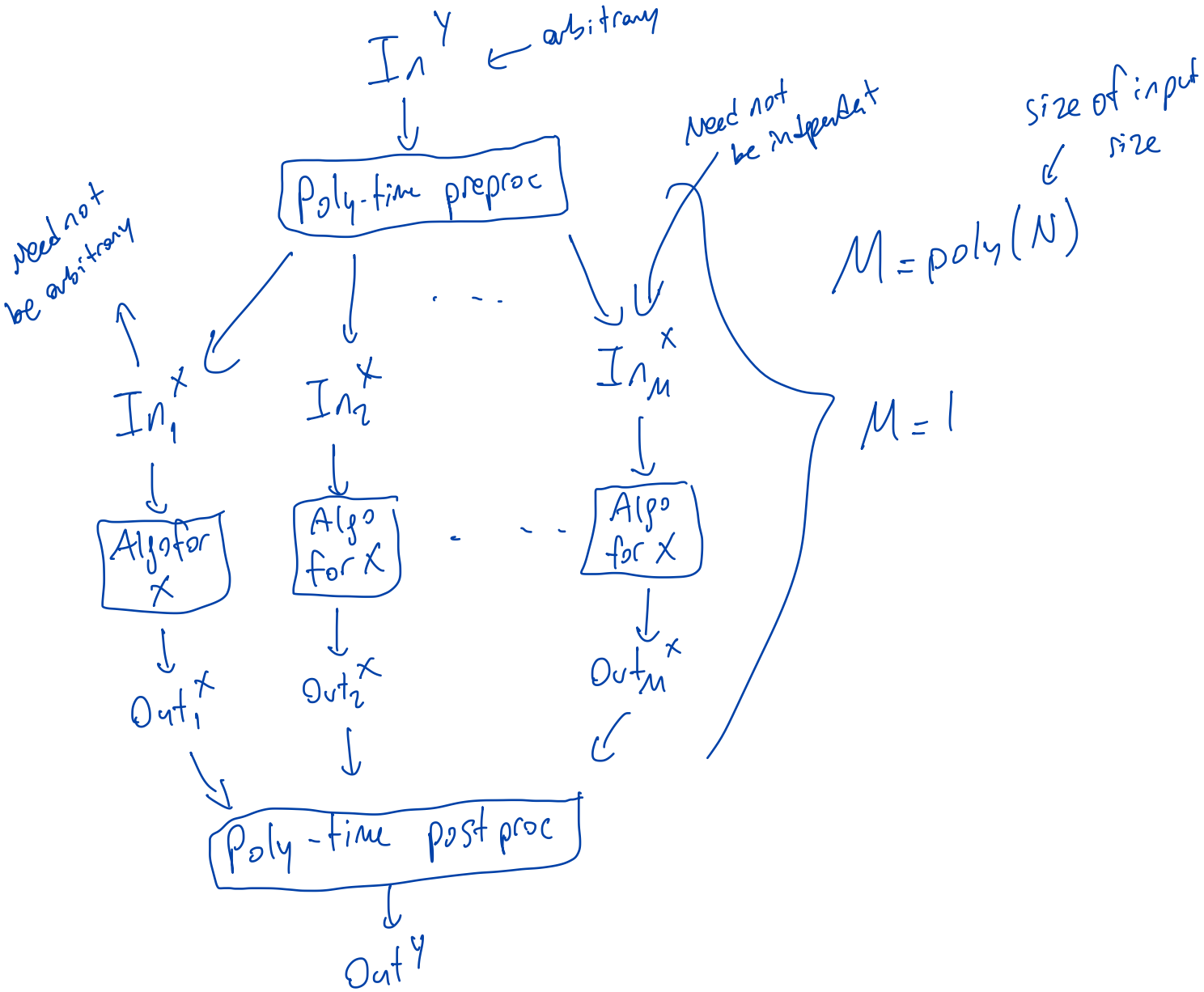
Apr 27

$Y \leq_p X$

→ Y is poly time reducible to X

≡ poly time reduction from Y to X

Solve In^Y - - - - → Out^Y

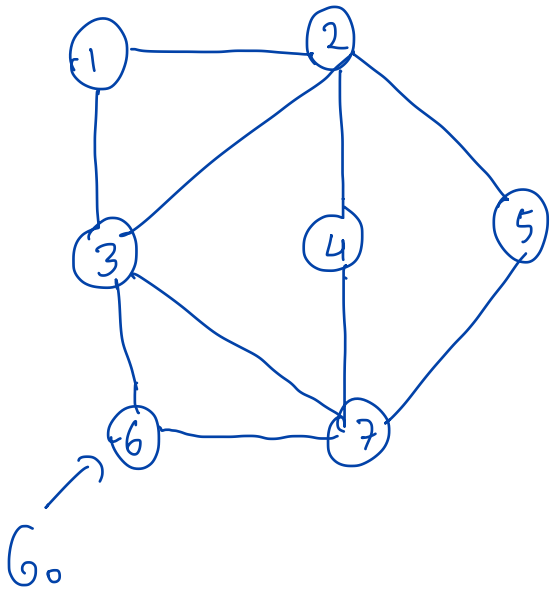


Ex. HW 2 Q2 $\leq_p$ Stable Matching ($M = 1$)

