Blocking in Large Mobile Cellular Networks with Bursty Traffic

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ABSTRACT

We consider large cellular networks. The traffic entering the network is assumed to be correlated in both *space* and *time*. The space dependency captures the possible correlation between the arrivals to different nodes in the network, while the time dependency captures the time correlation between arrivals to each node. We model such traffic with a Markov-Modulated Poisson Process(MMPP).

It is shown that even in the single node environment, the problem is not mathematically tractable. A model with an infinite number of circuits is used to approximate the finite model. A novel recursive methodology is introduced in finding the joint moments of the number of busy circuits in different cells in the network leading to accurate determination of blocking probability. A simple mixed-Poisson distribution is introduced as an accurate approximation of the distribution of the number ofbusy circuits.

We show that for certain cases, in the system with an infinite number of circuits in each cell, there is no effect of mobility on the performance of the system. Our numerical results indicate that the traffic burstiness has a major impact on the system performance. The mixed-Poisson approximation is found to be a very good fit to the exact finite model. The performance of this approximation using few moments is affected by traffic burstiness and average load. We find that in a reasonable range of traffic burstiness, the mixed-Poisson distribution provides a close approximation.

1. INTRODUCTION

The design of wireless cellular networks requires a thorough understanding of the traffic characteristics of new call arrivals into the network. Cellular networks must be designed with adequate capacity in order to provide an acceptable Quality of Service (QoS). At the call level, QoS is generally characterized by new call blocking (P_B), hand-off blocking (P_H), and forced-termination. New call blocking occurs when no free channel is available in the cell where the mobile subscriber initiates the call. Hand-off blocking occurs when a mobile user enters a new cell while a call is in progress and there is no available channel to carry the call. In order to minimize blocking, an increase in capacity can be achieved by increasing the number of channels in a cell, reducing the cell size, or introducing efficient power control algorithms.

Analytic models for traditional cellular networks are generally based on Poisson arrival processes [7],[8]. However, a Poisson arrival process does not adequately characterize arrival traffic with correlation. A Markov renewal process such as a Markov-Modulated Poisson Process (MMPP) that can characterize correlation in arrivals is a more appropriate model. Use of an MMPP in single-cell analysis has been shown by Sohraby[9], where a moment-matching technique is used to characterize arrival traffic. However, hand-off blocking, impact of mobility, and correlation of arrivals between cells cannot be analyzed through these single-cell models. Models which provide multiple-cell analysis as well as correlation between cells must be developed. However, no product-form solution for multiple-cell analysis exists, and the resulting state-space explosion prevents an exact analysis [3], [10]. Numerical approximation techniques for such systems must be developed. The fixed-point approximation (FPA) used by Kelly[3] for blocking provides a good approximation for large networks with low mobility. However, FPA is based on the assumption that all arrivals are Poisson and independent. Approximation techniques that do not require such an assumption must be developed.

We model the arrival process into a cellular network as an MMPP. Arrival rates into the network vary randomly over time, being governed by an underlying Markov chain. The MMPP allows us to capture correlation in time, that is correlation between arrivals into a single cell, and in the case of a multi-cell network, to capture correlation in space, that is correlation between arrivals to different cells, as well. Let us consider the case where the arrival rate into a cell is λ_j when the underlying Markov chain is in state j. As the

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MMPP changes state, there is a change in the arrival rates. Here, the arrivals are correlated in time. The arrivals may also be correlated in space, where there is correlation among the cells of the network. This can be seen in a case where as the MMPP changes state, there may be a change to a higher arrival rate in one cell, and to a lower arrival rate in the adjacent cell. This is a case of negative correlation among the cells. Similarly there may be positive correlation among cells when changes in arrival rates are similar among the cells. A simplification of correlation in space may be seen in the following example. Consider two large cells in a town, one in the downtown area and the other in a residential area. If most of the users are in the downtown cell during the day, there is a high arrival rate into that cell and a low arrival rate into the residential cell. At the end of the day as users travel back home, there is a high arrival rate in the residential cell and a low arrival rate in the downtown cell. Here, the arrivals into one cell are negatively correlated with arrivals into the other cell.

Arrivals into different cells might also be governed by separate Markov chains. In a multi-cell analysis, the individual MMPP's of the cells may be superimposed, resulting in a larger MMPP. Our model allows the handoff rates to also be controlled by an MMPP. In such a case, the time to handoff, for a call in each cell is exponential, with a rate that depends on the state of the modulating chain.

The cellular networks we consider consist of cells with a large number of circuits. Such a finite model of a large number of servers is not amenable to exact analysis. Since the cells contain a large number of circuits, we approximate the system by modeling it as a system with an infinite number of servers. A system with an infinite number of servers is often more tractable and we show that this approximation is in fact suitable for the size of cells we consider. For example, we may consider a cellular system of 12.5 MHz of total bandwidth, with 1.25 MHz allocated to CDMA. (12.5 MHz is the total spectral allocation for each service provider.) For this example, the reverse link may be able to support up to 132 users per cell [2]. Note that this with only 10% of the total bandwidth allocated to CDMA. The number of users supported in a cell can increase as more bandwidth is used.

We present two methods to approximate the steady-state probabilities of the number of busy circuits. In the single-cell environment, the steady-state probability of the number of busy circuits can be computed directly using a probability generating function. A recursive relation between the probabilities is also employed. However, the methods of using the generating function and the recursive equations are not easily extendible to multiple cells. We derive a recursive relation to find the joint moments of the number of busy circuits in each cell. This moment information is used to approximate the probability distribution of the number of busy circuits in each cell as a mixed-Poisson distribution. This distribution can be approximated by matching only a few moments, thus reducing the computational complexity.

The rest of this paper is organized as follows: In Section 2 we present our model. In Section 3 we develop the single-cell analysis, including a recursive relation for the moments. The mixed-Poisson approximation method that uses moment information is presented in Section 4. We present the analysis for the multiple-cell environment in Section 5. Section 6 includes numerical results that show the impact of modeling bursty traffic on blocking, and on the performance of the approximation. Section 7 concludes the paper.

2. MATHEMATICAL MODEL

We consider a large network consisting of K cells, with each cell having a large number of circuits. We model the arrival process of new calls to the network as a Markov Modulated Poisson Process (MMPP). The arrival rate of calls into the network is governed by an underlying Markov chain such that when this Markov chain is in state j, new calls arrive into cell i according to a Poisson process with rate λ_{ij}. The holding time of a call arriving in cell i is exponential with rate $\frac{1}{\mu_i}$. As a user moves along the network according to some mobility pattern, a call is handed off from cell i to cell k with rate γ_{ik}. Our general model allows handoff rates to be governed by an MMPP also. In such cases, the handoff rate between cell i and cell k is γ_{ikj} when the modulating chain is in state j.

