

Mar 10

Interval Scheduling Runtime Analysis

Recall $f(1) \leq f(2) \dots \leq f(n)$

Greedy alg.

$\mathcal{O}(n)$ { 0. $R = [n] \leftarrow \mathcal{O}(n)$
1. $S = \emptyset \leftarrow \mathcal{O}(1)$

2. While $R \neq \emptyset \leftarrow \leq n$

$\mathcal{O}(n)$ { $\mathcal{O}(n) \rightarrow$ (2.1) Pick $i \in R$ with the smallest index
 $\mathcal{O}(1) \rightarrow$ (2.2) Add i to S
 $\mathcal{O}(n) \rightarrow$ (2.3) Remove all j that conflict with i from R

3. Return $S^* = S \leftarrow \mathcal{O}(n)$

Overall runtime : $\mathcal{O}(n) + n \cdot \mathcal{O}(n) + \mathcal{O}(n)$
 $= \mathcal{O}(n) + \mathcal{O}(n^2) + \mathcal{O}(n)$
 $= \mathcal{O}(n^2)$

\hookrightarrow can we do better?

Shortest path problem

Input: Directed graph $G = (V, E)$
 $s \in V$

"length" $\rightarrow l_e \geq 0 \quad \forall e \in E$
 \uparrow
integer

Output: $\forall t \in V$, output $\stackrel{a}{=}$ shortest s - t path
 \uparrow
w.r.t. the length of the path

$$l(P) = \sum_{e \in P} l_e$$

Simpler version: Only output $d(t) \quad \forall t \in V$
 \uparrow
length of the shortest path