

Feb 15

THEOREM: For any input (M, W, Z) pref. lists
the GS algorithm outputs a stable matching.

\Rightarrow every input has a stable matching.

LEMMA 1: For every input, GS algo. terminates in $\leq n^2$ iters.

LEMMA 2: The output of GS algo (S) is a perfect matching

LEMMA 3: S has no instability

Lemmas 1+2+3 \Rightarrow Theorem

PF Idea Lemma 1: In each iteration, a new proposal is made

$$\begin{aligned} \Rightarrow \# \text{ iters} = \# \text{ proposals} &\leq \# \text{ pairs } (w, m) = |W \times M| \\ &= |W| \times |M| = n \cdot n = n^2 \end{aligned}$$

(PF details are on pg 7 in textbook)

Obs 0: S is a matching.

Obs 1: Once a man gets engaged, he keeps getting engaged to better woman.

Obs 2: If w proposes to m after m'
 $\Rightarrow m' > m$ in L_w

LEMMA 4: If at the end an iteration,
 w is free $\Rightarrow w$ has not proposed
to all men