We let $J(t)$ be the underlying Markov chain that governs the arrival rates and handoff rates, with state space $(1, 2, \ldots, J)$. The system $(N_1(t), N_2(t), \ldots, N_K(t), J(t))$ then is a continuous-time Markov chain, where $N_i(t)$ is the number of busy circuits in cell i at time t. The state space of this system is $\{(n_1, n_2, \ldots, n_K, j) : 0 \leq n_i \leq m_i, i = 1, 2, \ldots, K, 1 \leq j \leq J\}$ where m_i is the total number of circuits in cell i. Such a multidimensional continuous-time Markov chain does not have a product-form solution.

We show as an example, a simple network of two cells with m circuits in each cell. We assume the underlying Markov chain has two states. The state space of this system is $\{(n_1, n_2, j) : 0 \leq n_i \leq m, j = 1, 2\}$.

The infinitesimal generator Q for this chain has the following structure:

$$Q = \begin{bmatrix} B_0 & A_0 & & & & \\ D_1 & B_1 & A_1 & & & \\ & \ddots & \ddots & \ddots & & \\ & & & D_{m-1} & B_{m-1} & A_{m-1} \\ & & & & D_m & B_m \end{bmatrix} \quad (1)$$

The generator Q has dimensions $(J(m+1)^2) \times (J(m+1)^2)$, with blocks of size $(J(m+1)) \times (J(m+1))$. The blocks $A_i : 0 \leq i < m$ and $D_i : 0 < i \leq m$ are as such:

$$A_i = \begin{bmatrix} \Lambda_1 & & & & \\ \Gamma_{21} & \Lambda_1 & & & \\ & 2\Gamma_{21} & \Lambda_1 & & \\ & & \ddots & \ddots & \\ & & & m\Gamma_{21} & \Lambda_1 \end{bmatrix}$$

$$D_i = \begin{bmatrix} iM_1 & i\Gamma_{12} & & & \\ & iM_1 & i\Gamma_{12} & & \\ & & \ddots & \ddots & \\ & & & iM_1 & i\Gamma_{12} \\ & & & & i(M_1 + \Gamma_{12}) \end{bmatrix}$$

The diagonal blocks $B_i : 0 \leq i < m$, and B_m are as such:

$$B_i = \begin{bmatrix} C_0 & \Lambda_2 & & & \\ M_2 & C_1 & \Lambda_2 & & \\ & 2M_2 & C_2 & \ddots & \\ & & \ddots & \ddots & \Lambda_2 \\ & & & mM_2 & C_m \end{bmatrix}$$

$$B_m = \begin{bmatrix} C_0 & \Lambda_2 & & & \\ M_2+\Gamma_{21} & C_1 & \Lambda_2 & & \\ & 2(M_2+\Gamma_{21}) & C_2 & \ddots & \\ & & \ddots & \ddots & \Lambda_2 \\ & & & m(M_2+\Gamma_{21}) & C_m \end{bmatrix}$$

C_k in block B_i is defined as: $\tilde{Q} - 1_m\Lambda_1 - 1_m\Lambda_2 - i(M_1+\Gamma_{12}) - k(M_2+\Gamma_{21})$ where $1_m\Lambda_i = \Lambda_i$ if $n_i < m$, and $\mathbf{0}$ otherwise. We define $\Lambda_i = diag(\lambda_{i1}, \ldots, \lambda_{iJ})$, $M_i = \mu I$, where I is an identity matrix, $\Gamma_{ik} = diag(\gamma_{ik1}, \ldots, \gamma_{ikJ})$, and $\tilde{Q}$ is the infinitesimal generator for the underlying Markov chain.

Such a multi-dimensional Markov chain does not have a product form solution. The dimensionality of the system is $O(jm^K)$, making exact analysis complex. For even a small network with a large number of circuits, solving this system can become quite complex.

Analysis of a system with a finite number of servers is not usually tractable, and this is true even more so when we consider non-Poisson input processes such as MMPP. A system with an infinite number of servers, which is often more amenable to exact analysis, can be used to approximate the blocking probability in a comparable finite system. This approximation is more suitable for finite systems with a large number of servers, such as the cellular networks we consider. If each cell has a T1 line, offering voice channels at 64Kb/s, this results in 24 circuits in each cell, making this a suitable approximation for cellular networks. Blocking in the finite system is approximated by a corresponding probability in a system with an infinite number of servers and parameters equal to those in the finite system.

3. SINGLE-CELL ANALYSIS

We develop solution methods for a simple, but non-trivial one-cell case. Results for the one-cell analysis can be incorporated into the analysis of multi-cell networks, in the form of marginal analysis of the individual cells in the network.

In this paper, we consider the time blocking, that is the probability that the system is in a state of blocking in general time. Call blocking, on the other hand, is the probability that the system is in a state of being blocked to calls at arrival times. These two values are not identical here because the arrivals are not Poisson, and therefore do not see time averages. The call blocking can be calculated easily by conditioning the time blocking over arrival times.

The time blocking probability in a cell with m circuits is $P_{B,f} = \Pr(n = m)$. In order to approximate this blocking, we consider a corresponding system with an infinite number of servers. The blocking in the infinite system is represented as the probability that m circuits are busy, conditioned on m or less than m circuits being busy.

$$\begin{aligned} P_{B,i} &= \Pr(n = m | n \leq m) \\ &= \frac{\Pr(n = m, n \leq m)}{\Pr(n \leq m)} = \frac{\Pr(n = m)}{\Pr(n \leq m)} \end{aligned} \tag{2}$$

3.1 Poisson Arrivals

We begin with the simplified case of Poisson arrivals into one cell. New calls arrive into the network according to a Poisson process, at a rate of λ. A call has an exponential service time with average $1/\mu$. For a finite system with m circuits in a cell, the blocking probability for this cell is $P_{B,f} = \frac{\frac{\rho^m}{m!}}{\sum_{i=0}^{m} \frac{\rho^i}{i!}}$ where $\rho = \frac{\lambda}{\mu}$ is the offered load. The corresponding *blocking* in the infinite system is:

$$P_{B,i} = \frac{\Pr(n = m)}{\Pr(n \leq m)} = \frac{\frac{\rho^m e^{-\rho}}{m!}}{\sum_{v=0}^{m} \frac{\rho^v e^{-\rho}}{v!}} = \frac{\frac{\rho^m}{m!}}{\sum_{v=0}^{m} \frac{\rho^v}{v!}} \tag{3}$$

The approximation using the equivalent infinite system, in the case of Poisson arrivals then, is exact.

