Mar 12

$\boxed{\text{Special case}}$: $\ell_e = 1 \quad \forall e \in E$

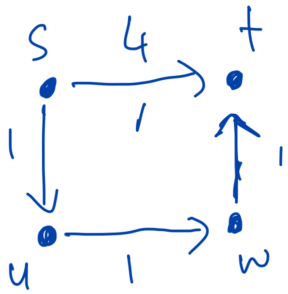
[also if $\ell_e = L \quad \forall e \quad (L \geq 1)$]

[Run BFS on s & layers of # $t = d(t)$]

<u>General case</u>: $\ell_e > 0 \quad \forall e \in E$

Idea: Reduce this to the case of $\ell_e = 1 \quad \forall e$

<u>Idea 1</u>: Ignore ℓ_e (i.e. just assume $\ell_e = 1 \ \forall e$ & run alg. from special case)

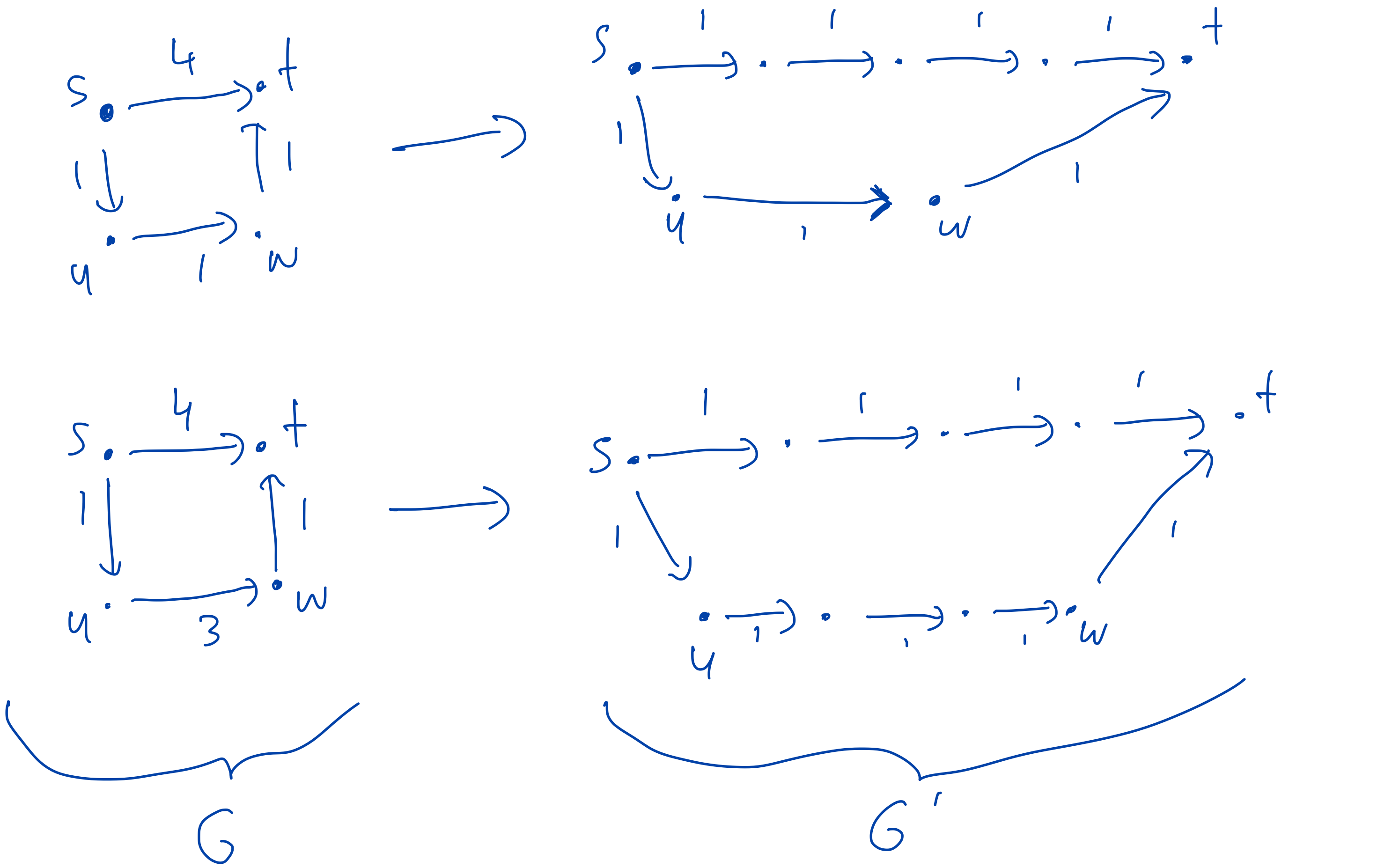


$d(t) = 3$

$\xrightarrow[\text{ignore } \ell_e]{\text{Run}}$ $s \longrightarrow t$

mistakenly claims $d(t) = 4$

Idea 2: Replace each edge $e \in E$ by a path of length l_e



Claim: a shortest path in $G \iff$ equivalent shortest path in G'

$\Longrightarrow$ Run alg. from special case on G'

Correctness: Claim + correctness of alg. from special case.

Runtime analysis: $l_{max} = \max_{e \in E} l_e$

Let n' & m' be the #nodes, #edges in G'

$O(n' + m')$

$= O(l_{max} \cdot (n+m))$

$m' \leq l_{max} \cdot (n+m)$

$n' \leq l_{max} \cdot (n+m)$

Recap/Aside: <u>RAM model</u>: unit space is a register

If you have n items, each register has $O(\log n)$ bits.

$\rightarrow$ All basic ops on $O(1)$ registers is $O(1)$ time.

$\ell_{max} = n^{100} \Rightarrow 100 \cdot \log n$ bits $\Rightarrow 100$ registers

Input size (# registers) $= O(m+n)$

runtime $= O(n^{100} \cdot (n+m))$

Assume $\ell_{max} = n^{O(1)} \Rightarrow O(1)$ registers for each ℓ_e

$\Rightarrow$ length $= O(m+n)$

<u>WANT</u>: Ideally runtime of $O(m+n)$