Lecture 5

CSE 331

Feb 10, 2021

Stable Matching

A perfect matching with no instability

What is instability?

Given

- 2n preference lists; L_m for each man m, L_w for each woman m
- a perfect matching S

A pair (m, w) $\notin$ S is instability, if

- m is ranked before w's husband in L_w

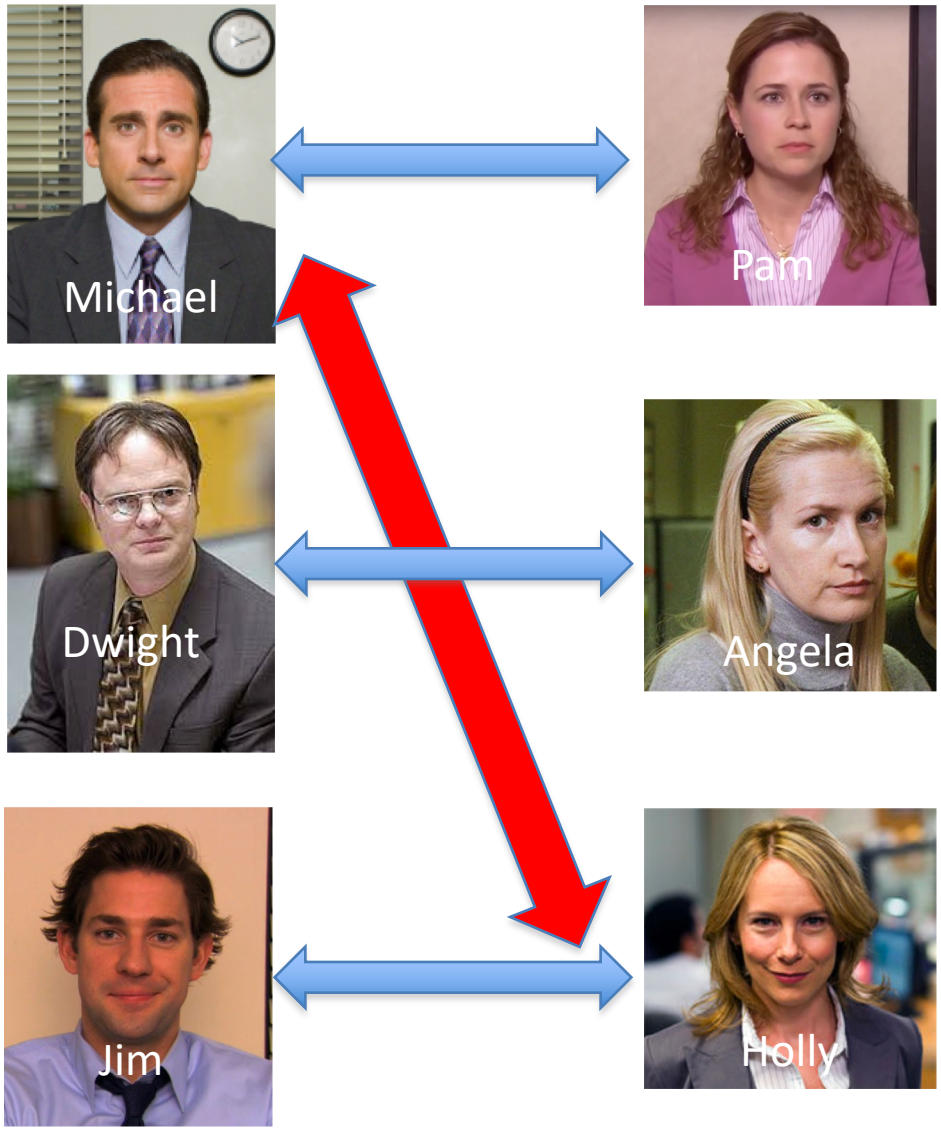
AND

- w is ranked before m's wife in L_m