3.2 Bursty Arrivals

Next we consider a single cell with new calls arriving according to an MMPP process. The new-call arrival rate is λ_j when the underlying Markov chain is in state j. Each call has an exponential service time with average $1/\mu$. The underlying Markov Chain $J(t)$ with J states is such that ω_{ij} is the infinitesimal rate of going from state i to state j. Such a system is modeled as a multi-dimensional Markov chain with state space $\{(n, j) : n \geq 0, 1 \leq j \leq J\}$ where n is the number of busy servers in the cell, and j is the state of the underlying Markov chain. We define $p(n, j)$ to be the probability that there are n circuits busy and that the MMPP is in state j. This probability distribution is such that $p(n, j) = 0$, for $n < 0$. The system of balance equations for this system is as follows:

$$\begin{aligned} \left(\lambda_j + i\mu + \sum_{1 \leq k \leq J, k \neq j} \omega_{jk} \right) p(i, j) \;&= \\ (i+1)\mu p(i+1, j) \;&+\; \lambda_j p(i-1, j) \\ +\; \sum_{1 \leq k \leq J, k \neq j} \omega_{kj} p(i, k) \quad & 1 \leq j \leq J, \;\; i > 0 \end{aligned} \tag{4}$$

The probability generating function of the system is defined as follows:

$$P(z) = \sum_{i=0}^{\infty} p(i) z^i \tag{5}$$

$$\text{where} \;\; p(i) = p(i, 1) + p(i, 2) + \cdots + p(i, J)$$

We rewrite this as

$$P(z) = \sum_{j=1}^{J} P_j(z) \tag{6}$$

where $P_j(z) = \sum_{i=0}^{\infty} p(i,j)z^i$ is the probability generating function for each phase in the MMPP.

We now follow the methodology for a system with a two-phase MMPP in order to keep the analysis tractable. From the balance equations (4), we obtain a set of first-order differential equations for $P_1(z)$ and $P_2(z)$:

$$\begin{aligned} (\lambda_1(1-z)+\omega_{12})P_1(z) \ + \ \mu(z-1)P_1'(z) \ &= \ \omega_{21}P_2(z) \\ (\lambda_2(1-z)+\omega_{21})P_2(z) \ + \ \mu(z-1)P_2'(z) \ &= \ \omega_{12}P_1(z) \end{aligned} \tag{7}$$

Here, $A'(z)$ denotes the derivate of $A(z)$, $\frac{dA(z)}{dz}$. A second-order differential equation for each phase can be obtained from the above equations, with a change of variable $\xi = \left(\frac{\lambda_1+\lambda_2}{\mu}\right)(z-1)$. The differential equation for $P_1(z)$ is shown, and the equation for $P_2(z)$ is similar, with parameters corresponding to phase two.

$$\xi G_1''(\xi) + (\alpha - \xi)G_1'(\xi) + (\beta\xi - \gamma)G_1(\xi) = 0 \tag{8}$$

where:

$$\alpha = \frac{\mu + \omega_{12} + \omega_{21}}{\mu}, \qquad \beta = \frac{\lambda_1\lambda_2}{(\lambda_1+\lambda_2)^2},$$

$$\text{and} \quad \gamma = \frac{\omega_{12}\lambda_1 + \omega_{21}\lambda_2 + \lambda_1\mu}{\mu(\lambda+1+\lambda_2)}$$

Solving Equation (8), we get:

$$P_1(z) = e^{\frac{-\lambda_1}{\mu}(1-z)} \begin{bmatrix} {}_1U\left(\frac{\omega_{12}}{\mu}, \alpha, \frac{(\lambda_1-\lambda_2)}{\mu}(1-z)\right) \\ +c_2F_1\left(\frac{\omega_{12}}{\mu}, \alpha, \frac{(\lambda_1-\lambda_2)}{\mu}(1-z)\right) \end{bmatrix} \tag{9}$$

where $U(a,b,c) = \dfrac{1}{\Gamma(a)} \displaystyle\int_0^{\infty} e^{-ct}t^{a-1}(1+t)^{b-a+1}dt$

is the confluent hyper-geometric function and

$$F_1(a,b,c) = \sum_{i=0}^{\infty} \frac{(a)_i c^i}{(b)_i i!}$$

is the Kummer confluent hyper-geometric function with

$$(a)_i = a(a+1)(a+2)\cdots(a+i-1), \ (a)_0 = 1$$

As proposed in [9], the initial conditions $P_1(1)$ and $P_1'(1)$ are used to determine c_1 and c_2 in $P_1(z)$. $P_1(1) = \frac{\omega_{21}}{\omega_{12}+\omega_{21}}$ is the steady-state probability that the MMPP is in state 1. $P_1'(1)$, the average number of busy circuits in cell 1 is determined by taking a derivative of Equation (4)

$$P_1'(1) = \frac{\omega_{21}\lambda^{(1)}(\mu+\omega_{21}) + \omega_{12}\lambda^{(2)}\omega_{21}}{(\omega_{12}+\omega_{21})\mu(\mu+\omega_{12}+\omega_{21})}$$

$P(z)$ simplifies for certain cases, when the parameters are integers, because of simplification in the hypergeometric series. For example, for $\lambda_1 = 10$, $\lambda_2 = 15$, $\omega_{12} = 1$, $\omega_{21} = 2$, and $\mu = 1$, we have:

$$P(z) = \frac{2}{125(z-1)^3} \begin{bmatrix} e^{15(z-1)} - e^{10(z-1)}(17 - 40z + 25z^2) \\ + e^{\frac{25}{2}(z-1)}10(z-1)\cosh(\frac{5}{2}(z-1)) \\ - e^{\frac{25}{2}(z-1)}4\sinh(\frac{5}{2}(z-1)) \end{bmatrix} \tag{10}$$

This numerical solution for $P(z)$ can then be expanded around $z = 0$. The coefficients of this expansion correspond to the probabilities such that $p(i)$ is the coefficient of z^i.

In general, obtaining exact solutions for $P_1(z)$ and $P_2(z)$ involves hyper-geometric functions which include infinite series. The numerical solution is not trivial due to the infinite series, and introducing many numerical errors. For certain simplifying cases of integer-valued parameters, $P_1(z)$ in Equation (9), and $P_2(z)$ reduce to a function involving exponentials. However, when $P(z)$ is in terms of hyper-geometric series, obtaining the probabilities from $P(z)$ is not trivial. The probability generating function must be evaluated at many values of z to obtain the probabilities, and this can be quite complex.

The probabilities may also be solved recursively. Let $\Lambda = diag(\lambda_1, \ldots, \lambda_J)$, $M = diag(\mu_1, \ldots, \mu_J)$, and $\tilde{Q}$ be the generator of the underlying Markov chain. We then write the balance equation (4) recursively as such:

$$\Pi_{i+1} = F_i\Pi_i - G_i\Pi_{i-1} \tag{11}$$

$\Pi_i = [p(i,1)\ldots p(i,J)]^T$ where ^{T} denotes the transpose,

$$F_i = \left(\frac{1}{i+1}\right) M^{-1}\left(\Lambda + iM - \tilde{Q}^T\right) \quad \text{and}$$

$$G_i = \left(\frac{1}{i+1}\right) M^{-1}\Lambda$$

To obtain a first-order recursion from this second-order equation, we let $W_i = [\Pi_i, \Pi_{i+1}]^T$. Then

$$W_i = H_iW_{i-1} \qquad i \geq 1 \tag{12}$$

where:

$$H_i = \begin{bmatrix} 0 & I \\ -G_i & F_i \end{bmatrix}, \quad S = \sum_{i=1}^{\infty}\prod_{j=1}^{i} H_j, \text{ and}$$

$$W_0 \text{ satisfies } (I+S)W_0 = \begin{bmatrix} C \\ C - \Pi_0 \end{bmatrix}$$

I is an identity matrix of appropriate dimensions, and $C = [p_1, \ldots, p_J]^T$ is the steady-state probability vector of the underlying Markov chain. We solve for W_i's recursively, then normalize according to W_0, computed with a truncated sum, $S_M = \sum_{i=1}^{M}\prod_{j=1}^{i} H_j$.

