Lecture 21

CSE 331

Mar 24, 2021

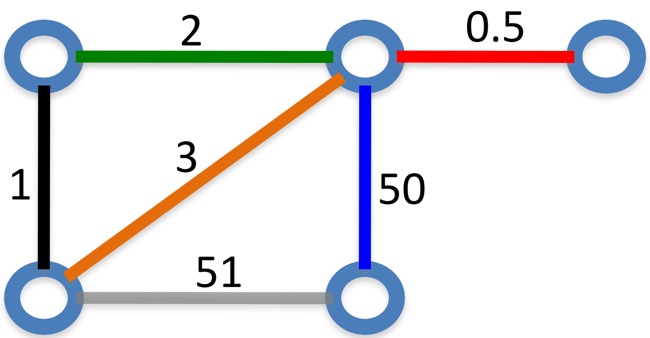
Minimum Spanning Tree Problem

Input: Undirected, connected $G = (V,E)$, edge costs c_e

Output: Subset $E' \subseteq E$, s.t. $T = (V,E')$ is connected
$C(T)$ is minimized

If all $c_e > 0$, then T is indeed a tree

Kruskal's Algorithm



Joseph B. Kruskal

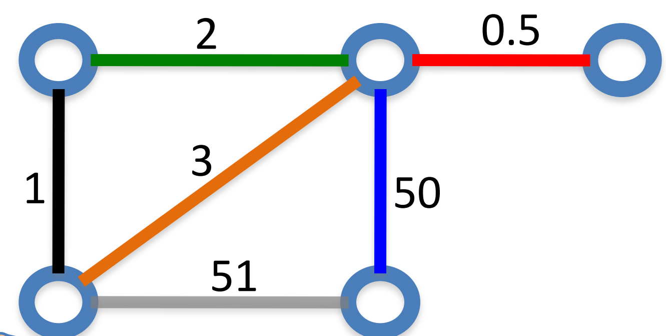
Input: G=(V,E), $c_e > 0$ for every e in E

