Fixed-Alternate Routing and Wavelength Conversion in Wavelength-Routed Optical Networks

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Abstract—Consider an optical network which employs wavelength-routing crossconnects that enable the establishment of wavelength-division-multiplexed (WDM) connections between node pairs. In such a network, when there is no wavelength conversion, a connection is constrained to be on the same wavelength channel along its route. Alternate routing can improve the blocking performance of such a network by providing multiple possible paths between node pairs. Wavelength conversion can also improve the blocking performance of such a network by allowing a connection to use different wavelengths along its route. This work proposes an approximate analytical model that incorporates alternate routing and sparse wavelength conversion. We perform simulation studies of the relationships between alternate routing and wavelength conversion on three representative network topologies. We demonstrate that alternate routing generally provides significant benefits, and that it is important to design alternate routes between node pairs in an optimized fashion to exploit the connectivity of the network topology. The empirical results also indicate that fixed-alternate routing with a small number of alternate routes asymptotically approaches adaptive routing in blocking performance.

Index Terms—Adaptive routing, alternate routing, lightpath, optical network, wavelength conversion, wavelength routing, WDM.

I. INTRODUCTION

A. Wavelength Routing, Wavelength Conversion, and Alternate Routing

WAVELENGTH-DIVISION multiplexing (WDM) divides the tremendous bandwidth of a fiber (potentially, a few tens of terabits per second) into many nonoverlapping wavelengths (WDM channels) [1]. Each channel can be operated asynchronously and in parallel at any desirable speed, e.g., peak electronic speed of several tens of gigabits per second. An access node may transmit signals on different wavelengths, which are coupled into the fiber using wavelength multiplexers. An optical signal passing through an optical switch may be routed from an input fiber to an output fiber without undergoing optoelectronic conversion. If wavelength converters are present in a switch, the input optical signal can be translated from one wavelength channel to another wavelength channel at the
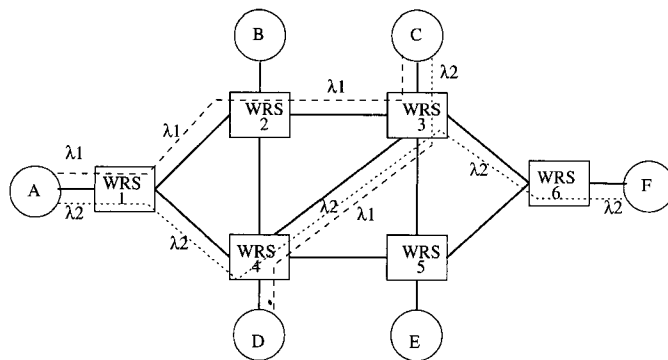
Fig. 1. Architecture of a wavelength-routed optical network.

output. A switch is capable of full wavelength conversion if a wavelength channel on any input port may be converted to any wavelength channel on any output port. A switch is capable of *limited* or *sparse* wavelength conversion if the switch has a limited number of wavelength conversion units, where a unit of wavelength conversion can be utilized to convert the wavelength channel of an optical signal passing through the switch. Fig. 1 illustrates a wavelength-routed optical network consisting of six access nodes (labeled A through F) and six switches (labeled 1 through 6).[1]

In such a network, a connection is set up by establishing a *lightpath* from the source node to the destination node. A lightpath is an optical channel which may span multiple fiber links to provide a circuit-switched interconnection between two nodes. In the absence of wavelength converters, a lightpath would occupy the same wavelength on all fiber links that it traverses. This is called the *wavelength-continuity constraint*. Two lightpaths on a fiber link must also be on different wavelength channels to prevent the interference of the optical signals. Fig. 1 shows two wavelength-continuous lightpaths: one between nodes A and C on wavelength λ_1, and another between nodes A and F on wavelength λ_2.

When wavelength converters [2] are present at switches, a lightpath may switch between different wavelengths on the route from source to destination. In Fig. 1, a wavelength-converted lightpath between nodes D and C is illustrated, where the wavelength-converted lightpath occupies wavelength λ_1 on links {D, 4} and {4, 3}, and wavelength λ_2 on link {3, C}, with wavelength conversion at switch 3. When alternate routing is implemented, the route for a lightpath can be one among a

[1]Note that, in this model, associated with a node, there is a switch and vice versa, e.g., node A and switch 1; for the simplicity of exposition, we will refer to the node–switch combination as an integrated unit, and continue to refer to this combination as a node.

set of alternative routes. Wavelength conversion and alternate routing are potentially beneficial schemes which can alleviate the wavelength-continuity constraint in optical networks. Wavelength conversion is a "hardware/software" solution in the sense that it requires the addition of wavelength converters in the network, as well as algorithms and protocols to manage the wavelength converters. Alternate routing is a "software" solution in the sense that it needs addition of signaling, control, and management protocols that can perform alternate routing. This work will examine the interplay between alternate routing and wavelength conversion in optical neworks.