When the offered load is very high, probabilities W_0 and W_i for i close to 0, may be very small. If the recursion is started at W_1, numerical inaccuracies may compound for probabilities of higher number of circuits. Also, if the probabilities near zero are very small, in the order of 10^{-10}, recursion might not continue for more than a few terms, leading to an inaccurate probability distribution after normalization. To alleviate this problem, we start near the average, and perform backward and forward recursions. The average is easily calculated from the z-transform of the probability distribution, as shown in the next section. We assume a value for W_{k^*} where k^* is close to the average. On the forward recursion, Equation (12) is used, stopping at W_M, where the probabilities are negligible. Backward recursion is performed using

$$W_i = W_{i+1} H_{i+1}^{-1}. \tag{13}$$

Using this recursive method, we avoid encountering numerical difficulties early in the recursion. The recursion is started near high probabilities so that the problem areas of very small probabilities are the stopping points for the recursion.

Generalizing this recursive procedure to a network with K cells is not possible. The probability vector for the system will have $K-1$ more dimensions. Calls can be transfered between cells, so each state will have transitions to many other states. Each cell has an infinite number of servers and the state transitions will not occur along a single dimension. Therefore, recursions of this type for a multi-cell network is not possible.

We must use approximations to solve for such systems. The fixed-point method, a widely used approximation, cannot be used here because the arrival process is MMPP. The fixed-point approximation inherently assumes the arrivals are Poisson ([3]).

We approximate the probabilities by using moments, which are relatively simple to compute. We demonstrate this technique now for the single-cell case. The differential equation for the generating function can be used to determine moments of the number in the system recursively.

Let $P(z) = (P_1(z), P_2(z), \ldots, P_J(z))$. The set of differential equations in Equation (7) can be written in matrix form as:

$$P(z)A(z) = P'(z)B(z) \tag{14}$$

$$\text{where} \quad A(z) = (z-1)\Lambda + \tilde{Q} \quad \text{and} \quad B(z) = (z-1)M$$

By repeatedly taking derivatives of this equation, moments of the system can be determined. We have the following recursive relation for the moments:

$$\begin{aligned} P^{(n)}(1) &= nP^{(n-1)}(1)A'(1)(nB'(1) - A(1))^{-1} \\ &= nP^{(n-1)}(1)\Lambda(nM - \tilde{Q})^{-1} \end{aligned} \tag{15}$$

where $P^{(n)}(1) = \frac{d^n}{dz^n}P(z)|_{z=1}$. Note that $P(1)$ is the steady-state probability of the underlying Markov chain satisfying $P(1)\tilde{Q} = 0$ and $P(1)e = 1$, e is a summing vector of 1s. $P'(1)e = E[n]$, and $P^{(k)}(1)$ are factorial moments, so that

$$P^{(k)}(1)e = E[n(n-1)\cdots(n-k+1)] \tag{16}$$

The factorial moments arise in the expansion of the probability generating function about $z = 1$. By equating the coefficients of z^n in this expansion, we obtain the the following relation ([1]), with $p(n) = \sum_{j=1}^{J} p(n,j)$:

$$p(n) = \sum_{r \geq n} (-1)^{r-n} \frac{m_{[r]}}{r!} \binom{r}{n} \tag{17}$$

where $m_{[r]} = P^{(r)}(1)e$ is the rth factorial moment. This infinite sum is truncated at k where $m_{[k]}/(k-n)!$ is negligible. In our experimentation, however, we have found that the computation of $\{p(n)\}$ can be numerically unstable. The summation in Equation (17) has an alternating series where the terms are very large. This presents high numerical errors in the computation of $p(n)$. Also, moments of very high order are required for an acceptable approximation of the blocking. This method cannot be used suitable to approximate the blocking.

In the next section we introduce an approximation without numerical instability, that requires fewer moments.

4. MIXED POISSON APPROXIMATION

We approximate the distribution of the number of busy circuits in a single cell for an infinite system, by a Mixed-Poisson distribution of L terms:

$$\tilde{p}(n) = \sum_{i=1}^{L} \frac{\alpha_i e^{-\beta_i} \beta_i^n}{n!} \quad \text{where} \quad \sum_{i=1}^{L} \alpha_i = 1 \tag{18}$$

In order to approximate $\{p(n)\}$ with an L-term mixed-Poisson distribution, only the first $2L-1$ moments are required. These moments for the single-cell case are calculated using Equation (15). We then find the parameters α_i and β_i that satisfy the given moment information. A system of $2L-1$ nonlinear equations is formed, using the expression for the factorial moments of $\{\tilde{p}(n)\}$:

$$m_{[r]} = \sum_{i=1}^{L} \alpha_i \beta_i^r \quad 0 \leq r \leq 2L-1 \quad \text{and} \quad m_0 = 1 \tag{19}$$

Using the given moment information, the above system of nonlinear equations is solved to obtain α_i and β_i. When $\tilde{p}(n)$ has a distribution of two terms, we obtain quadratic equations for the solutions of α_i and for the solutions of β_i. Specifically, α_i are the solutions to the following equation, quadratic in α

$$h\alpha^2 - h\alpha - (m_{[1]}^2 - m_{[2]})^3 = 0 \tag{20}$$

where $h = -3m_{[1]}^2 m_{[2]}^2 + 4m_{[1]}^3 m_{[3]} - 6m_{[1]}m_{[2]}m_{[3]} + 4m_{[2]}^3 + m_{[3]}^2$

When $m_{[1]}^2 = m_{[2]}$, as in the case of a Poisson distribution, the solutions to the above equation for α_i are 0 and 1. This corresponds exactly to the Poisson distribution, with $\alpha = 1$ and $\beta = m_{[1]}$, the average.

A three-term distribution for $\tilde{p}(n)$ results in cubic equations for the solutions of α_i and β_i respectively. It can be argued that the solution for a k-term $\tilde{p}(n)$ results in equations that are polynomial in α_i and β_i, of order k respectively. A good approximation to blocking in the finite system can be obtained by using relatively few moments, as shown in Section 6.

