Computing Approximate Blocking Probabilities in Wavelength Routed All-Optical Networks with Limited-Range Wavelength Conversion

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***Abstract*—In this paper, we propose a method to calculate the average blocking probability in all-optical networks using *limited-range* wavelength conversion. Previous works have shown that there is a remarkable improvement in blocking probability while using limited-range wavelength conversion, but these analytical models were either for a path or for a mesh-torus network. Using a graph-theoretical approach, we extend Birman's model for no wavelength conversion and derive an analytical expression to compute the blocking probabilities in networks for fixed routing. The proposed model is applicable to *any* network topology. We consider the case where an incoming wavelength can be converted to d adjacent outgoing wavelengths on either side of the input wavelength, in addition to the input wavelength itself, where d is the degree of conversion. When $d = 0$ and $d = ((C-1)/2)$, where C is the capacity of a link, the proposed model reduces to the model previously given for no wavelength conversion and the model previously given for full wavelength conversion respectively. Using this model we demonstrate that the performance improvement obtained by full wavelength conversion over no wavelength conversion can almost be achieved by using limited-range wavelength conversion with the degree of conversion, d, being only 1 or 2. In a few example networks we considered, for blocking probabilities up to a few percent, the carried traffic with limited conversion degree $d = 2$ was almost equal to the carried traffic for full wavelength conversion.**

Comparisons to simulations show that our analytical model is accurate for a variety of networks, for various values of the conversion degree ($d =$ 1, 2, 3), and hop length (1–4), and over a wide range of blocking probabilities ($>$0.0001). The method is also accurate in estimating the blocking probabilities on individual paths (and not just the average blocking probability in the network).

***Index Terms*—Blocking probability, reduced load approximation, wavelength conversion, wavelength-division multiplexing.**

I. Introduction

IN RECENT years, demand for high bandwidth has been growing at a very rapid pace led by Internet and multimedia applications. Networks which employ optical fiber for transmission are very attractive because fiber provides an enormous bandwidth (25 THz), low loss (0.2 dB/km) and very low bit error rate (10^{-9}–10^{-15}).

In all-optical networks, the data remain in the optical domain throughout their path except at the ends. Such paths are termed *lightpaths*. The currently favored technology to tap the huge bandwidth of optical fiber is wavelength division multiplexing (WDM). In WDM networks, the optical spectrum is divided into many different channels, and each channel corresponds to a different wavelength which can operate at the peak electronic speed. In wavelength routed WDM networks, we can reuse the wavelength provided no two lightpaths sharing a link are assigned the same wavelength.

In networks using full wavelength conversion, a call is accepted if on all the links on its route there is at least one free wavelength. With no wavelength conversion, a call is accepted on a route if there exists at least one wavelength which is simultaneously free on all the links of that route. This constraint is known as the *wavelength continuity constraint*. This means a call can be blocked even if there are free wavelengths (but not the same one) on all the links. Therefore, having full wavelength conversion is advantageous [3], [5], [8] in that it decreases the blocking probability.

However, implementing all-optical full wavelength conversion is quite difficult due to technological limitations. So, it is interesting to investigate whether we can do as well as full wavelength conversion in terms of blocking performance by using limited-range wavelength conversion, if not by using no wavelength conversion.

Limited wavelength conversion can imply a limit on the number of nodes with full wavelength conversion capability (*sparse wavelength conversion*) [7], or a limit on the range of wavelengths to which a given wavelength can be converted *(limited-range conversion)*.

The analysis presented in [1] and [2] for calculating blocking probability with limited-range conversion, though shown to be beneficial, is restricted to some specific network topologies. In this paper, we have extended the idea given in [3] to derive an *analytical expression* to compute the blocking probability of networks with *limited-range conversion* for fixed routing. The analysis can be used for *any* network topology.

The rest of the paper is organized as follows. In Section II, we present our analytical model to calculate the blocking probabilities for limited wavelength conversion. In Section III we present numerical results, and in Section IV we conclude.

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II. Limited-Range Wavelength Conversion

A. Traffic Model

In our case, we consider the *online blocking* model [9]. The lightpaths are set up and taken down on demand. These are analogous to setting up and taking down circuits in circuit-switched networks.

B. Assumptions

The following assumptions are used in our analytical model.

1) External calls arrive at each node according to an independent stationary Poisson process with rate λ.
2) Call holding time is exponentially distributed with unit mean.
3) Calls that cannot be routed in the network are blocked and lost.
4) The capacity of the links, denoted by C, is the same for all the links in the network. Each call requires a full wavelength on each link of its path.
5) Wavelengths are assigned uniformly randomly from the set of free wavelengths on the associated path.
6) Simplex connections are considered.
7) Existing lightpaths/calls cannot be reassigned different wavelengths to accommodate the new lightpath/call request.