Intelligent optical networks are expected to allow near-real-time dynamic provisioning of optical services. Such dynamic provisioning of optical connections corresponds to a network operations environment where the time scale for lightpath provisioning and teardown is of the order of seconds, and the lightpath holding time can be in the order of minutes. Dynamic lightpath provisioning in intelligent optical networks will enable the next generation of applications that require short connection durations. Therefore, when dynamic provisioning of lightpaths is enabled, the blocking probability of the optical network becomes a meaningful metric to analyze.

B. Previous Work

Routing and wavelength assignment in optical networks was introduced in [3], and was first analyzed in [4]. Routing strategies in wavelength-routed optical networks were considered in [5]–[10]. In [7], the authors reported that dynamic routing schemes such as least-loaded routing achieve significantly better blocking performance when compared with fixed shortest-path routing, in wavelength-continuous and wavelength-convertible networks. In [10], the authors examined adaptive wavelength routing and reported that adaptive routing outperforms constrained routing schemes such as alternate routing. The work in [5] examined three routing strategies and considered their impact on the dimensioning of the network. In [6], the authors proposed an analytical model for alternate routing, and considered the effect of blocking probability of paths with different numbers of hops and different wavelength-assignment policies. They also considered dynamic routing and compared the performance of alternate routing with dynamic routing.

The benefits of wavelength conversion have been a subject of interest in the past [7], [11]–[13]. It has been shown that, with fixed routing, wavelength conversion provides about 30%–40% improvement in blocking probability, and that most of the benefits can be obtained using sparse conversion. In [7], the authors reported that, with dynamic routing schemes, the wavelength-conversion gain is more than the wavelength conversion gain with fixed shortest path routing. Blocking probability models for a wavelength continuous path in optical networks were proposed in [19], [13]. Least-congested routing in wavelength-routed optical networks were examined in [19], [20]. Alternate routing has been extensively researched in loss networks [17], [21]–[25]. Fixed-point approximation models for loss networks with alternate routing was studied in [17], and for state-dependent routing in [21], [22], [25].

TABLE I
ROUTING TABLE AT NODE A FOR THE NETWORK IN FIG. 1,
WITH TWO ALTERNATE ROUTES TO EACH DESTINATION

Destination	Route 1	Route 2
B	A,1,2,B	A,1,4,2,B
C	A,1,2,3,C	A,1,4,3,C
D	A,1,4,D	A,1,2,4,D
E	A,1,4,5,E	A,1,2,3,5,E
F	A,1,2,3,6,F	A,1,4,5,6,F

Our work focuses on the interplay between fixed-alternate routing and different degrees of wavelength conversion. We develop a computational model that enhances earlier models with a model for fixed alternate routing. Our model utilizes an existing model for a wavelength-continuous path from [19]. Using the computational model, and with simulations, we examine the relative benefits of sparse wavelength conversion and alternate routing.

In the rest of this section, we provide precise algorithms for 1) fixed-alternate routing, 2) adaptive routing, 3) wavelength assignment, and 4) connection setup.

C. Fixed-Alternate Routing

Fixed-alternate routing requires that each access node in the network have a routing table, which contains an ordered list of a limited number of fixed routes to each destination node. When a connection request arrives, the source node attempts routes in sequence from the routing table, until a route with a valid wavelength assignment is found (the wavelength assignment algorithm is specified in Section I-E). If no available route is found from the list of alternate routes, then the connection request is blocked and lost. Fixed-alternate routing provides benefits such as 1) simplicity of control to setup and teardown lightpaths, and 2) fault tolerance upon link failures [17].

A *direct* route between a source node s and a destination node d is defined as the first route in the list of routes to d in the routing table at s. An *alternate* route between s and d is any route other than the first route in the list of routes to d in the routing table at s. The term "alternate routes" is also employed to describe all routes (including the direct route) from a source node to a destination node. As an example, Table I illustrates the routing table at node A for the network shown in Fig. 1. In this example, each source maintains one direct route and one alternate route, for a total of two alternate routes, to each destination node.

For the networks considered here, the routing tables at each node are ordered by the hop distance to the destination. Therefore, the shortest-hop path to the destination is the first route in the routing table. When there are ties in the hop distance between different routes, the ordering among them in the routing table is random.

D. Adaptive Routing

In adaptive routing, the route from a source node to a destination node is chosen dynamically, depending on the network state. The network state is determined by the set of all connections that are currently in progress. One form of adaptive routing

which we will consider in this work is adaptive-shortest-cost path routing under which each link in the network has a cost of 1 unit, and each wavelength-converter link (in the layered-graph model) will have a cost of c units. When a connection arrives, we determine the shortest-cost path between the source node and the destination node. If there are multiple paths with the same distance, one of them is chosen randomly. By choosing the wavelength conversion cost c appropriately, we can ensure that wavelength-converted routes are chosen only when wavelength-continuous paths are not available (e.g., we can choose c to be the cost of the longest wavelength-continuous path in the network). In shortest-cost adaptive routing, a connection is blocked only when there is no route (either wavelength-continuous or wavelength-converted) from the source node to the destination node in the network. Adaptive routing requires extensive support from the control and management protocols to continuously update the routing tables at access nodes.