5. MULTIPLE-CELL ANALYSIS

5.1 Poisson Arrivals

We consider a simple network of K cells with Poisson arrivals into the network. New calls arrive into cell i according to a Poisson process with rate λ_i. The call holding time for a call in cell i is exponential with mean $1/\mu_i$. A call is handed off from cell i to cell j with rate γ_{ij}, and $\gamma_{ii} = 0$. Each cell has an infinite number of servers. The system state is described by $(\vec{n}) = (n_1, n_2, \ldots, n_K)$, where n_i is the number of busy circuits in cell i. We denote by $p(\vec{n})$, the steady-state probability that the cells $(1, \ldots, K)$ have $n_1, \ldots, n_K)$ circuits busy, respectively. We introduce the notation: $a_i \triangleq (0, \ldots, n_i = a, \ldots, 0)$. This system has the following equilibrium equation:

$$p(\vec{n})\left[\sum_{i=1}^{K}\lambda_i + \sum_{i=1}^{K} n_i\mu_i + \sum_{i=1}^{K} n_i \sum_{\substack{j=1\\ j\neq i}}^{K}\gamma_{ij}\right] =$$
$$\sum_{i=1}^{K} p(\vec{n}-1_i)\lambda_i + \sum_{i=1}^{K} p(\vec{n}+1_i)(n_i+1)\mu_i$$
$$+\sum_{j=1}^{K}(n_i+1)\sum_{i=1}^{K} p(\vec{n}+1_i-1_j)(n_i+1)\gamma_{ij} \quad (21)$$

Such a system results in a continuous time Markov chain, with the following product-form steady-state probability distribution:

$$p(\vec{n}) = \prod_{k=1}^{K}\frac{\rho_i^{n_i}e^{-\rho_i}}{n_i!} \quad (22)$$

where $\rho_i = \frac{\Lambda_i}{\mu_i+\Gamma_i}$, $\Gamma_i = \sum_{j=1}^{K}\gamma_{ij}$, and Λ_i, the offered load is obtained through the following system of linear equations:

$$\Lambda_i = \lambda_i + \sum_{j=1}^{K}\frac{\gamma_{ji}}{\mu_j+\Gamma_j}\Lambda_j \quad (23)$$

The probability distribution of busy circuits in any particular cell is dependent on other cells only through the offered load, Λ. The probability of the number of busy circuits in cell i, then is independent of the distribution in other cells, as follows:

$$p(n_i) = \frac{\rho_i^{n_i}e^{-\rho_i}}{n_i!} \quad (24)$$

We consider cellular systems with 24 channels in each cell, based on a T1 line with 64Kb/s voice channels. For such a large number of circuits, the infinite-approximation of the finite system performs well, as shown in Table 1, for a two-cell network. In addition, we consider systems where suitable blocking is in the range of 10^{-2} to 10^{-3}. Table 1 shows the accuracy of the approximation for various values of offered load.

Table 1: Approximation of blocking in a finite system with the corresponding probability in the infinite system, for Poisson Arrivals.

Offered Load	Blocking		Relative Error
	Finite System	Infinite System	
ρ_1	$P_{B,f}$ ($\times 10^{(-2)}$)	$P_{B,i}$ ($\times 10^{(-2)}$)	%
14	0.397103	0.432942	9.03
14.5	0.552335	0.611177	10.65
15	0.746494	0.839351	12.44
15.5	0.982649	1.123796	14.36
16	1.262687	1.469818	16.40
16.5	1.587239	1.881385	18.53
17	1.955739	2.360932	20.72

The results in Table 1 are for the new call blocking experienced in one cell of a two-cell network. The system parameters are as such: $\lambda_i = \lambda$, $\mu_i = 1$, and $\gamma_{ij} = 1$ for $i \neq j$. The infinite approximation is an overestimate, and performs better for smaller values of offered load, ρ. This is because at higher loads, the probability distribution of the number of busy servers has a greater mass at higher values of n, possibly higher than m since there is no limit on the number of servers. So $\Pr(n = i)$ for $i > m$, now have larger values, impacting on the approximation of $P_{B,f}$. In the range of blocking suitable in cellular systems, 10^{-2} to 10^{-3}, the infinite approximation of the finite system is acceptable.

5.2 Bursty Arrivals

We now consider bursty arrivals into a multi-cell network. We assume a network of K cells with an infinite number of circuits in each cell. The arrival rate of new calls into the network and the handoff rates between cells are governed by a Markov chain. When the underlying chain is in state j, new calls arrive into cell i with a Poisson rate λ_{ij} and the handoff rate between cell i and cell k is γ_{ikj}. The call holding time of a call in cell i is exponential with rate μ_i. This system is modeled as a continuous-time Markov chain with state space $\{(n_1, n_2, \ldots, n_K, j) : n_i \geq 0, 1 \leq j \leq J\}$ where n_i is the number of busy circuits in cell i, and j is the state of the underlying Markov chain.

The arrival and handoff rates in the cells may be controlled by different Markov chains, such that the rates in cell i are controlled by the modulating chain $J_i(t)$. This refers to the case where each cell has a independent pattern of correlation in the arrivals and handoffs. In addition, the handoff rates and arrival rates may be governed by separate chains. These various MMPP-s may be superimposed, resulting in one MMPP with an expanded state-space, for the whole network. Suppose we have n individual MMPP-s with generators $\tilde{Q}_i$ and rate matrices Λ_i. The generator $\tilde{Q}$ and rate matrix Λ of the composite MMPP is as follows:

$$\tilde{Q} = \tilde{Q}_1 \oplus \tilde{Q}_2 \oplus \cdots \oplus \tilde{Q}_n$$

$$\Lambda = \Lambda_1 \oplus \Lambda_2 \oplus \cdots \oplus \Lambda_n$$

where $\oplus$ is the Kronecker sum. The Kronecker sum is defined as follows:

$$A \oplus B = I_B \otimes A + B \otimes I_A$$

where $\otimes$ is the Kronecker product.

We assume $p(n_1, n_2, \ldots, n_K) = 0$ when $n_i < 0$ for any i. The following balance equations satisfy the continuous-time Markov chain resulting from a two-cell network:

$$\begin{bmatrix} \lambda_{1j} + \lambda_{2j} + n_1\mu_1 + n_2\mu_2 + \\ n_1\gamma_{12j} + n_2\gamma_{21j} + \sum_{k=1}^{J} \omega_{jk} \end{bmatrix} p_j(n_1, n_2) \tag{25}$$

$$\begin{aligned} &= \lambda_{1j} p_j(n_1 - 1, n_2) + \lambda_{2j} p_j(n_1, n_2 - 1) \\ &+ (n_1 + 1)\mu_1 p_j(n_1 + 1, n_2) + (n_2 + 1)\mu_2 p_j(n_1, n_2 + 1) \\ &+ \sum_{k=1}^{J} \omega_{kj} p_k(n_1, n_2) + (n_1 + 1)\gamma_{12j} p_j(n_1 + 1, n_2 - 1) \\ &+ (n_2 + 1)\gamma_{21j} p_j(n_1 - 1, n_2 + 1) \quad n_1, n_2 \geq 0,\ 1 \leq j \leq J \end{aligned}$$

The recursive method used in the one-cell analysis to solve for the steady-state probabilities cannot be used here. From the balance equation above, it is clear that $p(n_1, n_2, j)$ is in terms of $p(n_1, n_2, k)$ for $k \neq j$, $p(n_1 - 1, n_2, j)$, $p(n_1, n_2 - 1, j)$, $p(n_1 + 1, n_2, j)$, $p(n1, n_2 + 1, j)$, $p(n_1 + 1, n_2 - 1, j)$, and $p(n_1 - 1, n_2 + 1, j)$. Both cells have an infinite number of circuits and the state transitions occur along multiple dimensions. Therefore,we cannot write recursive relations between the Π_{n_1,n_2}s. Approximation techniques must be applied to networks with more than one cell. The moment information for this system can be used to approximate the probabilities.