C. Analytical Model for Limited-Range Conversion

We assume that for any given input wavelength, it is possible to translate it to a limited range of output wavelengths. More precisely, it is assumed that a wavelength can be converted to d adjacent wavelengths on either side of the input wavelength, in addition to the input wavelength itself, where d is the degree of conversion. Hence, any wavelength can be converted to $(2d+1)$ wavelengths. For example, incoming wavelength λ_i can be converted to any of the outgoing wavelengths $\lambda_{i-d}, \cdots, \lambda_i, \cdots, \lambda_{i+d}$. We also assume that the conversions are circularly symmetric.

Let $p_m(x_1, x_2, \cdots, x_N)$ denote the probability of having m choices for the outgoing wavelengths on an N–hop path given that $x_1, \cdots, x_N$ wavelengths are free on links $1, \cdots, N$ respectively. If $\bar{\boldsymbol{x}} = (x_1, x_2, \cdots, x_N)$,

$$p_m(\bar{\boldsymbol{x}}) = \Pr[X_R = m | X_1 = x_1, \cdots, X_N = x_N] \qquad (1)$$

where X_R is a random variable denoting the number of choices for the outgoing wavelengths on route $R = \{1, 2, \cdots, N\}$ and the random variables X_i denote the number of free wavelengths on link i. Let us first consider the case of a two-hop route $R = \{i, j\}$, for which

$$p_m(x, y) = \Pr[X_{(i,j)} = m | X_i = x, X_j = y]. \qquad (2)$$

This is the probability of having m possible outgoing wavelengths on a two link route given that x and y wavelengths are free on the first and the second link, respectively. We can think of having a bipartite graph $(\boldsymbol{X}, \boldsymbol{Y})$, where the set of vertices $\boldsymbol{X}$ and $\boldsymbol{Y}$ represent the set of wavelengths available on the first and second link, respectively; hence, the cardinality of both the sets is equal to the capacity of the link C, i.e., $|\boldsymbol{X}| = |\boldsymbol{Y}| = C$. Each vertex $u_k \in \boldsymbol{X}$ has an edge incident with the "facing" vertex $v_k \in \boldsymbol{Y}$ and d adjacent vertices on either side of v_k (see Fig. 1). Therefore, each vertex of $\boldsymbol{X}$ will have a degree of $(2d+1)$. The $(2d+1)$ vertices in set $\boldsymbol{Y}$ which are adjacent to a vertex in set $\boldsymbol{X}$ are called its neighbors. For example, the neighbors of vertex $2 \in \boldsymbol{X}$ are vertices 1, 2, and $3 \in \boldsymbol{Y}$ as shown in Fig. 1. The edges denote the possible conversion from one wavelength to another wavelength. For the last (respectively, first) vertex, we will have edges to the immediately higher (respectively, lower) d vertices and the first (respectively, last) d vertices from the top (respectively, bottom), i.e., the wavelength conversion is assumed to be circularly symmetric. This is merely for the sake of analytical convenience, as it distributes the load uniformly among all the wavelengths on a link. Let $\boldsymbol{X}_i \subseteq \boldsymbol{X}$ and $\boldsymbol{Y}_j \subseteq \boldsymbol{Y}$ denote those vertices corresponding to which we have free wavelengths on link i and link j respectively. The cardinalities of sets $\boldsymbol{X}_i$ and $\boldsymbol{Y}_j$ are x and y (free wavelengths on link i and j, respectively).

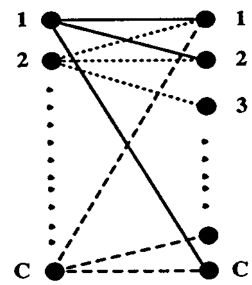

Fig. 1. Possible wavelength conversions at each node for $d = 1$. The conversions are circularly symmetric.

Let $\Gamma(\boldsymbol{X}_i)$ denote the neighbors of the vertices in set $\boldsymbol{X}_i$. Then we are interested in finding the probability of having m such neighbors of vertices in $\boldsymbol{X}_i$ which are incident with the vertices in $\boldsymbol{Y}_j$. Then, for $1 \le x, y \le C$

$$\begin{aligned} p_m(x, y) &= \Pr[|\Gamma(\boldsymbol{X}_i) \cap \boldsymbol{Y}_j| = m] \\ &= \sum_{l:|\Gamma(\boldsymbol{X}_i)|=l, |\boldsymbol{X}_i|=x} \frac{\binom{|\Gamma(\boldsymbol{X}_i)|}{m}\binom{C-|\Gamma(\boldsymbol{X}_i)|}{y-m}}{\binom{C}{y}} \\ &\quad \times \Pr(|\Gamma(\boldsymbol{X}_i)| = l / |\boldsymbol{X}_i| = x) \\ &= \sum_{l=\min[C, x+2d]}^{\min[C,(2d+1)x]} \frac{\binom{l}{m}\binom{C-l}{y-m}}{\binom{C}{y}} \\ &\quad \times \Pr(|\Gamma(\boldsymbol{X}_i)| = l / |\boldsymbol{X}_i| = x). \end{aligned} \qquad (3)$$

The last equality in (3) has the summation running from $\min[C, (x+2d)]$ to $\min[C, (2d+1)x]$. This is because the minimum cardinality (neighbors) of $\Gamma(\boldsymbol{X}_i)$ will be $(x+2d)$ or C, depending on whichever is smaller, and the maximum cardinality (neighbors) can be either C or $(2d+1)x$, depending on whichever is smaller, as the number of neighbors cannot be more than C (the capacity of the link).

