Mar 22

Proof of Lemma 1

Lemma 1: At the end of each iteration of the while loop $\forall u \in R$, P_u is a shortest s-u path

By induction on $|R|$

Base case: $|R| = 1$, $R = \{s\}$, $d(s) = 0$

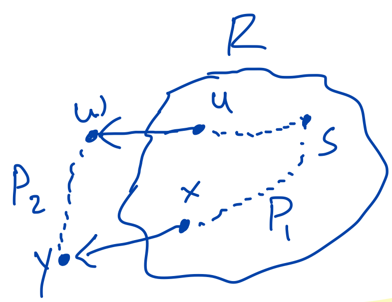
I.H. Assume Lemma is true for $|R| = k$ $(k \geq 1)$

I.S. Argue $|R| = k+1$

Assume w is $(k+1)^{th}$ vertex added to R.

Assume w is "discovered from" u $(d(w) = d(u) + \ell_{(u,w)})$

$P_w = P_u, w$



Goal: Argue P_w is a shortest s-w path

Pf: By contradiction.

$\exists$ a s-w path P'_w s.t. $\ell(P'_w) < \ell(P_w)$ $(*)$

As $s \in R$ and $w \notin R \Rightarrow P'_w$ "crosses" R at some point

$\Rightarrow \exists\, x \in R, y \notin R$ s.t. $(x,y) \in E$

$P'_w = P_1, x, y, P_2 \Rightarrow \ell(P'_w) = \ell(P_1) + \ell_{(x,y)} + \ell(P_2)$

def. of $d(x)$ $\longrightarrow$ $\geq \underbrace{d(x) + \ell_{(x,y)}} + \ell(P_2)$

$\geq d'(y) + \ell(P_2)$

$$d'(y) \geq d'(w) = d(w) = \ell(P_w)$$

since w is chosen before y

alg. defn

$\ell(P'_w) \geq \ell(P_w) + \ell(P_2) \Rightarrow$ contradiction with $(*)$ $\blacksquare$

Minimum Spanning Tree (MST)

Input: $G = (V, E)$, $c_e \geq 0 \;\; \forall e \in E$

connected, undirected

[for convenience only]

Output: (i) $E' \subseteq E$ s.t. $T = (V, E')$ is connected

sub-graph of G

(ii) min $c(T)$

$$\Downarrow \quad c(T) = \sum_{e \in E'} c_e$$