We derive a partial differential equation for the probability generating function for the number of busy servers in each cell in the system. Let $\Lambda_i = \text{diag}(\lambda_{i1}, \lambda_{i2}, \ldots, \lambda_{iJ})$, $\Gamma_{ik} = diag(\gamma_{ik1}, \gamma_{ik2}, \ldots, \gamma_{ikJ})$ and $\tilde{Q}$ is the generator of the underlying Markov chain. We denote the following:

$$\vec{z} = (z_1, z_2, \ldots, z_K)$$

$$P_j(\vec{z}) = \sum_{\substack{n_i \geq 0 \\ i=1,\ldots,K}} p(n_1, \ldots, n_K, j) z_1^{n_1} \cdots z_K^{n_K}$$

$$P(\vec{z}) = (P_1(\vec{z}), \ldots, P_J(\vec{z}))$$

From the balance equations for a K-cell network, similar to Equation (26), for this system, we get the following partial differential equation for the probability generating function:

$$P(\vec{z})A(\vec{z}) = \sum_{i=1}^{K} \frac{\partial}{\partial z_i} P(\vec{z}) B_i(\vec{z}) \tag{26}$$

$$\text{where} \quad A(\vec{z}) = \sum_{i=1}^{K} (z_i - 1)\Lambda_i + \tilde{Q},$$

$$B_i(\vec{z}) = (z_i - 1)M_i + \sum_{\substack{k=1 \\ k \neq i}}^{K} (z_i - z_k)\Gamma_{ik} \quad i = 1, \ldots, K$$

There is no closed-form solution for $P(\vec{z})$ in this case. We can use this equation to obtain a recursive relation for the joint moments of the network. We demonstrate this for a two-cell network to keep the analysis simple yet non-trivial. Note that $P(1, 1)$ is the steady state probability vector of the underlying Markov chain. We define some notation:

$$A^{(i,j)} = \frac{\partial^i \partial^j}{\partial z_1^i \partial z_2^j} A(z_1, z_2)|_{z_1 = z_2 = 1}$$

Then from Equation (26) we get:

$$P^{(i,j)}(A^{(0,0)} - iB_1^{(1,0)} - jB_2^{(0,1)}) \tag{27}$$

$$- \quad P^{(i-1,j+1)}(iB_2^{(1,0)}) - P^{(i+1,j-1)}(jB_1^{(0,1)}) =$$

$$- \quad P^{(i-1,j)}(iA^{(1,0)}) + P^{(i,j-1)}(jA^{(0,1)}) \tag{28}$$

Let $N(i) = (P^{(i,0)}, P^{(i-1,1)}, \ldots, P^{(0,i)})$ be the ith order moment. We then have the following recursive relation:

$$N(i) = N(i-1)\Lambda(i)(-B(i))^{-1} \qquad i > 0 \tag{29}$$

$$\text{where } N(0) = P(1, 1), \quad \Lambda(1) = (\Lambda_1, \Lambda_2),$$

$$\Lambda(i) = \begin{bmatrix} i\Lambda_1 & \Lambda_2 & & & \\ & (i-1)\Lambda_1 & 2\Lambda_2 & & \\ & & \ddots & \ddots & \\ & & & 2\Lambda_1 & (i-1)\Lambda_2 & \\ & & & & \Lambda_1 & i\Lambda_2 \end{bmatrix}$$

and $B(i)$ has the following structure:

$$\begin{bmatrix} \tilde{Q} - iK_1 & \Gamma_{12} & & & \\ i\Gamma_{21} & \tilde{Q} - (i-1)K_1 - K_2 & & & \\ & (i-1)\Gamma_{21} & \ddots & & \\ & & \ddots & (i-1)\Gamma_{12} & \\ & & \ddots & \tilde{Q} - K_1 - (i-1)K_2 & i\Gamma_{12} \\ & & & \Gamma_{21} & \tilde{Q} - iK_2 \end{bmatrix}$$

$$K_1 = M_1 + \Gamma_{12} \quad \text{and} \quad K_2 = M_2 + \Gamma_{21}$$

The probability of the number of busy circuits in the system, $p(n_1, \ldots, n_K) = \sum_{j=1}^{J} p(n_1, \ldots, n_K, j)$ can be obtained using the joint moment information calculated using Equation (29). Let $m_{[r_1, r_2, \ldots, r_K]} = P^{(r_1, r_2, \ldots, r_K)}(\mathbf{1})$. Then, by writing the Taylor series expansion of $P(\vec{z})$ at $(z_1 - 1)(z_2 - 1) \cdots (z_K - 1)$ and equating the coefficients of $z_1^{n_1} z_2^{n_2} \cdots {}_K^{n_K}$, the probabilities are obtained as follows:

$$p(n_1, \ldots, n_K) = \sum_{\substack{r_i = n_i \\ i=1,\ldots,K}}^{\infty} \frac{m_{[r_1, \ldots, r_K]}}{r_1! \cdots r_K!} \prod_{i=1}^{K} (-1)^{r_i - n_i} \binom{r_i}{n_i} \tag{30}$$

As in the case of the single-cell network, we encounter numerical difficulties in computing $p(n_1, \ldots, n_K)$ using this method. It is impractical to use Equation (30) in obtaining the joint probabilities of the number of busy circuits and performance measures such as blocking. We focus on new-call blocking as the performance measure of interest, and use the marginal information on the number of busy circuits in each cell to this end. From Equation (29) we obtain the marginal moments. For a two-cell network, $m_{1\,[r_1]} = m_{[r_1, 0]}$

and $m_{2[r_2]} = m_{[0,r_2]}$. We then use the Mixed Poisson distribution presented in Section 4 to approximate the marginal probabilities of busy circuits in each cell, i.e., $p(n_1) = \sum_{n_2 \geq 0} p_1(n_1, n_2)$

5.3 Impact of Mobility

We consider the impact of mobility in the above models with an infinite number of circuits in each cell. We find that in certain cases, mobility has no effect on $p(\vec{n})$. When arrivals are Poisson, we observe this result in the case where the network is symmetric, such that $\lambda_i = \lambda$, $\mu_i = \mu$, and that the handoff rate *into* a cell is equal to the handoff rate *out of* that cell, that is, Γ_i and $\sum_{j=1}^{K} \gamma_{ji}$ are equal for all i. For such conditions, ρ_i in Equation (22) is independent of mobility and reduces to $\frac{\lambda}{\mu}$. The probability distribution is thus independent of the mobility parameter, and the distribution in each cell is as for the single-cell case with no mobility.

