

Mar 3

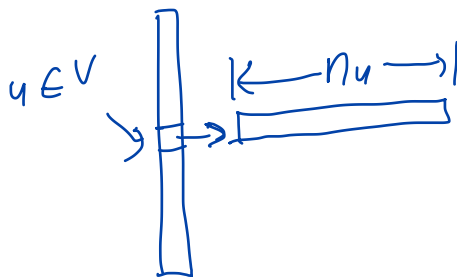
$$2m = \sum_{u \in V} n_u$$

$$n_u = \# \text{ neighbors of } u = \left| \{w \mid (w,u) \in E\} \right|$$

(degree)

Adj. list

$$\# \text{ pointers} = |V| = n$$



$$\text{Sum of lists} = \sum_{u \in V} n_u = 2m$$

$$\text{Overall space} = n + 2m = O(n + m)$$

$\downarrow \quad \downarrow$
 $O(n) \quad O(n^2)$

$$0 \leq m \leq \binom{n}{2}$$

\downarrow
 $\frac{n \cdot (n-1)}{2}$
 $\leq O(n^2)$

BFS (G, s) // G is in adj. list format

- $O(n)$ {
0. $CC[s] = T$ and $CC[u] = F \quad \forall u \neq s \in V$
 1. $i = 0$
 2. $L_0 = \{s\}$
 3. While $L_i \neq \emptyset$ $\rightarrow T_1: \# \text{ times this loop is run}$
 - 3.1. $L_{i+1} = \emptyset \rightarrow O(1)$
 - 3.2. For all $u \in L_i$ $\rightarrow T_{12}: \# \text{ alg gets here}$
 for all $(u, w) \in E$ $\rightarrow T_{123}: \# \text{ alg gets here}$

$\left. \begin{array}{l} \text{if } CC[w] == F \\ \quad CC[w] = T \\ \quad \text{Add } w \text{ to } L_{i+1} \end{array} \right\} O(1)$
 - 3.3. $i++ \rightarrow O(1)$

$O(n) \leftarrow$ 4. Return CC

(pass by value)

$$\begin{aligned}
 \text{Total runtime} &= O(n) + \underset{\substack{\uparrow \uparrow \\ 3.1 \quad 3.3}}{T_1} \cdot O(1) + \underset{\substack{\uparrow \\ 3.2.}}{T_{123}} \cdot O(1) + O(n) \\
 &\leq O(n) + T_{123} \cdot O(1) + T_{123} \cdot O(1) = \underline{O(n) + O(T_{123})}
 \end{aligned}$$

Goal: Bound total

Analysis 1: $T_{123} = O(n^3) \Rightarrow$ overall $O(n) + O(n^3) = O(n^3)$

Analysis 2: $T_{123} \leq n^2$ (Obs. every vertex u appears in ≤ 1 L_i)

\uparrow
Claim: $T_{12} \leq n \iff T_{123} \leq T_{12} \cdot n = n \cdot n = n^2$