Apr 15

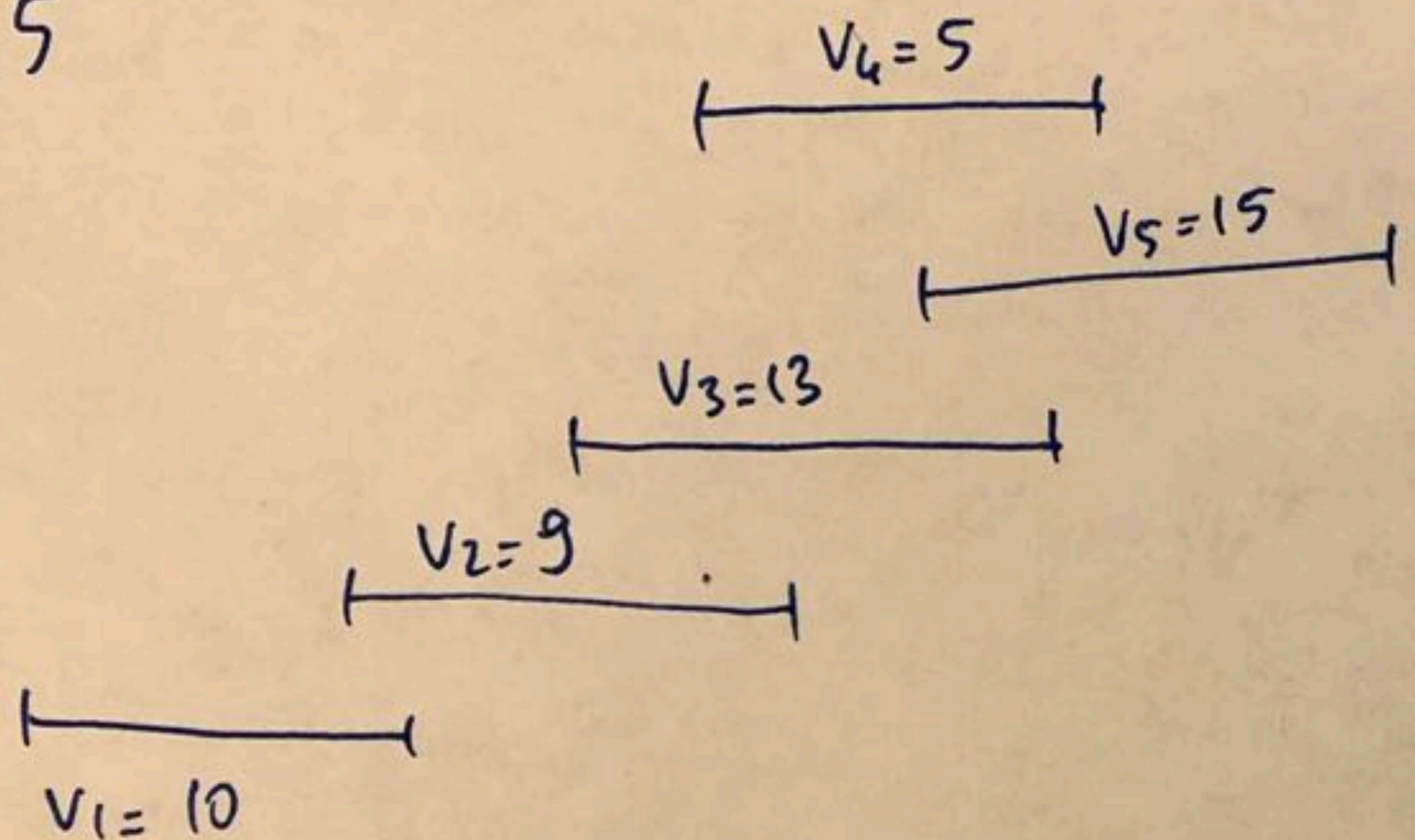
$M[0] = 0$

for $J = 1 \dots n$

$M[J] = \max\{v_J + M[p(J)],\ M[J-1]\}$

- $M[0..n]$
- have access to $p(1) \dots p(n)$

$n = 5$



$p(4) = 1$

$p(5) = 2$

$p(3) = 1$

$p(2) = 0$

$p(1) = 0$

	0	1	2	3	4	5
$J=0$	0					
$J=1$	0	10				
$J=2$	0	10	10			
$J=3$	0	10	10	23		
$J=4$	0	10	10	23	23	
$J=5$	0	10	10	23	23	25

$p(J)$

$J-1$

$M[0] = 0$

$M[1] = \max\{v_1 + M[0],\ M[0]\}$
$= \max\{10 + 0,\ 0\} = 10$

$M[2] = \max\{v_2 + M[0],\ M[1]\}$
$= \max\{9 + 0,\ 10\} = 10$

$M[3] = \max\{v_3 + M[1],\ M[2]\}$
$= \max\{13 + 10,\ 10\} = 23$

$M[4] = \max\{v_4 + M[1],\ M[3]\}$
$= \max\{5 + 10,\ 23\} = 23$

$M[5] = \max\{v_5 + M[2],\ M[4]\}$
$= \max\{15 + 10,\ 23\} = 25$

Compute an optimal solution O_J: an optimal solution for $[J]$

$n=5$ $\quad 5 \overset{?}{\in} O_5 : 25 > 23 \Rightarrow 5 \in O_5$

$p(5)=2$: Consider $O_5 \setminus \{5\} = O_2 \subseteq [2]$. $2 \overset{?}{\in} O_2 : 9 < 10 \Rightarrow 2 \notin O_2$

Consider $O_2 = O_1 = [1] \Rightarrow 1 \in O_1 \Rightarrow \{1, 5\}$ is an optimal solution

MSchedule $(n; M, p)$

If $n = 0$ return $\emptyset$
If $(v_n + M[p(n)] > M[n-1])$
    return $\{n\} \cup$ MSchedule $(p(n); M, p)$
else
    return MSchedule $(n-1; M, p)$

SUBSET SUM Problem

Input: n integers $w_1, w_2, \ldots, w_n$; $w_i > 0$

Budget $W \geq 0$

Output: A subset $S \subseteq [n]$ s.t.

(i) $\sum_{i \in S} w_i \leq W$

(ii) $\max W(S) = \sum_{i \in S} w_i$

Ex. $n = 3$; $w_A = 1$ $w_B = 3$ $w_C = 3$

(i) $W = 7 \Rightarrow$ opt soln $\{A, B, C\}$

(ii) $W = 6 \Rightarrow$ opt soln $\{B, C\}$

(iii) $W = 5 \Rightarrow$ opt solns $\{A, B\}$
$\{A, C\}$

Simpler Q: max $|S|$ (instead of $W(S)$)

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