For a system with bursty arrivals and an infinite number of servers, we see a similar result when the marginal densities of the number of busy circuits in the cells are equal. This includes the condition that the network be symmetric, as described above, for each phase in the MMPP. Under such conditions, the recursive relation for the marginal moments of the network reduces to that of a single-cell case, with no mobility parameters. For example, for a two-cell network, the generating function for the marginal distribution of busy servers in cell 1 is:

$$F(z) = (F_1(z), \dots, F_J(z)) = \sum_{n1=0}^{\infty} \sum_{n2=0}^{\infty} P(n_1, n_2) z^{n_1}$$

We consider the marginal information about cell 1 from the balance equations shown in Equation (26). We find that if the marginal densities of the cells are equal, we arrive at the following for the recursive relation of the moments:

$$G^{(n)}(1) = nG^{(n-1)}(1)\Lambda_1(nM_1 - \tilde{Q})^{-1} \tag{31}$$

This is identical to Equation (14), the recursive relation for the single-cell case. All the moments of marginal distribution are thus are independent of the mobility parameter. Therefore, the marginal distribution of the number of busy circuits in the cells is independent of mobility when conditions mentioned above are met and there are an infinite number of circuits in each cell, even when the arrivals follow an MMPP.

We claim that in spite of the above observation, the infinite model is still a good approximation of the finite system. The model with an infinite number of circuits serves to model systems with a large number of circuits. As mentioned earlier, when we consider cellular networks with even just 10% allocation of bandwidth to CDMA, up to 132 channels may be supported in one cell. When we consider CDMA cellular networks, we can easily expect a very large number of channels per cell. In order to study numerically, the impact of mobility in a system with a very large number of circuits, we look at the system with 24 circuits, and a low average load. A very low average load in a system with 24 circuits is analogous to a system with very large number of circuits and moderate load, in terms of system utilization. We look at the point probability $\Pr(n = 0)$ to study the impact of mobility on the system. We show only one point in the probability distribution, however note that this trend is seen for the distribution as a whole. Table 2 shows $\Pr(n = 0)$ with increasing mobility for a 24-circuit system, an average load of 4. We represent mobility with the parameter $r_i = \sum_{i=1}^{K} \gamma_{ij}$, and here, $r = r_i$ for all cells i. This table shows that mobility does not have a big impact on $\Pr(n = 0)$. This supports the claim that for cells with a very large number of circuits, mobility has less of an impact on the system.

Table 2: Impact of mobility on $\Pr(n = 0)$

Mobility	$\Pr(n=0)$ ($\times 10^{-3}$)
0	9.276712663
2	9.276712924
4	9.276713126
6	9.276713325
8	9.276713521
10	9.276713714

In the next section we provide numerical results for new-call blocking in the network.

6. NUMERICAL RESULTS

6.1 Single-Cell Network

We assume one cell in the network, with an infinite number of servers. The MMPP process has two states such that ω_{ij} is the infinitesimal rate of going from state i to state j. The arrival rate is λ_i in state i, and the average load is $\bar{\lambda} = \lambda_1 p_1 + \lambda_2 p_2$ where p_i is the steady-state probability of the underlying Markov chain being in state i. The call holding time, $1/\mu$, is normalized to 1. We study the accuracy of the mixed-Poisson approximation at various loads, and the effect of burstiness in traffic on the new call blocking and on the performance of the approximation.

We study the two-term mixed-Poisson approximation (MPA-2) for blocking when arrivals are non-Poisson, by setting $\lambda_1/\lambda_2 = 0.8$, and varying the average offered load, $\bar{\lambda}$. Table 3 compares the MPA-2 with blocking in the finite system with 24 circuits, in the range of 10^{-2} to 10^{-3}. The approximation is very good, and becomes more accurate for smaller values of average offered load. Even at a relatively high average load of 17, the approximation is very accurate as compared to blocking in the finite model, with only 1.88% relative error. The two-term MPA provides a close approximation to blocking in the finite system for blocking in the range of 10^{-2} to 10^{-3} and average load in the range of 14 to 17, for $\lambda_1/\lambda_2 = 0.8$.

To describe the burstiness in the arrival traffic, we use the following measure, as defined in [6]:

$$\theta = \frac{\omega_{12}\omega_{21}(\lambda_1 - \lambda_2)^2}{(\omega_{12} + \omega_{21})^3} \tag{32}$$

We use θ as a measure of deviation from a Poisson process with the same average arrival rate, $\bar{\lambda}$. For a Poisson process, it is apparent from the above equation that $\theta = 0$. The arrival traffic is characterized as increasingly bursty, as the value of θ is increased.

Table 3: **Mixed-Poisson Approximation for a Single-Cell Network**

Offered Load	Blocking Finite System	Blocking Infinite, MPA	Relative Error
λ	$P_{B,f}$ ($\times 10^{-2}$)	$P_{B,i}$ ($\times 10^{-2}$)	%
14	0.502509	0.497613	0.97
14.5	0.698195	0.690621	1.08
15	0.944740	0.933270	1.21
15.5	1.247618	1.230643	1.36
16	1.611185	1.586661	1.52
16.5	2.038460	2.003881	1.70
17	2.531007	2.483400	1.88

For the above two-phase MMPP, all four parameters λ_1, λ_2, ω_{12}, and ω_{21} affect the arrival process. In order to keep the numerical analysis tractable, we consider here, cases of balanced mean where $\lambda_1 p_1 = \lambda_2 p_2$. We find that the triplet $(s, \bar{\lambda}, T)$ can fully characterize the source, where $s = \frac{\lambda_1}{\lambda_2}$, $\bar{\lambda}$ is the average load, and $T = 1/\omega_{12}$ is the average time spend in state 1 of the MMPP before transition to state 2. For further numerical results, we set $T = 1$ and vary s and $\bar{\lambda}$. From $(s, \bar{\lambda}, T)$, we set the parameters of the system as follows:

$$\omega_{12} = \frac{1}{T}, \quad \omega_{21} = \frac{1}{sT}, \quad \lambda_1 = \frac{\bar{\lambda}(1+s)}{2}, \quad \lambda_2 = \frac{\lambda_1}{s}$$

Figure 1 shows the effect of time correlation in the arrival process on blocking in a single-cell network. The MPA-2 refers to approximation in the model with an infinite number of servers and the exact finite result is for the finite system with 24 circuits in each cell. We observe that as arrivals are more highly correlated, new calls to the cell experience higher blocking. As traffic becomes increasingly bursty, a bursty period will result in a higher number of circuits being occupied. This causes an increased blocking not only during this bursty period, but also following it, since call holding times are unaffected. We also observe from this figure that the MPA-2 performs very well in approximating the blocking in the finite system. The approximation becomes relatively poorer as traffic burstiness increases. However, for reasonable correlation, in the range of $\theta = 0$ to 12, the relative error is less than 10%. The MPA-2 provides a good approximation of blocking in the finite model.

