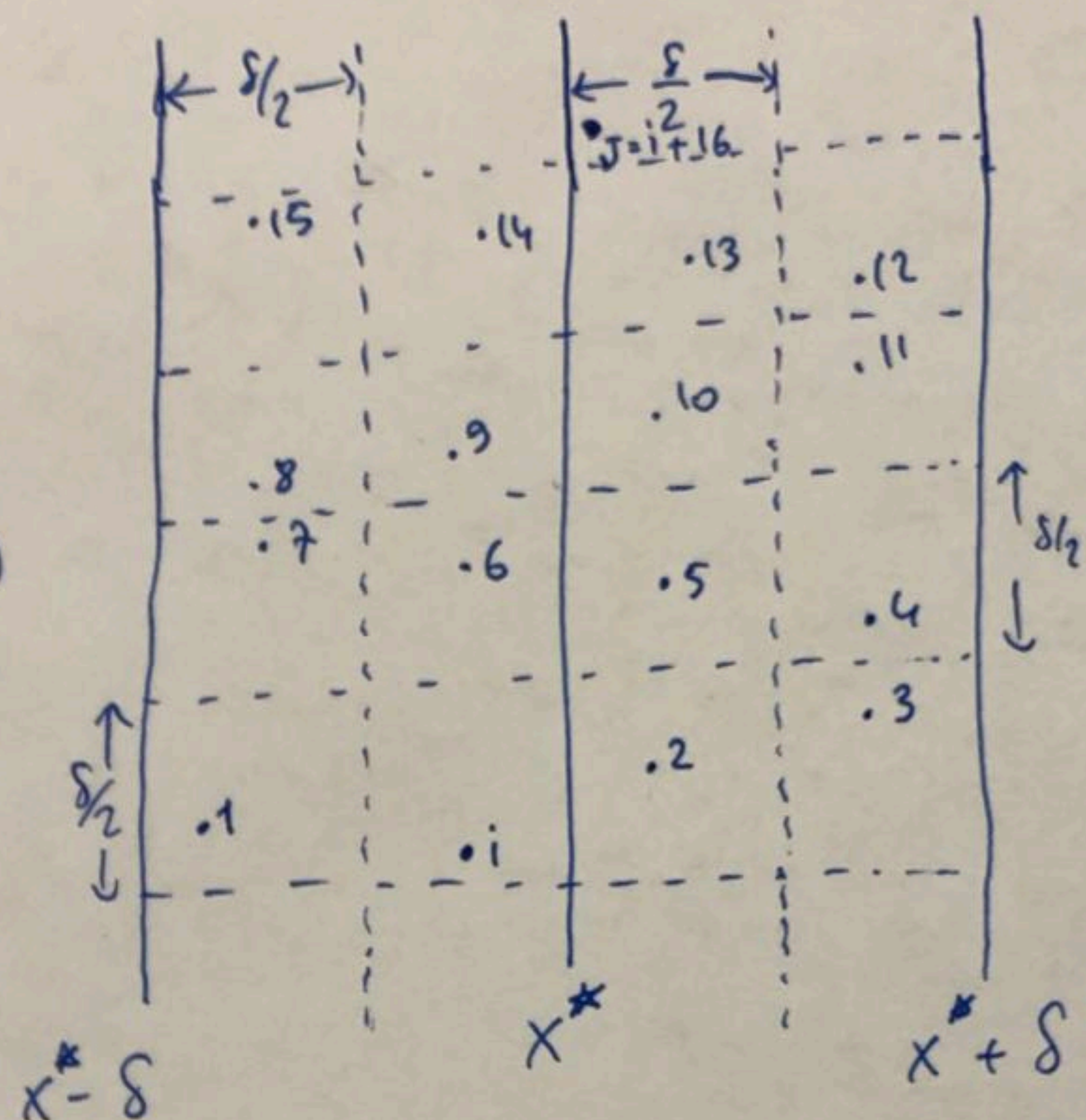
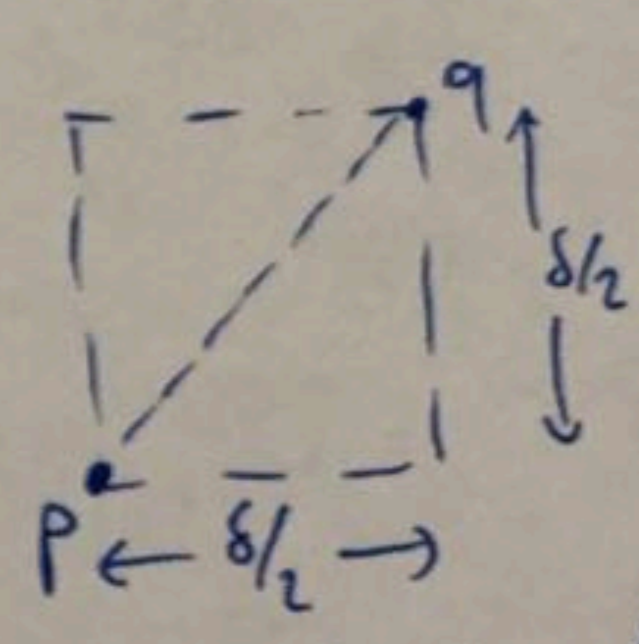


Claim: Every $\delta/2 \times \delta/2$ square has <u>at most 1</u> pt from S in it

Pf (idea): If not assume $\exists$ pts p & q inside one square

$$d(p,q) = \sqrt{\left(\frac{\delta}{2}\right)^2 + \left(\frac{\delta}{2}\right)^2} = \sqrt{\frac{\delta^2}{4} + \frac{\delta^2}{4}}$$

$$= \sqrt{\frac{\delta^2}{2}} = \frac{\delta}{\sqrt{2}} < \delta$$



Ex. p & q are furthest apart if on diagonal pts

contradicts the defn of δ as each square is in Q or R.