When there is only one wavelength (vertex) free belonging to the set $\boldsymbol{X}_i$, i.e., $|\boldsymbol{X}_i| = 1$, then the number of neighbors of that vertex will be exactly $(2d+1)$, and in this case the lower and upper limits of summation coincide. In the case when one has exactly x wavelengths (vertices) free in the set $\boldsymbol{X}_i$, i.e., $|\boldsymbol{X}_i| = x$, and if all the x wavelengths are adjacent, then we have only $\min[(x+2d), C]$ distinct neighbors. This one extreme case constitutes the lower limit. Had all these x free wavelengths (vertices) been sufficiently apart from one another so that each contributes $(2d+1)$ distinct neighbors then we will have $(2d+1)x$ distinct neighbors provided $(2d+1)x$ is less than C; otherwise this number will be C. This other extreme case constitutes the upper limit of the summation. For all other cases, the number of neighbors lies between these two limits. Note that in general $p_m(x,y) \neq p_m(y,x)$ but interestingly, it can be shown that $p_0(x,y) = p_0(y,x)$.[1] When $d=0$, our model reduces to Birman's model [3] for no wavelength conversion. In this case, $p_m(x,y) = p_m(y,x)$, by symmetry. We can rearrange the links such that the links of the path have free wavelengths in increasing order and then we can use (3) to compute $p_m(x,y)$. When $d = ((C-1)/2)$, the method reduces to that given in [4] for full wavelength conversion.

The probability term, $\Pr(\cdot)$ in (3) is given by

$$\begin{aligned} \Pr[|\Gamma(\boldsymbol{X}_i)| &= l / |\boldsymbol{X}_i| = x] \\ &= \Pr[|\Gamma(\boldsymbol{X}_i)| \leq l / |\boldsymbol{X}_i| = x] \\ &\quad - \Pr[|\Gamma(\boldsymbol{X}_i)| \leq l-1 / |\boldsymbol{X}_i| = x]. \end{aligned} \tag{4}$$

First, note that $\Pr[|\Gamma(\boldsymbol{X}_i)| \leq l \, | \, |\boldsymbol{X}_i| = x] = 1$ if $l = C$ or $l = (2d+1)x$ since the number of neighbors cannot exceed either number. Also, $\Pr[|\Gamma(\boldsymbol{X}_i)| \leq l \, | \, |\boldsymbol{X}_i| = x] = 0$ if $l < x + 2d \leq C$ since there are at least $x + 2d$ neighbors, in this case. Hence, for the remainder of the discussion, we assume $x + 2d \leq l < \min[C, (2d+1)x]$.

Consider the probability that $|\Gamma(\boldsymbol{X}_i)| \leq l$, given that the vertices $\Gamma(\boldsymbol{X}_i)$ lie in some contiguous range of l vertices (and $|\boldsymbol{X}_i| = x$). In this range, some of the vertices may not be neighbors of $\boldsymbol{X}_i$. Therefore, the total number of neighbors is at most l. Clearly there are cases when we have no more than l neighbors but they do not lie in some contiguous range of l vertices. This yields the inequality

$$\begin{aligned} \Pr[|\Gamma(\boldsymbol{X}_i)| &\leq l / |\boldsymbol{X}_i| = x] \\ &\geq \Pr[|\Gamma(\boldsymbol{X}_i)| \leq l, l \text{ contiguous } / |\boldsymbol{X}_i| = x]. \end{aligned}$$

We have

$$\Pr[|\Gamma(\boldsymbol{X}_i)| \leq l, l \text{ contiguous}] = \Pr[\cup_{k=1}^{C} \Gamma(\boldsymbol{X}_i) \subseteq l_k] \tag{5}$$

where each l_k is a contiguous set of vertices of size l, and k varies from 1 to C as we can position l contiguous vertices in C ways because the conversion is assumed to be circularly symmetric.

