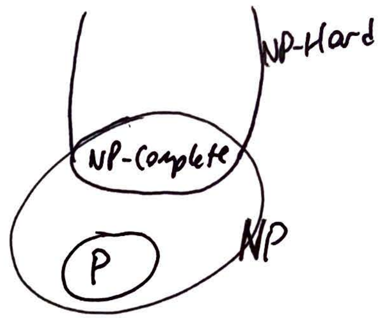
May 4

NP-Complete Problems

Def: X is NP-Complete if

(i) $X \in NP$

(ii) $\forall\, Y \in NP$, $Y \leq_p X$ $\rightarrow$ Just (ii) $\Rightarrow$ X is NP-Hard.



Lemma 1: Let X be an NP-Complete problem

If $X \in P \Rightarrow P = NP$

Lemma 2: Let Y be an NP-Complete problem. $X \in NP$

If $Y \leq_p X \Rightarrow X$ is also NP-Complete

THM: 3-SAT is NP-Complete. $\Rightarrow$ COR: IS is NP-Complete

3-SAT $\leq_p$ IS

General strategy: to prove X is NP-Complete

Step 1: $X \in NP$ ($X =$ IS)

Step 2: Identify an NP-Complete problem Y ($Y =$ 3-SAT)

Step 3: Prove $Y \leq_p X$ (3-SAT $\leq_p$ IS)

Overview of 3-SAT is NP-Complete

Step 1: Define Circuit-SAT

Step 2: Prove Circuit-SAT is NP-Complete

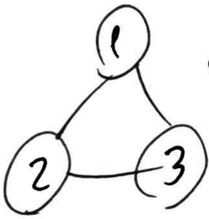
Step 3: Circuit-SAT $\leq_p$ 3-SAT

k-colorability (k-coloring)

$G=(V,E)$

Def: A k-coloring of G if $c: V \to \{1, \dots, k\}$

s.t. $\forall (u,w) \in E$, $c(u) \neq c(w)$



3-colorable but NOT 2-colorable

Def: G is k-colorable if $\exists$ a k-coloring for it.

Def: (k-coloring (k-colorability problem)

i/p: G, k

o/p: T if G is k-colorable

F o/w