Feb 26

<u>Prop:</u> Let T be a BFS tree for $G = (V, E)$

If $(u,w) \in E$ s.t. $u \in L_i$ and $w \in L_j$

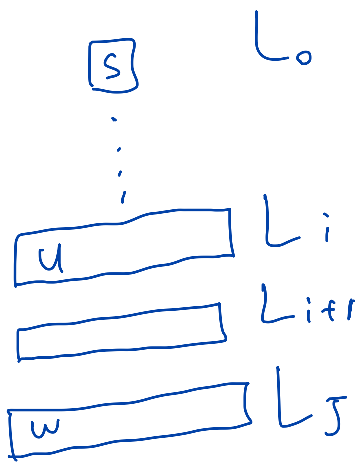
$\Rightarrow |i - j| \leq 1 \iff i \in \{j-1, j, j+1\}$

<u>Pf idea:</u> Pf by contradiction.

w.l.o.g. assume $i \leq j$ [o.w. switch i & j]

for contradiction assume $|i-j| > 1 \Rightarrow j > i+1$

$\iff j \geq i+2$



Consider situation when BFS is creating L_{i+1}

$\Rightarrow u \in L_i$, $w \notin L_0, L_1, \ldots, L_{i-1}$

$\Rightarrow (u,w) \in E$

$\Rightarrow$ By BFS defn. $w \in L_{i+1}$

$\Rightarrow$ contradicts $w \in L_j$ for $j \geq i+2$ ∎

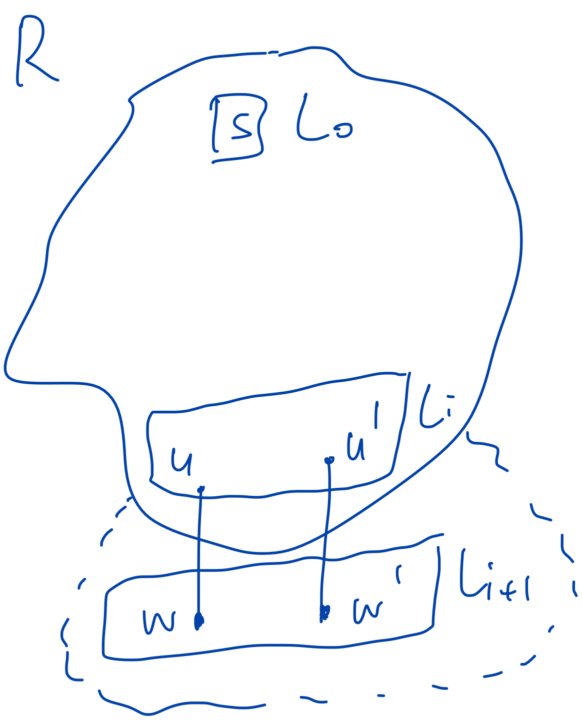
Explore(s)

0. $R = \{s\}$
1. While $\exists (u,w) \in E$ s.t. $u \in R$ and $w \notin R$

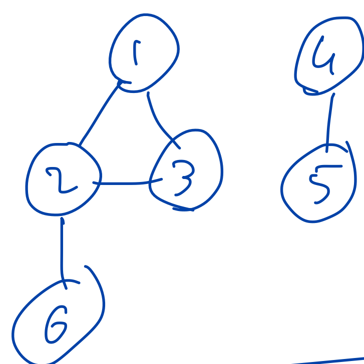
   Add w to R

2. Output $R^* = R$

Lem 0: Explore always terminates } Ex.



Def: Set of all vertices connected to s is called its connected component $CC(s)$



$CC(2) = \{2, 1, 3, 6\}$

$CC(4) = \{4, 5\}$

Theorem: For all G, start vertices s, $R^* = CC(s)$

General trick: to show $A = B \iff A \subseteq B$ and $B \subseteq A$

Lemma 1: $R^* \subseteq CC(s)$

Lemma 2: $CC(s) \subseteq R^*$

→ Ex: By induction

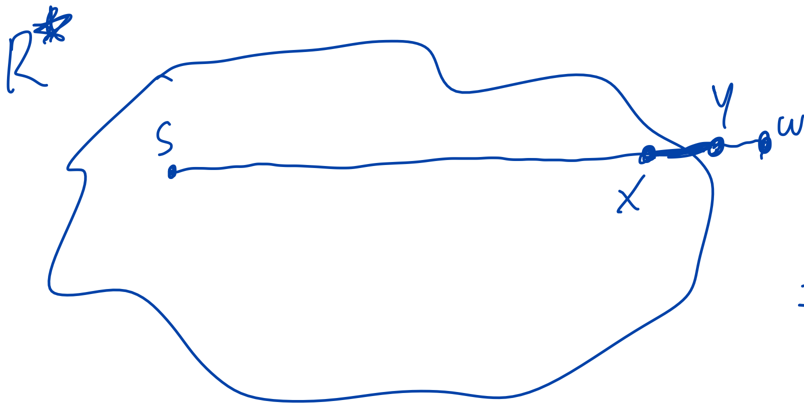
Lemma 1 + Lemma 2 ⇒ Thm

Thm ⇒ BFS is correct

Pf. idea of Lemma 2: Pf. by contradiction

Assume $CC(s) \not\subseteq R^*$ $\Rightarrow \exists\, w \in CC(s)$ BUT $w \notin R^*$

$\Leftrightarrow \exists$ s–w path p in G but $w \notin R^*$
[$w \in CC(s)$]



Since p starts inside of R^* but ends outside of R^*

$\Rightarrow \exists\, (x,y) \in p$ s.t. $x \in R^*, y \notin R^*$

$\Rightarrow y$ should have been added to R by Explore

$\Rightarrow$ Algo has not terminated

$\Rightarrow$ contradicts with the existence of R^* ■