[1] $p_0(x,y)$ is the probability that $\Gamma(X_i) \cap Y_j = \phi$. We will prove that whenever $\Gamma(X_i) \cap Y_j = \phi$, $\Gamma(Y_j) \cap X_i = \phi$. We prove this by contradiction. Suppose $\Gamma(Y_j) \cap X_i \neq \phi$. Then there exists a vertex $u_i \in X_i$ s.t. $u_i \in \Gamma(Y_j) \cap X_i$. Since $u_i \in \Gamma(Y_j)$, there exists a vertex $v_j \in Y_j$ s.t. $\Gamma(v_j) = u_i$. Therefore, $\Gamma(u_i) = v_j$ which contradicts $\Gamma(X_i) \cap Y_j = \phi$.

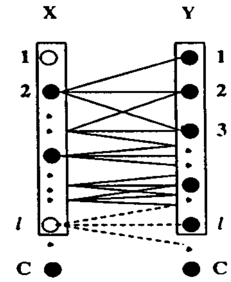

Fig. 2. This figure is for limited conversion with degree $d = 1$. If all the neighbors must lie within a contiguous range of l vertices, we cannot choose the first and last d vertices from the set X. Here we cannot choose the first and the lth vertices shown by the hollow circles. For example, if we choose the lth vertex its neighbors fall outside the range l as shown in the figure above.

Let us denote the event $\Gamma(\boldsymbol{X}_i) \subseteq l_k$ by Z_k (given that $|\boldsymbol{X}_i| = x$). Then the right-hand side of (5) is given by

$$\begin{aligned} \Pr[\cup_{k=1}^{C} \Gamma(\boldsymbol{X}_i) \subseteq l_k] &= \Pr[Z_1 \text{ or } Z_2 \text{ or } \cdots \text{ or } Z_C] \\ &\leq \binom{C}{1} \Pr[Z_k] \end{aligned} \tag{6}$$

because we can choose l contiguous vertices in $\binom{C}{1}$ ways. Now consider one such contiguous set of l vertices in the set $\boldsymbol{Y}_j$, say set $\mathbf{L}_j$. As per our assumption, the x vertices in set $\boldsymbol{X}$ are contained in the "facing" vertices of the vertices in set $\mathbf{L}_j$. We denote this set of "facing" vertices as set $\boldsymbol{X}_l$ which has cardinality l. We want to choose x vertices in $\boldsymbol{X}_l$ such that their neighbors are in set $\mathbf{L}_j$. If any vertex is in the first d vertices or the last d vertices of the set $\boldsymbol{X}_l$, some neighbors must fall outside the range of set $\mathbf{L}_j$ as shown in Fig. 2. Therefore, x vertices can be chosen only from $(l-2d)$ vertices of set $\boldsymbol{X}_l$. The total number of ways in which set $\boldsymbol{X}_i$, such that $|\boldsymbol{X}_i| = x$, can be formed is $\binom{C}{x}$. Therefore, the probability that $\Gamma(\boldsymbol{X}_i)| \subseteq l_k$, when l_k is a set of contiguous vertices of size l, is given by

$$\Pr[Z_k] = \frac{\binom{l-2d}{x}}{\binom{C}{x}}. \tag{7}$$

Therefore, from (5)–(7), we get

$$\Pr[|\Gamma(\boldsymbol{X}_i)| \leq l, l \text{ contiguous}] \leq \frac{\binom{C}{1}\binom{l-2d}{x}}{\binom{C}{x}}. \tag{8}$$

We have already mentioned that $\Pr[|\Gamma(\boldsymbol{X}_i)| \leq l]$ is *lower bounded* by the left-hand side of (8) given that $|\boldsymbol{X}_i| = x$. Thus, this does not yield a bound on $\Pr[|\Gamma(\boldsymbol{X}_i)| \leq l]$. We assume the following approximation holds for $x + 2d \leq l < \min[C, (2d+1)x]$:

$$\Pr[|\Gamma(\boldsymbol{X}_i)| \leq l / |\boldsymbol{X}_i| = x] \approx \frac{\binom{C}{1}\binom{l-2d}{x}}{\binom{C}{x}}.$$

We will see from numerical results later that this yields a good approximation to the blocking probabilities, at least for small values of the conversion degree d.

For the general case of an N-hop route, $N \geq 3$, let x_j be the number of the idle wavelengths on the jth hop. We condition on the set of disjoint events $\{X_{\overline{R}} = k | k = m, x_1, x_2, \cdots x_{N-1}\}$, where $\overline{R} = \{1, \cdots, N-1\}$. Recall that X_R is a random variable denoting the number of possible outgoing wavelengths on route R. We thus obtain the recursive relation (assume the first $(N-1)$ links to be the first link and the last link to be the second link):

$$p_m(x_1, \cdots, x_N) = \sum_{k=m}^{x_{N-1}} p_k(x_1, \cdots, x_{N-1}) p_m(k, x_N) \quad (9)$$

where $p_m(k, x_N)$ is given by (3).

