Feb 12

Gale-Shapley algorithm

n men
n women
2n pref. lists

(0) Initially, all men & women are free

(1) In a loop: (in book, men propose)

A free woman proposes to a man

(2) You have n matched pairs.

Initial state: All n men & n women are free

(1) Let w be a free woman

Q1: Which man should w propose to?

A1: The man on the top of her preference list

→ w will propose to m

Q2: What should m do?

A2: case 2.1 : m accepts the proposal
Issue → m could get a better person later on

case 2.2 : m rejects the proposal
Issue → m might not get a better proposal later on

Case 2.3 : m conditionally accepts

⇒ (m, w) are engaged.

General state: All men/women are either free or engaged

(1) All n men & n women are engaged

→ Alg. terminates & outputs the engaged pairs as the final output

ELSE

(2) $\exists$ a free woman w

Q1: Who should w propose to?

A1: Propose to the best man m she HAS NOT proposed yet

→ w proposes to m

Q2: What should m do?

A2: case 2.1: m is free → (m, w) are engaged

case 2.2: m is not free → (m, w') are already engaged

case 2.2.1: $w' > w$ in L_m (no changes)

case 2.2.2: $w > w'$ in L_m ⇒ (m, w) are engaged, w' is free