6.2 Multiple-Cell Network

We study the effect of space correlation on blocking in a multi-cell network. Figure 2 shows the results on blocking for a pair of negatively correlated cells. The arrivals to the two cells are correlated such that when the arrival rate in cell 1 increases, the arrival rate in cell 2 decreases, and vice versa. In Figure 2, when the MMPP is in state 1, the arrival rates to cells 1 and 2 are $\frac{\bar{\lambda}_1(1+s)}{2}$ and $\frac{\bar{\lambda}_1(1+s)}{2s}$, respectively, and vice versa when the MMPP is in state2. The average load in cell 1 is kept constant, at 16. Due to correlation between the cells, the average load in cell 2 increases as θ increases. The figure shows that as correlation among cells increases, blocking increases. There is a greater increase in blocking in cell 2 because its average load increases as θ increases. The difference in blocking between the two cells is greater at higher values of θ. The higher the correlation between

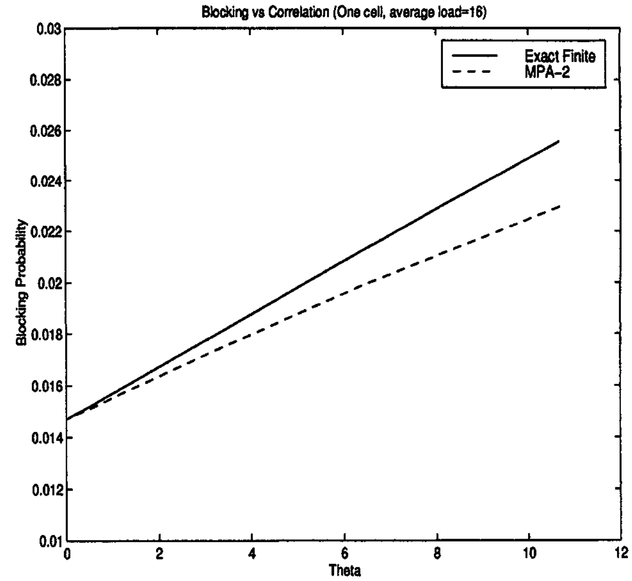

Figure 1: Blocking vs Correlation in Arrivals

arrivals to the two separate cells, the greater the difference in blocking experienced by users of these two cells. The MPA-2 performs fairly well, although it provides a relatively poorer approximation as the cells become more highly correlated. The relative error in the approximation for the higher values of θ, is still acceptable, at 10%-15%. We observe that MPA-2 is quite accurate in the range of $\theta = 0$ to 20.

We now study the impact of mobility on the finite model with 24 circuits in each cell. The average load is kept constant at 16. Figure 3 shows the impact of mobility on blocking for two values of θ. As mobility increases, blocking decreases in both cases. Blocking is higher for larger values of θ. The difference between the blocking at the two values of θ decreases as mobility is increased. At very high rates of mobility, correlation in the arrival traffic has a smaller effect on blocking.

7. CONCLUSION

We have modeled large cellular networks with MMPP arrivals to capture correlation in new-call traffic. Models of networks containing multiple cells result in multi-dimensional Markov chains, which do not have product-form solutions. We approximate the finite system with an equivalent infinite system. This approximation is exact for a single-cell case with Poisson arrivals, and very good for a multi-cell network with Poisson arrivals. We have introduced the methodology of recursively computing moments of a multiple-cell network with bursty arrivals. These moments are then used to approximate the steady-state probabilities. We have introduced the mixed-Poisson distribution as a good approximation of the distribution of the number of busy circuits for multiple-cell networks with bursty arrivals.

For a system with an infinite number of circuits in each cell, we have found that for certain cases, there is no effect of mobility on the steady-state distribution of busy circuits.

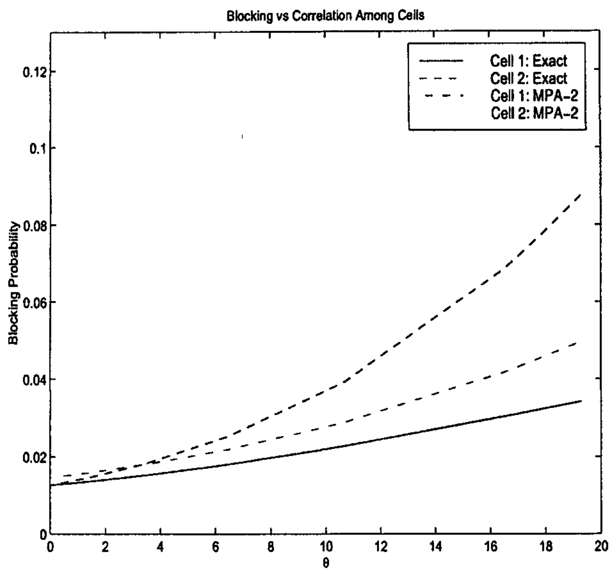

Figure 2: Blocking vs Correlation in Arrivals Among Cells

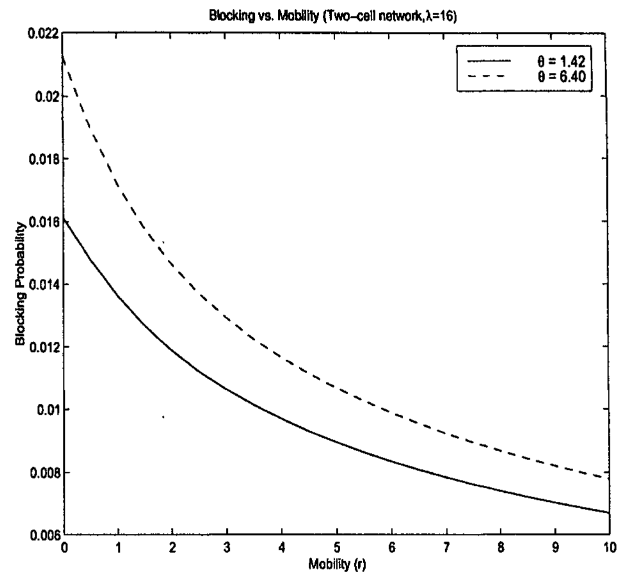

Figure 3: Blocking vs Mobility

For Poisson arrivals, this result is seen when the network is symmetric. For bursty arrivals, we observe this result when the marginal densities of busy circuits in the cells are equal, and the network is symmetric. This important result shows that for a network with a very large number of circuits in each cell, this is very little impact of mobility on the performance of the system.

We have found the two-term mixed-Poisson approximation to be a very good fit to the exact finite model. Even at high loads and moderate correlation in arrival traffic, the relative error in approximation is less than 5%. For the range of blocking we are concerned with, 10^{-2} to 10^{-3}, the mixed-Poisson distribution is a very close approximation. In the exact analysis of a single cell, we found that an increase in burstiness in the arrival traffic increases blocking. The mixed-Poisson approximation is quite accurate for reasonable traffic burstiness, that is θ in the range of 0 to 15. In a multiple-cell network where arrivals into separate cells are negatively correlated, we found that blocking increases as the cells more highly correlated. Also, for a two-cell network, the higher the correlation, the greater the difference in blocking between the two cells. We have also studied the impact of mobility of users in a network with few circuits, on blocking. As users are increasingly mobile, they experience a smaller blocking. In addition, at very high mobility, the correlation structure of the arrivals has a smaller impact on blocking.

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