T = Ø

Sort edges in increasing order of their cost

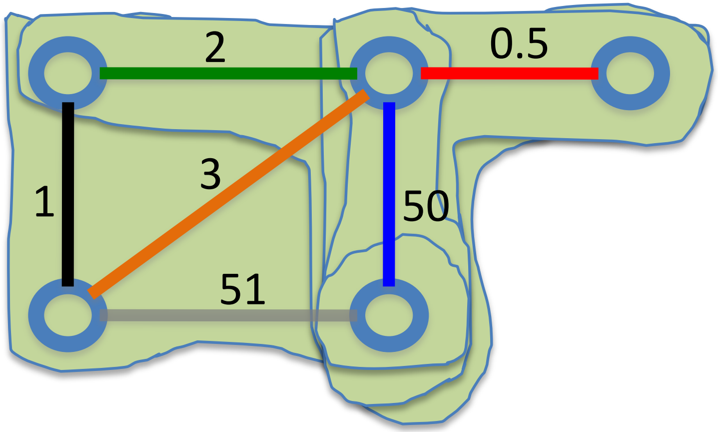
Consider edges in sorted order

If an edge can be added to T without adding a cycle then add it to T

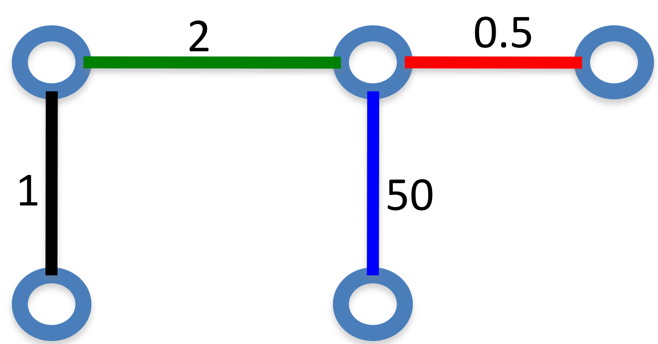


Prim's algorithm

Similar to Dijkstra's algorithm



Robert Prim



Input: G=(V,E), c_e> 0 for every e in E

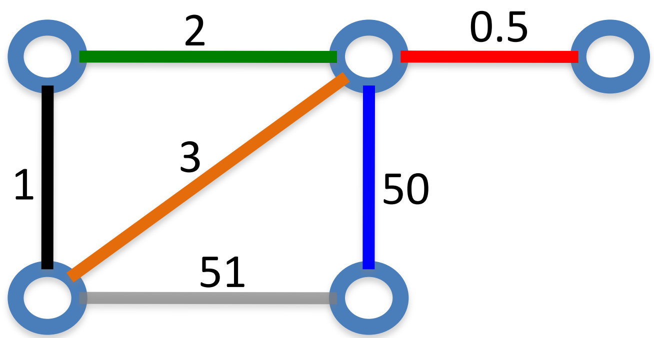
S = {s}, T = Ø

While S is not the same as V

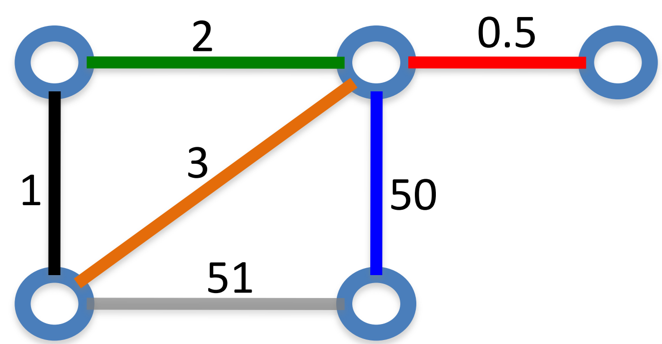
Among edges e= (u,w) with u in S and w not in S, pick one with minimum cost

Add w to S, e to T

Reverse-Delete Algorithm



Input: G=(V,E), $c_e > 0$ for every e in E



T = E

Sort edges in decreasing order of their cost

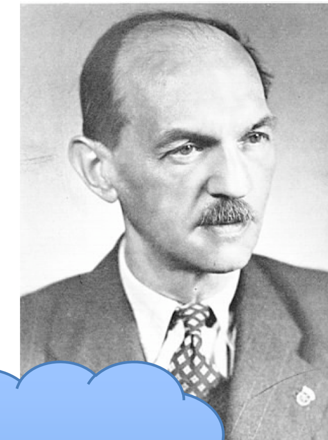
Consider edges in sorted order

If an edge can be removed T without disconnecting T then remove it

(Old) History of MST algorithms

1920: Otakar Borůvka

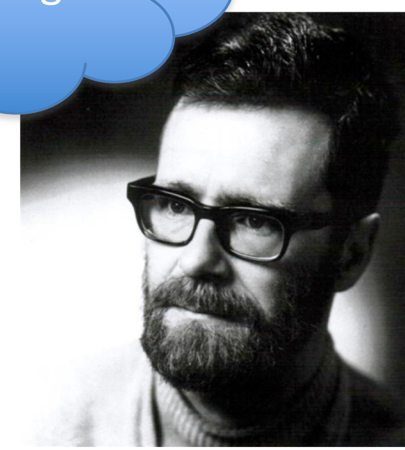
1930: Vojtěch Jarník



Same algo!

1956: Kruskal

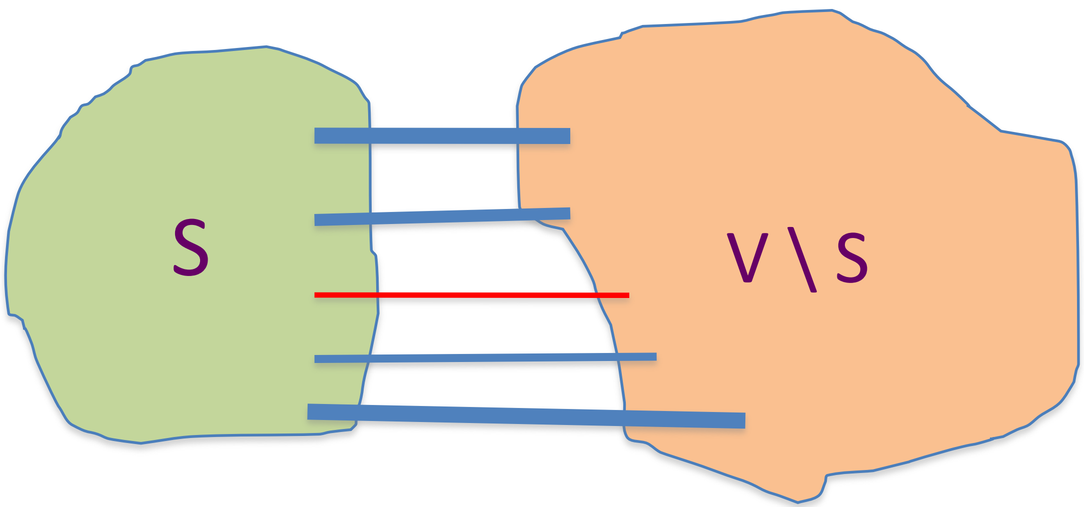
1957: Prim



1959: Dijkstra

Cut Property Lemma for MSTs

Condition: S and V\S are non-empty



Cheapest crossing edge is in **all** MSTs

Assumption: All edge costs are distinct

Today’s agenda

Optimality of Prim’s algorithm

Prove Cut Property Lemma

Optimality of Kruskal’s algorithm

Remove distinct edge weights assumption