D. Fixed Wavelength Routing

We consider a network with an arbitrary topology with J links and C wavelengths on each link. A route R is a subset of links $\{1, \cdots, J\}$. Calls arrive for route R as a Poisson stream with rate a_R. An incoming call on route R is set up if it finds a free wavelength on all the links from the possible choices of outgoing wavelengths with the given degree of limited wavelength conversion. If such a combination of wavelengths is not possible on the links constituting the path, then the call is blocked and lost. If the call is accepted, it simultaneously holds the wavelength/wavelengths on all the links on route R for the duration of the call. The holding times of all the calls are assumed to be exponentially distributed with unit mean.

Let X_j be the random variable denoting the number of idle wavelengths on link j in equilibrium. Let $\boldsymbol{X} = (X_1, \cdots, X_J)$ and let

$$q_j(w) = \Pr[X_j = w]; \qquad w = 0, \cdots, C$$

be the idle capacity distribution. Throughout the following approximations are made.

1) The random variables $X_1, X_2, \cdots, X_J$ are mutually independent. Then

$$q(\mathbf{w}) = \prod_{j=1}^{J} q_j(w_j)$$

where $\mathbf{w} = (w_1, w_2, \cdots, w_J)$.

2) When there are w idle wavelengths on link j, the time until the next call is set up on link j is exponentially distributed with parameter $\alpha_j(w)$. This parameter is the call set-up rate on link j when w wavelengths are free on link j.

From the approximation (2), it follows that the number of idle wavelengths on link j can be viewed as a birth-and-death process, and therefore we have

$$q_j(w) = \frac{C(C-1)\cdots(C-w+1)}{\alpha_j(1)\alpha_j(2)\cdots\alpha_j(w)} q_j(0) \qquad w = 1, \cdots, C \quad (10)$$

where

$$q_j(0) = \left[1 + \sum_{w=1}^{C} \frac{C(C-1)\cdots(C-w+1)}{\alpha_j(1)\alpha_j(2)\cdots\alpha_j(w)}\right]^{-1}. \quad (11)$$

The call set-up rate on link j, when there are w idle wavelengths on link j, $\alpha_j(w)$, is obtained by combining the contributions from the request streams to routes of which link j is a member.

$$\begin{aligned}\alpha_j(w) &= 0, \quad \text{if} \quad w = 0 \\ &= \sum_{R: j \in R} a_R \Pr[X_R > 0 | X_j = w] \\ &\qquad w = 1, \cdots, C.\end{aligned} \quad (12)$$

If the route consists of a single link, then the probability term $\Pr(\cdot)$ under the summation sign in (12) will be equal to 1. If the route consists of two links, let $R = \{i, j\}$. The term $\Pr(\cdot)$ can be further simplified by conditioning it on the set of disjoint events $\{X_i = l | l = 0, \cdots, C\}$.

$$\begin{aligned}&\Pr[X_{\{i,j\}} > 0 | X_j = w] \\ &\quad = \sum_{l=1}^{C} \Pr[X_i = l | X_j = w] \Pr[X_R > 0 | X_j = w, X_i = l] \\ &\quad = \sum_{l=1}^{C} q_i(l)(1 - p_0(w, l))\end{aligned} \quad (13)$$

where $p_0(w, l)$ is given by (3).

E. Computation of Blocking Probability

The blocking probability for calls to route R is

$$\begin{aligned}L_R &= \Pr[X_R = 0] \\ &= q_i(0), \qquad \text{if } R = \{i\} \\ &= \sum_{l=0}^{C} \sum_{w=0}^{C} q_i(l) q_j(w) p_0(l, w), \quad \text{if } R = \{i, j\}.\end{aligned} \quad (14)$$

Blocking probability for routes with more hops can also be calculated similarly.

F. Algorithm for Computation of Blocking Probability

The algorithm below uses a fixed-point method to compute the approximate blocking probabilities for the traffic on all the routes and the (average) blocking probability of the network.

1) Initialization. For all the routes R let $\hat{L}_R = 0$. For $j = 1, \cdots, J$, let $\alpha_j(0) = 0$, and let $\alpha_j(m)$ be chosen arbitrarily, $m = 1, \cdots, C$.
2) Determine $q_j(\cdot)$ from (10) and (11).
3) Obtain new values of $\alpha_j(\cdot), j = 1, \cdots, J$, using (12). (Note that (13) must be used in (12) for 2-hop paths, and suitable generalizations for paths with more hops.)
4) Calculate L_R, for all routes R, using (14). If $\max_R |L_R - \hat{L}_R| < \epsilon$ (where ϵ is suitably small positive quantity), then terminate. Otherwise let $\hat{L}_R = L_R$, and go to step 2.
5) The (average) blocking probability of the network is then given by

$$Pb_{network} = \frac{\sum_{r=1}^{R} a_r L_r}{\sum_{r=1}^{R} a_r}.$$

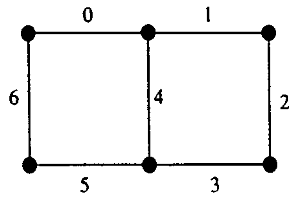

Fig. 3. Example network with six nodes and seven links.