I.e., if a man AND a woman would be both happier in a new marriage

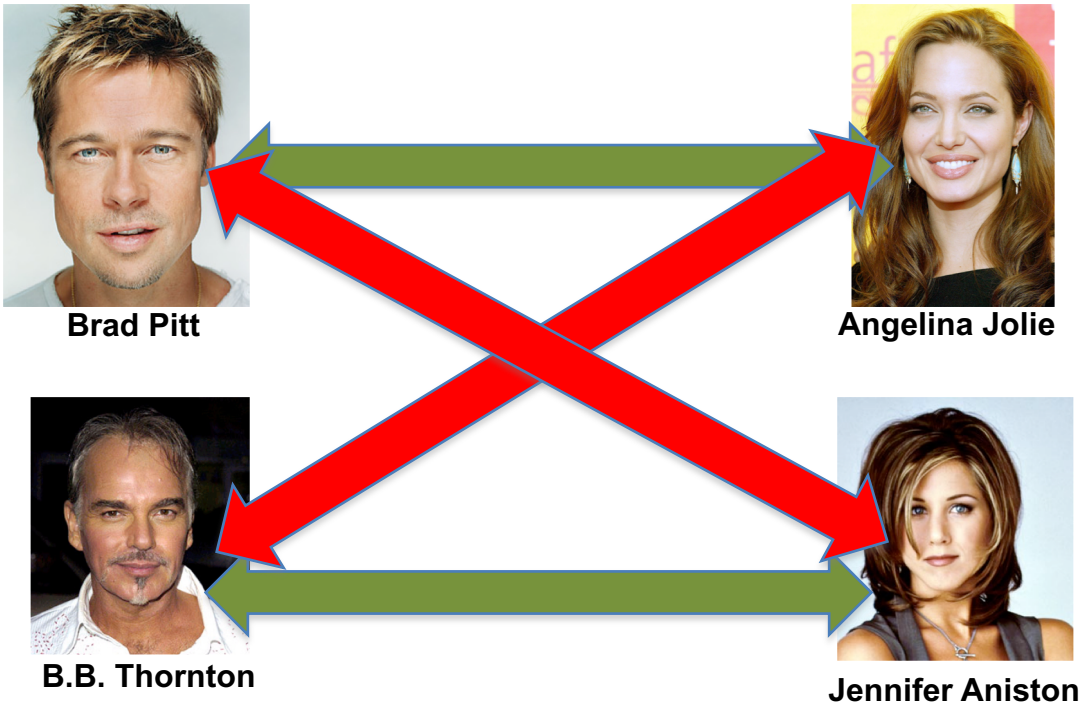
Preferences

Instability

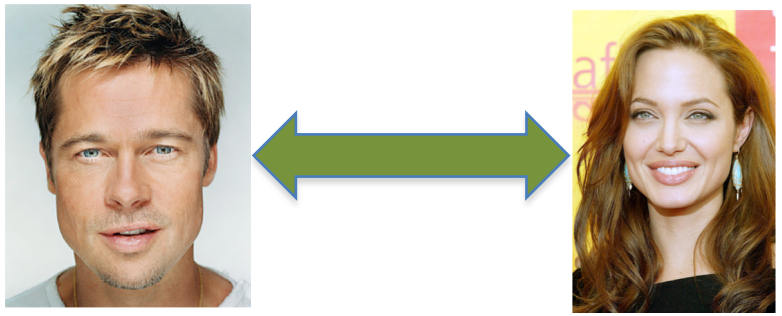


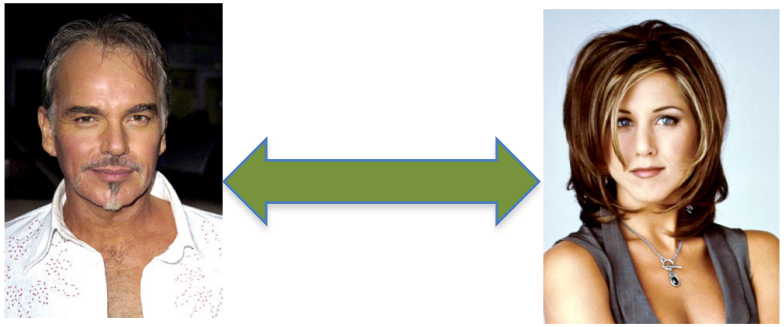
A stable marriage

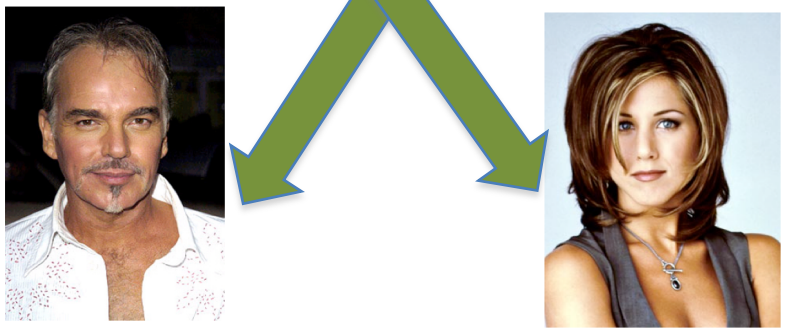
Even though BBT and JA are not very happy



Two stable marriages possible!







