Pigeon-hole principle

If $\leq n-1$ pigeons are put in n holes

$$\Rightarrow \exists \text{ at least one empty hole.}$$

Lemma 4 : At the end of ~~the~~ any iteration (of GS alg) if woman w is free $\Rightarrow w$ has NOT proposed to all men.

Pf. idea: Pf by contradiction $\left(\begin{array}{c} \text{Pigeon-hole principle +} \\ \text{Obs 1 + Alg defn} \end{array} \right)$

Pf. details : Assume free woman w who has proposed to <u>all</u> men.

$$\underset{\text{Obs 1 + Alg def}}{\Longrightarrow} \text{ all } n \text{ men are engaged } (*)$$

Since w is free $\Rightarrow n-1$ women are engaged

$$\Longrightarrow n-1 \text{ men are engaged.} \Rightarrow \text{contradicts with } (*)$$

$\left(\begin{array}{l} \text{PHP} \\ \text{hole :: men} \\ \text{pigeon : women} \\ \text{assign :: engaged} \end{array} \right.$