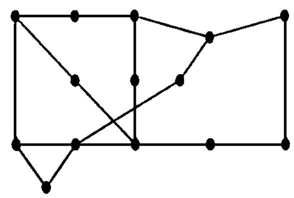

Fig. 4. Mesh network with 14 nodes and 18 links.

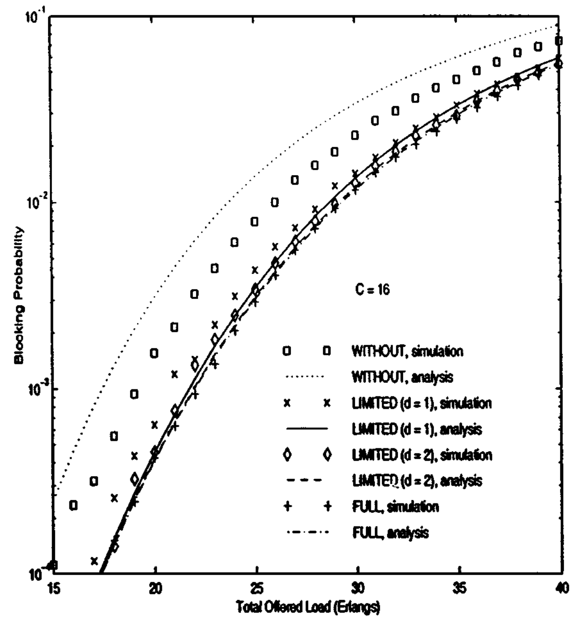

Fig. 5. Average blocking probability in the example network with six nodes and seven links versus the total offered load, for $C = 16$ wavelengths per link. The plot shows the analytically calculated values and simulation values for no, full, and limited wavelength conversion with conversion degree $d = 1$ and $d = 2$.

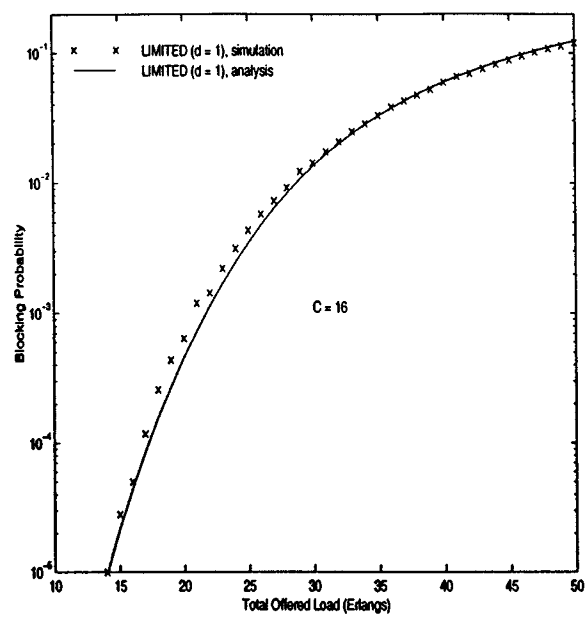

Fig. 6. Comparison of the results obtained through analysis and simulation for limited wavelength conversion with degree $d = 1$ for the example network with six nodes and seven links.

III. Numerical Results

We present simulation and analytical results for an example network with 6 nodes and 7 links (Fig. 3), a 6-node ring network, and a mesh network (Fig. 4) for three different cases: no wavelength conversion, limited wavelength conversion with degree $d = 1, 2$, and full wavelength conversion. For the first two networks, we consider connections between all possible node pairs, so that the number of possible routes is 15. For the mesh network, we have taken 80 routes. (These are the routes for which the minimum hop path is unique.) The offered traffic on each route is assumed to be equal (uniform traffic) and we plot the (average) blocking probability (over all routes) versus the total offered load to the network.

In simulation, for limited wavelength conversion, we choose a wavelength out of the free wavelengths on the first hop uniformly randomly and at each subsequent hop look for the possible outgoing wavelengths with the given degree of wavelength conversion. If more than one such wavelength is available, then once again we choose a wavelength uniformly randomly on this hop. This is repeated on subsequent hops. If, at some hop (other than the first), there is no wavelength free which is in the possible subset of outgoing wavelengths, then we fall back to the previous hop and choose a wavelength out of the free wavelengths minus the earlier chosen free wavelength/wavelengths uniformly randomly. If we exhaust all the free wavelengths on the first hop, and still cannot find any possible outgoing wavelength on some hop then we block the call.

