Apr 7

KICKASS PROPERTY LEMMA

$S = \{(x,y) \in P \mid |x - x^*| \le \delta\}$

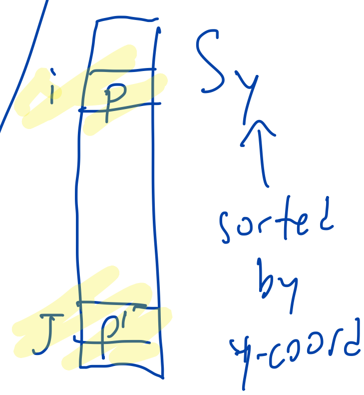
For every $p \neq p' \in S$ s.t. $d(p,p') < \delta$

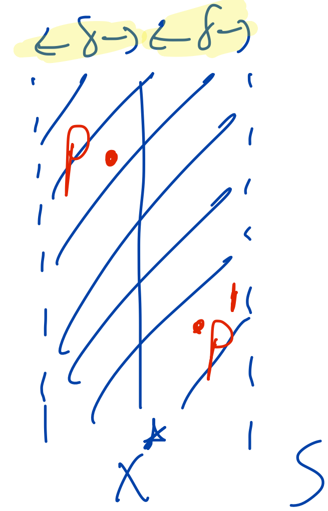
s.t. $S_y[i] = p$

$S_y[j] = p'$

then

$|i - j| \le 15$





Note: (1) "15" can be made to "9" (can be as small as 7)

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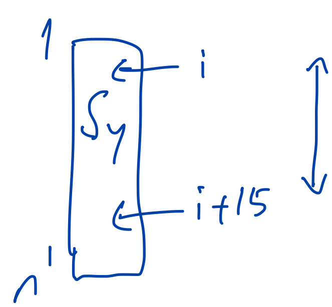
for $i = 1 \dots n' - 1$

Let (p_i, p_i') among $(S_y[i], S_y[i+1]), (S_y[i], S_y[i+2])$ .... $(S_y[i], S_y[\underbrace{i+15}_{\min(i+15,\, n')}])$ with the smallest distance



Let (p, p') be the closest pair of points among $(p_1, p_1'), (p_2, p_2'), \dots, (p_{n'}, p_{n'}')$

If $d(p, p') < \delta$

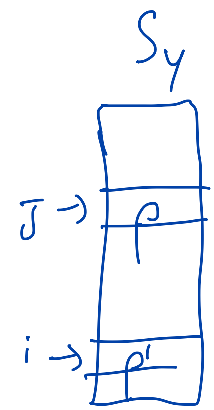
return (p, p')

else

return NULL

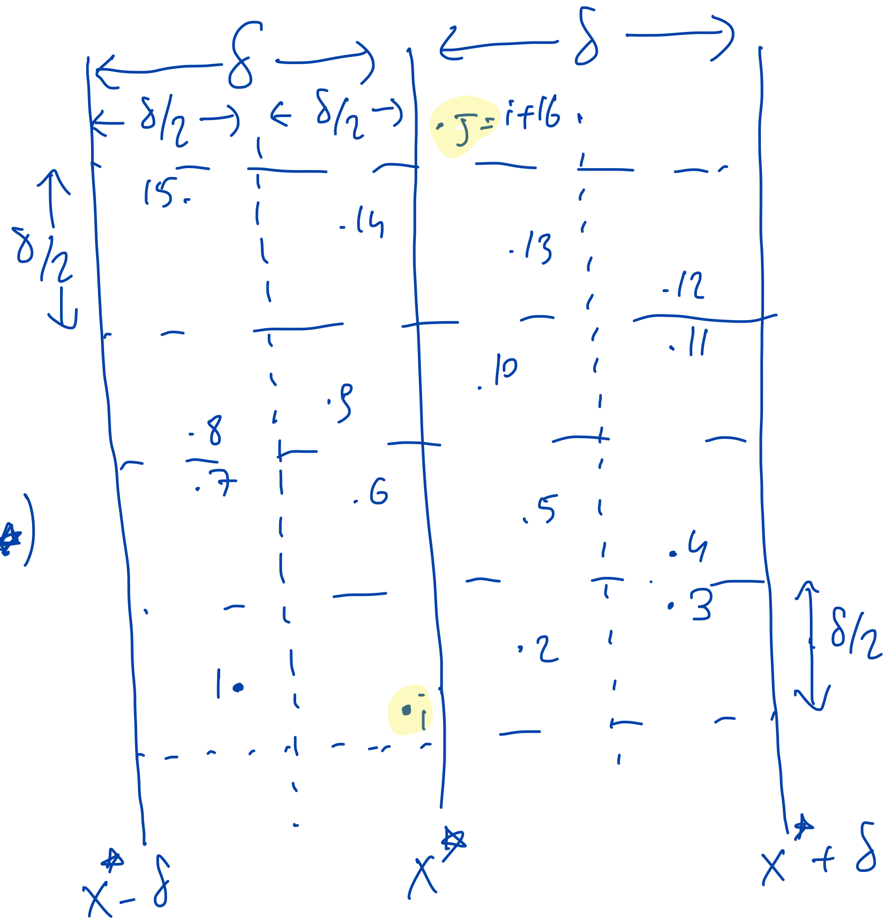
Pf (idea) of Kickass Property Lemma:

For contradiction assume $|i - j| \geq 16$



$d(p,p') < \delta$

$(\bigstar)$



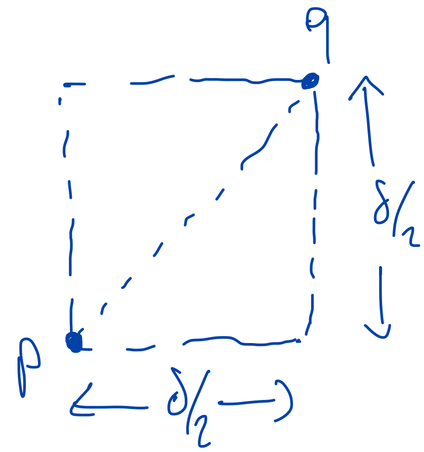
$d(p,p') \geq \frac{3\delta}{2}$

$> \delta$

$\Rightarrow$ contradicts $(\bigstar)$

Claim: Every $\frac{\delta}{2} \times \frac{\delta}{2}$ square has at most 1 pt from S in it.

pf (idea): If not assume $\exists$ pts p & q inside one square



$$d(p,q) = \sqrt{\left(\frac{\delta}{2}\right)^2 + \left(\frac{\delta}{2}\right)^2}$$

$$= \sqrt{\frac{\delta^2}{4} + \frac{\delta^2}{4}}$$

$$= \sqrt{\frac{\delta^2}{2}} = \frac{\delta}{\sqrt{2}} < \delta$$

contradicts the defn of δ as each square is in Q or R.