Stable Marriage problem

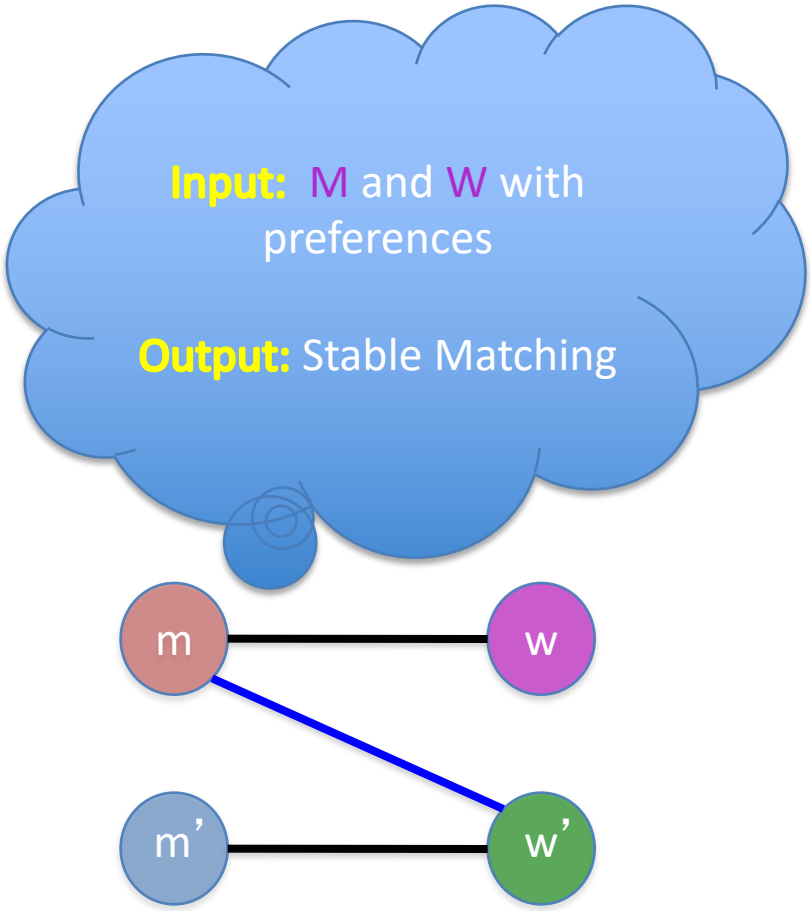
Set of men M and women W

Preferences (ranking of potential spouses)

Matching (no polyandry/gamy in M X W)

Perfect Matching (everyone gets married)

Instability



Stable matching = perfect matching + no instability

Questions/Comments?

Two Questions

Does a stable marriage always exist?

