Apr 5

Closest pair of points

Input: n points: $p_1, p_2, \dots, p_n$; $p_i = (x_i, y_i)$

Output: p_i p_j s.t. $d(p_i, p_j)$ is minimized

$$d(p_i, p_j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

ASSUMPTIONS:

(i) Given p_i, p_j ; can compute $d(p_i, p_j)$ in $O(1)$ time

$\rightarrow$ WLOG can ignore the $\sqrt{}$ (square root)

$d(p_i, p_j)$ is min $\Longleftrightarrow$ $d(p_i, p_j)^2 \rightarrow (x_i - x_j)^2 + (y_i - y_j)^2$

(ii) All the x_i s are distinct
All the y_i s are distinct

If not
(i) "rotate" all points slightly
(ii) modify the subsequent alg. to handle the general case

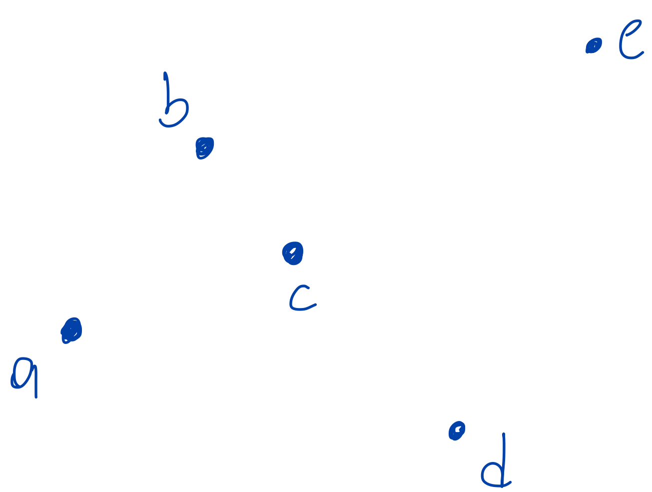
Notation:

P be the set of points

$\Theta(n \cdot \log n)$ by sorting

P_x: points in P sorted in increasing order of x values

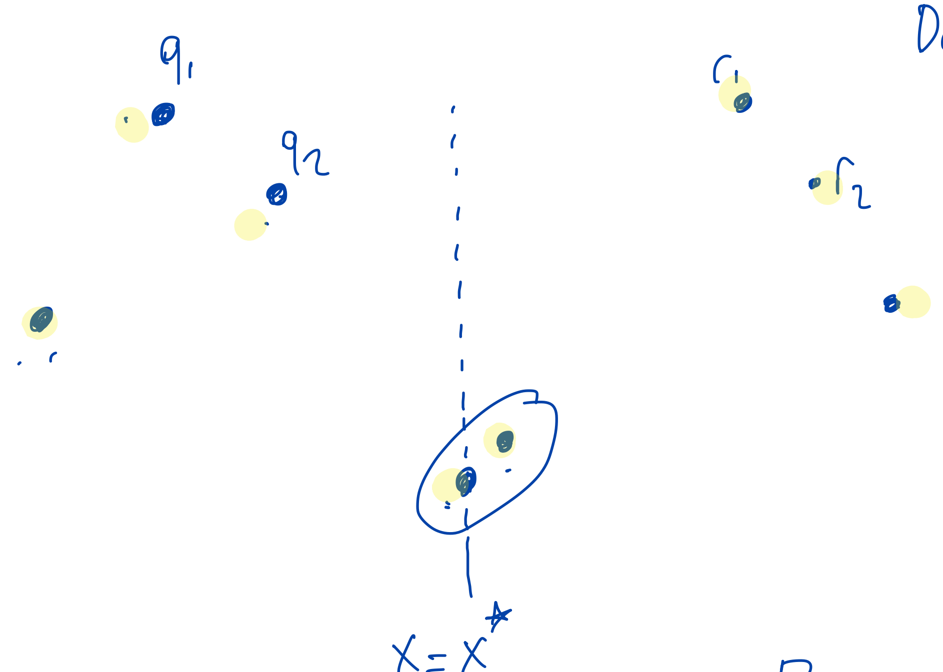
P_y: points in P sorted in increasing order of y values



$P_x = [a, b, c, d, e]$

$P_y = [d, a, c, b, e]$

$n = 8$



Define

$(x^*, y^*) = P_x\left[\left\lceil \frac{n}{2} \right\rceil\right]$

$Q = \{(x, y) \in P \mid x \leq x^*\}$

$R = \{(x, y) \in P \mid x > x^*\}$

By recursion find (i) $q_1, q_2 \rightarrow$ closest pair of points in Q

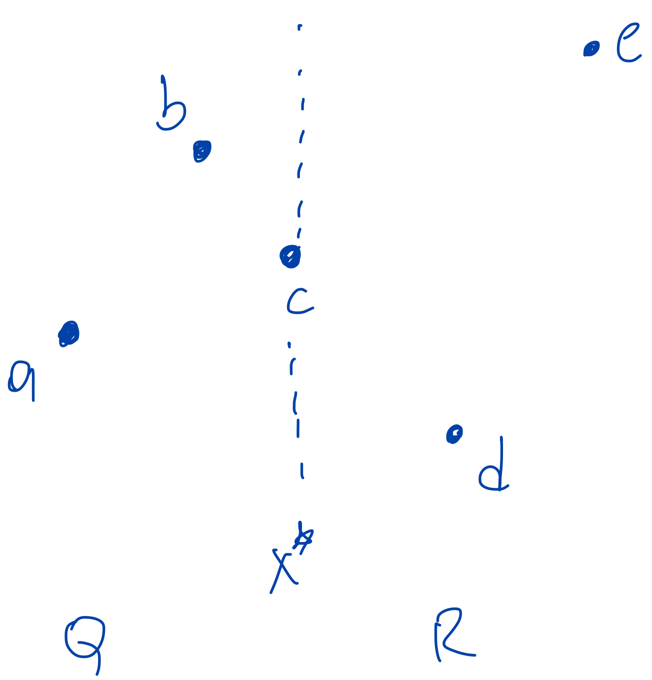
(ii) $r_1, r_2 \rightarrow$ closest pair of points in R

<u>ASIDE</u> : Given P_x, P_y; compute Q_x, Q_y, R_x, R_y in $O(n)$ time

$$Q_x = P_x\left[1 : \left\lceil \frac{n}{2} \right\rceil\right] \qquad R_x = P_x\left[\left\lceil \frac{n}{2} \right\rceil + 1 : n\right]$$

$\rightarrow$ scan (x, y) in order of P_y; if $x \leq x^*$, add (x, y) to Q_y

else, add (x, y) to R_y



$P_x = [a, b, c, d, e]$

$P_y = [d, a, c, b, e]$

$Q_x = \{a, b, c]$

$R_x = [d, e]$

$Q_y = [a, c, b]$

$R_y = [d, e]$