Feb 24

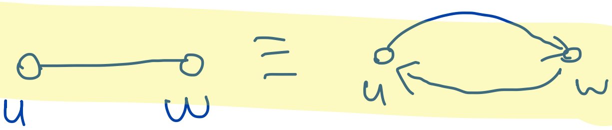
$G = (V, E)$ $E \subseteq V \times V$

V: set of vertices/nodes

E: set of edges

Default: $n = |V|$; $m = |E|$

Def: G is undirected $\iff \forall u, w \in V, (u,w) \in E \iff (w,u) \in E$ s.t. $u \neq w$



o.w. G is directed

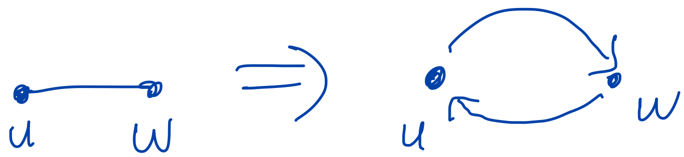
(.) Friendship network (u)

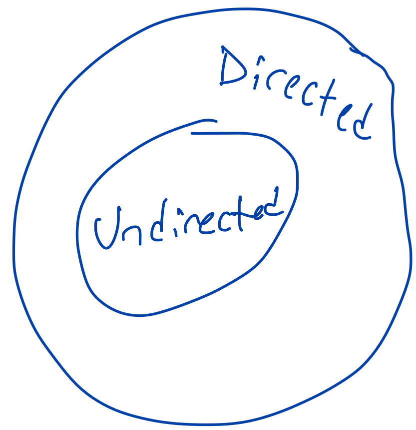
(.) Wikipedia page (d)

Default: G is undirected

Claim: Every undirected graph is also directed.

Pf. idea



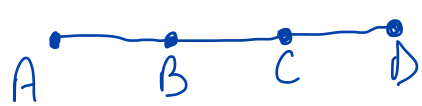


Def: A path in $G=(V,E)$ is sequence of vertices $u_1, u_2 \ldots u_k$ s.t. $\forall i \in [k-1]$,

$$(u_i, u_{i+1}) \in E$$

Notes: (i) u_i need not be distinct (ii) holds for directed graphs

---



D, C, B, A ✓	D, C, B, A ✗
A, B, C, D ✓	A, B, C, D ✓
A, B, C, B ✓	A, B, C, B ✗
A, C, D ✗	A, C, D ✗

Def: A simple path does NOT have any repeated vertices.

By default: All paths are simple.

Def: length of a path = # edges in the path

$len(A, B, C, D) = 3$

Q: What is the max length of a (simple) path of n nodes?

A: $n-1$ Ex (hint: use PHP)

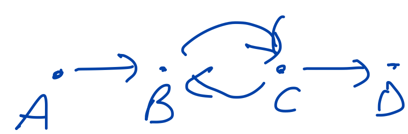
Def: A cycle $u_1, u_2, \ldots, u_k$ is a path s.t.

(1) $u_1, \ldots, u_{k-1}$ are distinct

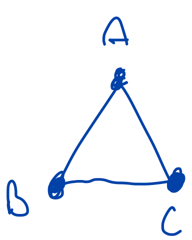
(2) $u_1 = u_k$

(3) . G is undirected: $k \geq 4$

. G is directed: $k \geq 3$

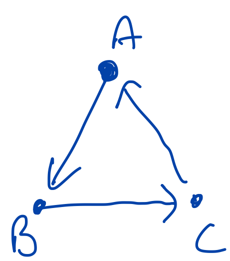


B, C, B ✓



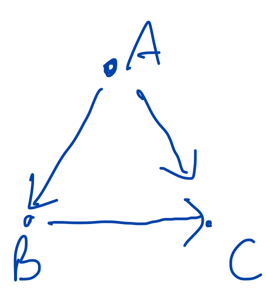
A, C, B, A ✓

A, B, C, A ✓



A, C, B, A ✗

A, B, C, A ✓



A, C, B, A ✗

A, B, C, A ✗

⇒ Directed Acyclic Graph (DAG)

Def: u & w are connected if $\exists$ u-w path (undirected)

u & w are strongly connected if $\exists$ u-w path
w-u path

Def: A (directed) graph is (strongly) connected if $\forall$ $u \neq w$, u & w are (strongly) connected