

Mar 15

# Dijkstra's algorithm

$$d'(w) = \min_{\substack{u \in R \\ u, w \in E}} \{d(u) + l_{u,w}\}$$

0.  $R = \{s\}$ ,  $d(s) = 0$

1. While  $\exists (u, x) \in E$  s.t.  $u \in R$  and  $x \notin R$

Pick  $w$  among all such  $x$ 's with the smallest  $d'(w)$  value

Add  $w$  to  $R$

$$d(w) = d'(w)$$

Def: Let  $P_u$  be the  $s$ - $u$  path in "Dijkstra tree"

THM:  $\forall u \in V$ ,  $P_u$  is the  $\rightarrow^a$  shortest  $s$ - $u$  path

$\Rightarrow$   $d(u)$  values are computed correctly  $\Rightarrow$  Dijkstra is correct

$\uparrow$   
Ex.

Lemma 1: At the end of each iteration of the while loop  
 $\forall u \in R$ ,  $P_u$  is  $\underline{a}$  shortest  $s$ - $u$  path

Lemma 2:  $u \in U$  s.t.  $\exists s$ - $u$  path  $\Leftrightarrow u \in R$  at the end

$\uparrow$  Ex.

Lemmas 1+2  $\Rightarrow$  THM