For the six-node example network (Fig. 3), we plot the graphs for 16 wavelengths showing the performance of full, no, and limited wavelength conversion. From Fig. 5, we see that the performance obtained by limited wavelength conversion with degree $d = 1$ is close to the performance of full conversion and with degree $d = 2$ it almost matches the full wavelength conversion performance. In Fig. 6, we show that our analysis results for limited wavelength conversion with degree $d = 1$ is in good agreement with the results of simulations. In Fig. 7, we plot the curves for 16 wavelengths for a six-node ring network and with conversion degree $d = 2$, we can virtually achieve the same performance as that of full conversion. In Fig. 8, we compare the results of our analysis and simulations for $d = 2$ and see that they match quite well. For the mesh network (Fig. 4), we consider 18 one-hop, 29 two-hop, 25 three-hop, and eight four-hop routes. We show graphs for 10 wavelengths. From Fig. 9, we note that the blocking performance of limited wavelength conversion with degree $d = 2$ is very close to that of full wavelength conversion. Our method gives accurate results for higher values of d as well, for example, $d = 3$ as shown in [11].

Table I shows path-wise blocking probabilities for the network shown in Fig. 3. Simulation results are given as 95% confidence intervals estimated by the method of batch means. The number of batches is 20. We consider all the 15 possible routes

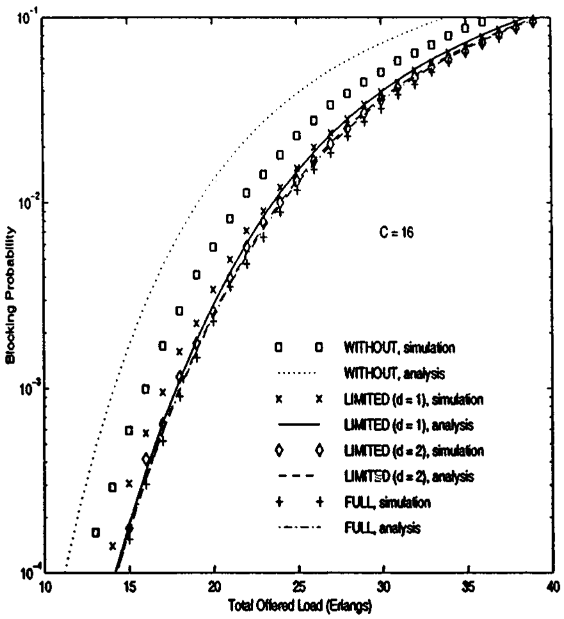

Fig. 7. Average blocking probability in the six-node ring network versus the total offered load, for $C = 16$ wavelengths per link. The plot shows the analytically calculated values and simulation values for no, full, and limited wavelength conversion with conversion degree $d = 1$ and $d = 2$.

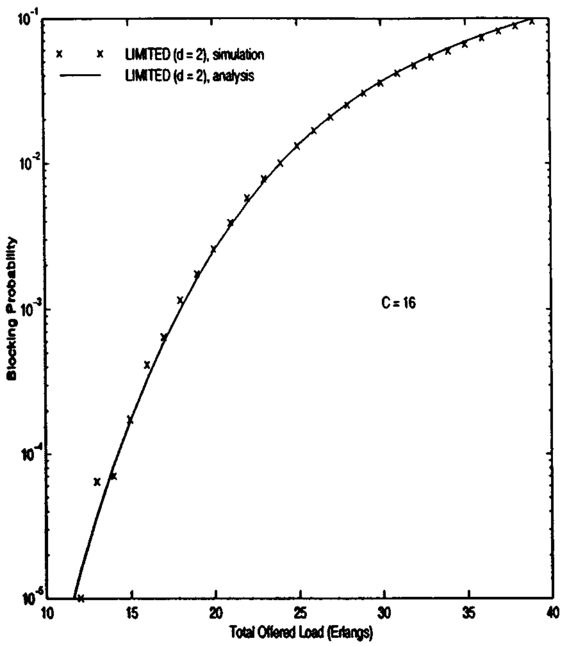

Fig. 8. Comparison of the results obtained through analysis and simulation for limited-wavelength conversion with degree $d = 2$ for the six-node ring network.

