Feb 8

Stable matching/marriage

n women $W = \{w_1, w_2, \ldots w_n\}$

n men $M = \{m_1, m_2, \ldots m_n\}$

$n=2$

$W = \{JA, AJ\}$

$M = \{BP, BBT\}$

Def (matching) A subset $S \subseteq M \times W = \{(m,w), m \in M, w \in W\}$ is a matching if

(i) $\forall w \in W$, $\exists$ at most one men $m \in M$, $(m,w) \in S$

AND

(ii) $\forall m \in M$, $\exists$ at most one women $w \in W$, $(m,w) \in S$

Def (perfect matching) at most $\leftarrow$ EXACTLY

BP — JA

BBT — AJ

M ✓ P ✓

BP — JA

BP — AJ

BBT

M ✗ P ✗

BP JA

BBT AJ

M ✓ P ✗

Def: (Preference list)

$\forall w \in W$, L_w: total ranking of all men $m \in M$

$\forall m \in M$, L_m: total ranking of all women $w \in W$

Ex: $n=2$

$M = \{BP, BBT\}$ $\quad W = \{JA, AJ\}$

L_{BP}: $AJ > JA$

L_{BBT}: $JA > AJ$

L_{JA}: $BP > BBT$

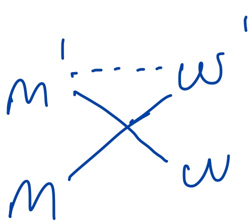
L_{AJ}: $BP > BBT$

$2n$: # pref lists,

$2n \times n = 2n^2$

Def: A stable matching is a perfect matching with no instability

Def (Instability): Given the $2n$ preference lists, a perfect matching S;



we say a pair $(m', w') \notin S$ is an instability

IF ① $m' > m$ in $L_{w'}$

AND

② $w' > w$ in $L_{m'}$

Ex:

BP — JA

BBT — AJ

NOT a stable matching

Q1: Is (BBT, JA) an instability?

NO

Q2: Is (BP, AJ) an instability?

YES