Mar 26

THM: Kruskal's algo is correct.

(Consider all edges in increasing order of c_e & add e if adding it does not introduce a cycle)

Pf. idea: Consider the case when $e=(u,w)$ is being added to T.

Goal: Show e is the cheapest crossing edge for some cut $(S, V\setminus S)$

Q: What is S?

A: Let S be the vertices connected to u using ONLY the edges in T so far.

Perturbation Trick

Assume all c_e's are integers (Ex: without this assumption)

Idea: Add to i^{th} edge an extra $\frac{i}{2 \cdot m \cdot n}$ $1 \le i \le m$

$$c'_{e_i} = c_{e_i} + \frac{i}{2 \cdot m \cdot n}$$

Ex. All c'_e's are distinct

Q: By how much does the MST cost change?

edges in tree T is $n-1$

$\Rightarrow$ max change in $c(T) \le (n-1) \cdot \frac{m}{2m \cdot n}$

$$= \frac{n-1}{2n} \le \frac{n}{2n} = \frac{1}{2}$$

$\Rightarrow$ cannot "confuse" 2 spanning trees of diff. costs since

if $c(T_1) \neq c(T_2) \Rightarrow |c(T_1) - c(T_2)| \ge 1$

as all c_e are integers.