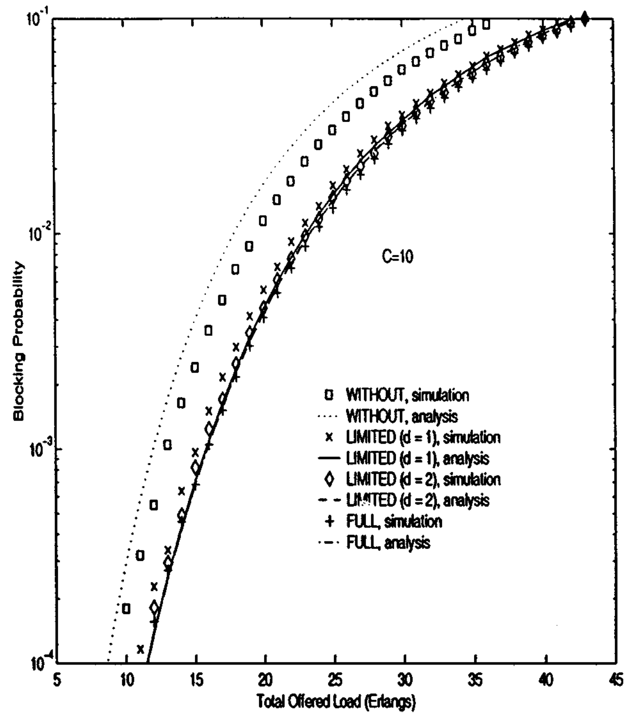

Fig. 9. Average blocking probability in the mesh network (Fig. 4) versus the total offered load, for $C = 10$ wavelengths per link. The plot shows the analytically calculated values and simulation values for no, full, and limited wavelength conversion with conversion degree $d = 1$ and $d = 2$.

TABLE I
WE CONSIDER THE NETWORK SHOWN IN FIG. 3 WITH A TOTAL OFFERED LOAD OF SIX ERLANGS. ALL THE 15 ROUTES ARE CONSIDERED. SO EACH ROUTE HAS 0.4 ERLANGS OF LOAD. THE DEGREE OF WAVELENGTH CONVERSION IS $d = 2$. THE NUMBER OF WAVELENGTHS CONSIDERED IS 8. R: ROUTES (SET OF LINKS), L_R^{sim}: BLOCKING PROBABILITY OBTAINED BY USING SIMULATIONS, L_R: ANALYTICALLY CALCULATED BLOCKING PROBABILITY

R	$L_R^{sim}(\%)$	$L_R(\%)$
{0}	(0.0155, 0.0261)	0.0214
{1}	(0.0139, 0.0255)	0.0213
{2}	(0.0741, 0.0969)	0.0857
{3}	(0.0146, 0.0220)	0.0213
{4}	(0.0010, 0.0070)	0.0032
{5}	(0.0144, 0.0300)	0.0214
{6}	(0.0000, 0.0000)	0.0000
{0, 1}	(0.0451, 0.0639)	0.0430
{0, 4}	(0.0174, 0.0324)	0.0247
{1, 2}	(0.1035, 0.1229)	0.1077
{4, 5}	(0.0207, 0.0309)	0.0247
{2, 3}	(0.1054, 0.1288)	0.1077
{3, 5}	(0.0398, 0.0596)	0.0430
{0, 1, 2}	(0.1261, 0.1761)	0.1291
{2, 3, 5}	(0.1223, 0.1631)	0.1287

and each link has a capacity of eight wavelengths. The total offered load to the network is six Erlangs and $d = 2$. Since the load is uniformly distributed on all the paths, each path has a load of 0.4 Erlangs. We observe that analytically calculated blocking probabilities are in good agreement with the simulation results. We have shown the result for low load as the calculation of blocking probability using our algorithm becomes more accurate as the load is increased. The reason for this is that the algorithm presented in the paper is a reduced load algorithm, and it is well known that it gives accurate results as the load is increased [4]. The graphs plotted also show that at higher load, simulation results match very well with the analytical results.

We have presented three examples to show the accuracy of our analytical method in estimating the blocking probabilities. In all these examples, limited conversion provides a marked improvement in the blocking performance of the network as compared to no wavelength conversion. Furthermore, the performance obtained by limited conversion with small values of the conversion degree, such as $d = 1$ or $d = 2$, is very close to the blocking performance of the network with full wavelength conversion.

The only drawback of our analytical model for limited wavelength conversion presented above is that its computational requirements are significant: exponential in terms of the number of hops. The complexity of calculating the blocking probability L_R is of the order of $O(C^H)$, where H denotes the number of hops in route R. Thus, when we have to consider a large number of wavelengths on each link, or when the diameter of the network is large, our method will be intractable. We note, however, that this computational complexity is the same as that of the model for no wavelength conversion [3]. Very recently, a method of significantly reducing the computational complexity in the no wavelength conversion case has been described by Sridharan and Sivarajan [10]. The adaptation of this reduced complexity model for the limited wavelength conversion case, considered in this paper, is the subject of future research.

IV. Conclusion

In this paper, we have proposed a method to calculate the average blocking probability in optical networks using *limited-range* wavelength conversion. The proposed analytical model in this paper is applicable to *any* topology. Using this model, we have demonstrated that the performance improvement obtained by full-wavelength conversion over no-wavelength conversion can be achieved by using limited-wavelength conversion with the degree of conversion, d, being only 1 or 2.

The model presented in this paper, especially that used for computing the $p_m(\cdot)$, can also be used for alternate routing with limited-range wavelength conversion by extending the method presented in [6]. This is also a topic for further research.

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