Going forward: ONLY consider problems with Boolean output

Problem 1: Independent Set (IS) $G=(V,E)$

$S \subseteq V$ is an independent set if NO edges exist between nodes in S



G_0

Eg: $\{1,4\}$ ✓ $\{6\}$ ✓

$\{3,7\}$ ✗ $\{1,4,5,6\}$ ✓

$\{1,4,7\}$ ✗

$\{3,4,5\}$ ✓

Decision Problem: Input: $G=(V,E)$; $0 \leq k \leq n$

Output: True iff $\exists$ an IS of size $\geq k$

E.g. $G_0, 2$: ✓ $G_0, 4$: ✓ $G_0, 5$: ✗

Problem 2: Vertex Cover (VC) $G=(V,E)$

$C \subseteq V$ is a vertex cover if ALL edges in E have ≥ 1 end-point in C.

E.g. G_0 : $\{1,2,3,4,5,6,7\}$ ✓

$\{1,2,3,4,5,6\}$ ✓ ⇒ Any subset of size $n-1$ is a VC

$\{1,2,6,7\}$ ✓

$\{2,3,7\}$ ✓

$\{1,7\}$ ✗

Decision Problem: Input: $G=(V,E)$; $0 \le k \le n$

Output: True iff $\exists$ a VC of size $\le k$

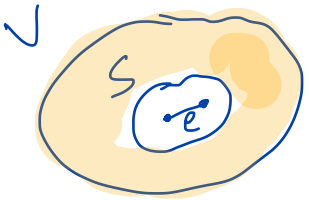
E.g. G_0 ; 6 ✓ G_0 : 3 ✓ G_0 : 2 ✗

THM: (1) $IS \le_p VC$

(2) $VC \le_p IS$

Lemma: $G=(V,E) \Rightarrow S \subseteq V$ is an IS iff $V \setminus S$ is a VC.

Pf: $\Rightarrow$ Let S be an IS. Assume $V \setminus S$ is NOT a VC

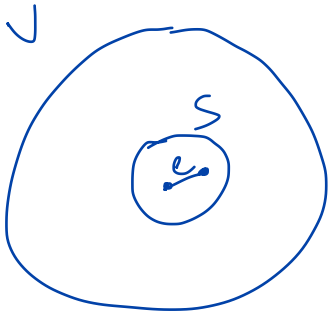


$\Rightarrow \exists$ an edge e w/ no endpoint in $V \setminus S$

$\Rightarrow$ Both end points of e are in S

$\Rightarrow S$ is not an IS $\Rightarrow$ Contradiction

$\Leftarrow$ Let $V \setminus S$ be a VC. But S is NOT an IS



$\Rightarrow \exists$ an edge "inside" S

$\Rightarrow V \setminus S$ is not a VC $\Rightarrow$ Contradiction

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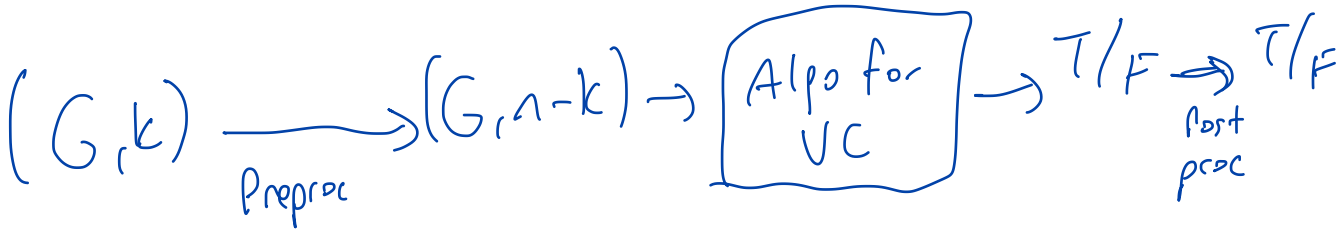
COR: G has an IS of size $\ge k \iff G$ has a VC of size $\le n-k$

$\rightarrow IS \le_p VC$

Pf: Input : (G, k) for IS

$\Rightarrow (G, n-k)$ for VC



$\Rightarrow$ Similarly can show $VC \leq_p IS$