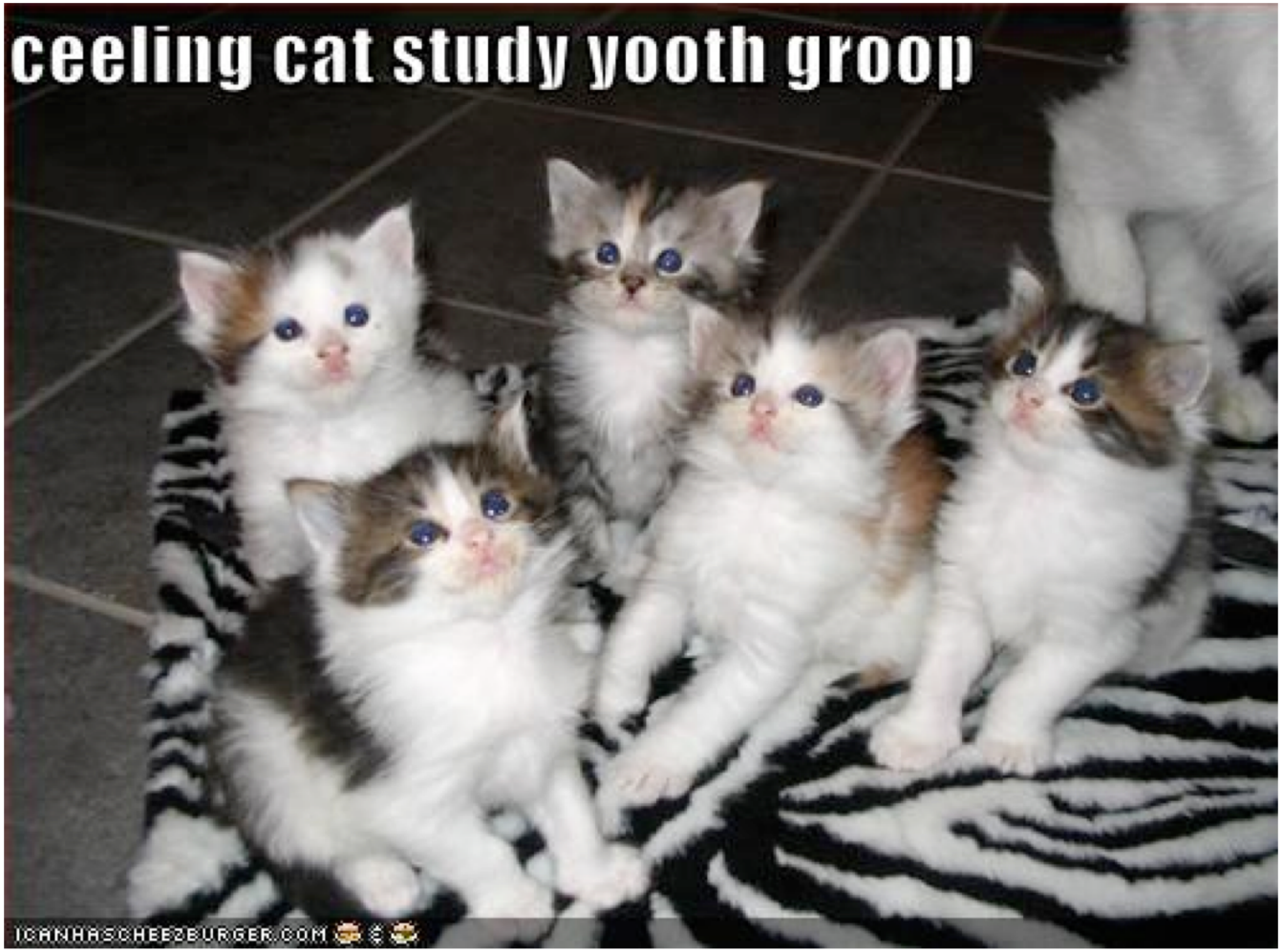
If one exists, how quickly can we compute one?

Today's lecture

Naïve algorithm

Gale-Shapley algorithm for Stable Marriage problem

Discuss: Naïve algorithm!



The naïve algorithm

Incremental algorithm to produce all n! perfect matchings?

Go through all possible perfect matchings S

If S is a stable matching

then Stop

Else move to the next perfect matching

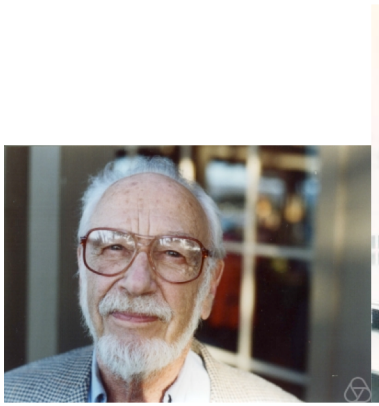
Gale-Shapley Algorithm

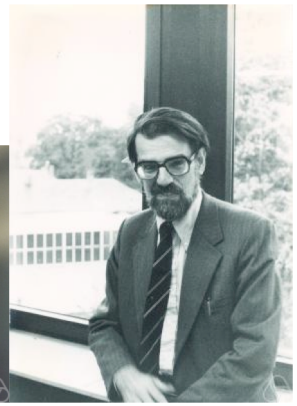
David Gale

Lloyd Shapley

$O(n^3)$ algorithm

Moral of the story…





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Questions/Comments?

Rest of today's agenda

GS algorithm

Run of GS algorithm on an instance

Prove correctness of the GS algorithm