Apr 29

<u>SAT formula</u>: AND (or conjunction) of clauses

$C_1, C_2 \ldots C_m$

$\equiv C_1 \wedge C_2 \wedge \ldots \wedge C_m$ $\qquad X = \{x_1, \ldots x_n\}$

<u>Clause:</u> OR (or disjunction) of literals

$t_1 \vee t_2 \vee \ldots t_\ell$ $\qquad$ Each $t_i \in \{x_1, \ldots x_n, \bar{x}_1, \ldots, \bar{x}_n\}$

<u>Eg.</u> $(x_1 \vee \bar{x}_2), (\bar{x}_1 \vee \bar{x}_3), (x_2 \vee \bar{x}_3)$ — (*) $m=3$

<u>Assignment:</u> $v: X \to \{0,1\}$

(2^n possible assignments)

$x_1=0$	1	0
$x_2=0$	1	0
$x_3=0$	1	1

An assignment <u>satisfies</u> a formula if the formula evaluates to true under the assignment.

$\equiv$ the assignment satisfies ALL clauses

$\to$ An assignment satisfies a clause if the clause evaluates to true under the assignment

Assignment: $(0,0,0) \Rightarrow (0,0,0)$ satisfies (*)

$(x_1 \vee \bar{x}_2) = 0 \vee \bar{0} = 0 \vee 1 = 1$

$(\bar{x}_1 \vee \bar{x}_3) = \bar{0} \vee \bar{0} = 1 \vee 1 = 1$

$(x_2 \vee \bar{x}_3) = 0 \vee \bar{0} = 0 \vee 1 = 1$

$\times\ (1,1,1) = (1 \vee \bar{1}) \wedge (\bar{1} \vee \bar{1}) \wedge (1 \vee \bar{1}) = 0$

$\times\ (0,0,1) = (0 \vee \bar{0}) \wedge (\bar{0} \vee \bar{1}) \wedge (0 \vee \bar{1}) = 0$

Q: Given a SAT formula, does it have a satisfying assignment?

eg. $\{x_1, x_2\}$

$(x_1 \vee x_2) \wedge (x_1 \vee \bar{x}_2) \wedge (\bar{x}_1 \vee \bar{x}_2) \wedge (\bar{x}_1 \vee \bar{x}_2)$

3-SAT problem: Same as the SAT problem with the extra restriction that each clause has EXACTLY 3 literals.

THM: 3-SAT $\le_p$ Independent Set

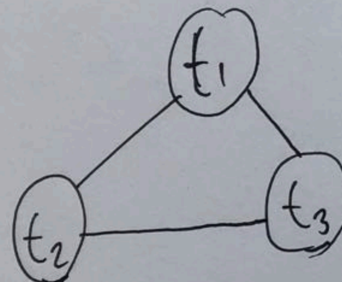
→ Use "gadgets"

2 equivalent ways of looking at 3-SAT:

→ Make independent 0/1 choices for $X_1, \ldots, X_n$ s.t. you satisfy at least one literal in each clause.

→ Pick one literal for each clause s.t. all picked literals do not conflict ← you do NOT pick both X_i & $\overline{X_i}$

gadget: $C = t_1 \vee t_2 \vee t_3$



Each of the 3 IS $\{t_1\}$ or $\{t_2\}$ or $\{t_3\}$ correspond to which literal will choose from clause C

Redux: Given a 3-SAT formula: $C_1 \ldots C_m \longrightarrow (G, m)$

Claim/design: s.t. 3-SAT formula is satisfiable $\Longleftrightarrow$ G has an ind. set of size $\ge m$

Claim: Done if we can compute such G in poly time

(G, k) for IS

Redux:

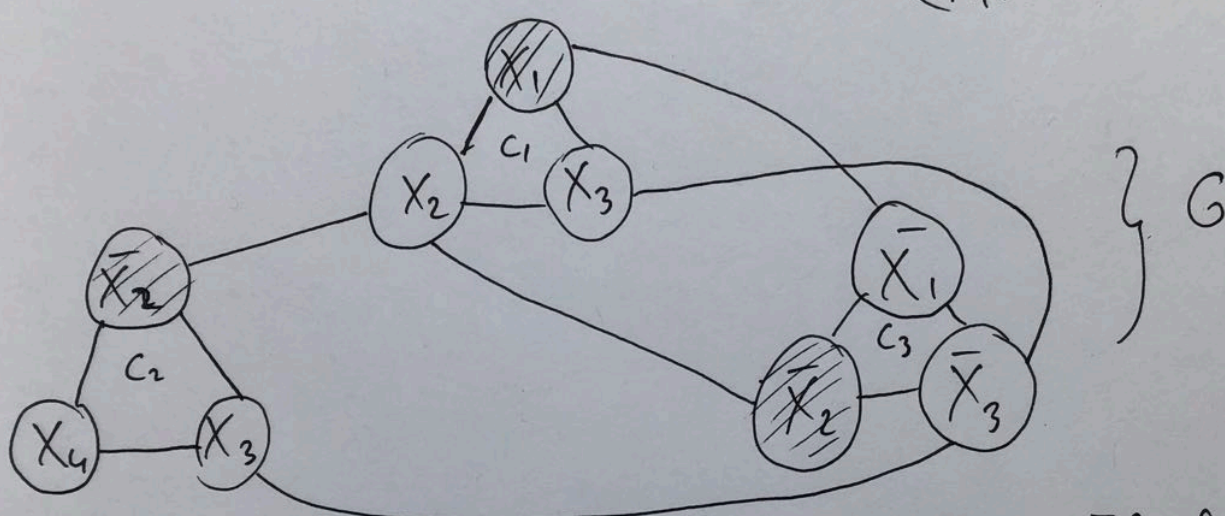
Step 1: Replace each clause C_i by its "triangle"

Step 2: Add an edge between X_i & $\bar{X}_i$ if they occur in the formula

$(X_1 \vee X_2 \vee X_3), = C_1$

$(\bar{X}_2 \vee X_3 \vee X_4), = C_2$

$(\bar{X}_1 \vee \bar{X}_2 \vee \bar{X}_3), = C_3$



To argue: 3-SAT formula is satisfiable $\iff$ G has an IS of size $\cancel{\geq}$ m $=$

Since all IS in G have size $\leq m$

Pf sketch:

$\Rightarrow$: Given SAT formula that is satisfiable

$\Rightarrow$ G has an IS of size m

each clause has at least one true literal

$\Rightarrow$ pick any one

Ex: $x_1 = 1$, $x_2 = 0$, $x_3 = 1$, $x_4 = 1$

Claim: The m literals that we pick ~~form~~ an IS in G.