E. Wavelength Assignment

The wavelength-assignment algorithm assigns a wavelength to each link in the route, with appropriate wavelength conversion. This work assumes the following random wavelength-assignment algorithm. Let R be the *wavelength reservation parameter*, which is defined implicitly in the wavelength-assignment algorithm. The wavelength reservation parameter may be used to prevent alternate routes from consuming wavelengths that might otherwise be used by direct routes. Given a route r to which we need to assign wavelength(s), let S be the set of idle wavelengths available on the route, i.e., each wavelength $w \in S$ is free on each fiber link of the route. Consider the following two scenarios.

- If there are no wavelength converters in the network:
    If r is a direct route, and if S is nonempty, choose a random wavelength from S. If r is a direct route, and if S is empty, the route is blocked. If r is an alternate route, and if $|S| > R$, then choose a random wavelength from S. If r is an alternate route, and if $|S| \le R$, then the route is blocked.

- If there are wavelength converters present in the network:
    Try to assign wavelengths without utilizing any wavelength converters, as above. If not possible, (i.e., if r is a direct route and S is empty, or if r is an alternate route and $|S| \le R$), divide the route r into subpaths, $r_1, r_2, \ldots, r_n$, depending on wavelength converter availability at intermediate nodes of the route. Let $S_1, S_2, \ldots, S_n$ be the set of idle wavelengths available on subpaths $r_1, r_2, \ldots, r_n$, respectively. If r is a direct route, and if $S_i > 0$, for $1 \le i \le n$, choose a random wavelength from each S_i; otherwise, the route is blocked. If r is an alternate route and $|S_i| > R$, choose a random wavelength from each S_i; otherwise, the route is blocked.

The above algorithm is "naive" in the sense that it may utilize more wavelength converters than may be necessary to establish a lightpath. This is because the above algorithm does not exploit the possibility that certain adjacent subpaths in a lightpath may have common free wavelengths and hence a wavelength converter need not be used in going between those subpaths.

However, the performance of the above algorithm provides an upper bound on the performance of any wavelength-assignment scheme. The work in [16] examines wavelength-assignment algorithms in the presence of sparse wavelength-conversion that minimize the number of wavelength converters needed to establish a wavelength-converted lightpath.

F. Connection Setup

The procedure for connection setup involves the following steps.

1) *Routing*: Find a route from the source to the destination. Route finding can involve: selecting a route from a list of prespecified routes such as in fixed-alternate routing; route selection can also be performed dynamically, depending on network state, as in adaptive routing. Our study focuses on fixed-alternate routing, and compares empirically the performance of fixed-alternate routing with adaptive-shortest-cost path routing.

2) *Wavelength Assignment*: Our study assumes that wavelength assignment is performed as described in Section I-E.

3) *Connection Setup Signaling*: After the route selection and wavelength assignment are performed for a lightpath, connection setup involves reserving resources along the lightpath route, and then configuring the switches and other network elements appropriately. We assume that the control and management software at the switches and access nodes implement the connection setup and teardown procedures (see, for example, [14], [15]).

G. Outline of Remaining Sections

Section II discusses the system architecture and states our assumptions. Our analytical model is presented in Section III. Section IV elaborates on the approach to solve the analytical model. Section V presents numerical results for three representative network topologies. Section VI concludes our study with a discussion of the main contributions of this work.

II. NETWORK ARCHITECTURE AND ASSUMPTIONS

- The network consists of nodes and links interconnected in an arbitrary mesh interconnection pattern. There are N nodes in the network, labeled $1, 2, \ldots, N$. The (unidirectional) links in the network are labeled $1, 2, \ldots, E$.
- Each link can have at most C wavelengths.
- A lightpath r consists of a subset of $1, 2, \ldots, E$ links that form a path, with an assignment of a wavelength to each link.
- A lightpath connection request is denoted by a (s, d) pair, where s is the source node and d is the destination node. We label a (s, d) pair with an integer, so that there are $N \times (N - 1)$ possible (s, d) node pairs in the network.
- Calls for node pair i arrive according to a Poisson process with rate A^i. The holding time for a call is exponentially distributed with mean 1 (i.e., all time units are normalized to the holding time of a call). The rate of calls will be

denoted in units of Erlangs, where 1 Erlang is defined to be the number of calls per unit call-holding time.

- $r_i(1)$, $r_i(2)$, ..., $r_i(M_i)$ is the ordered list of alternate routes for node pair i. $r_i(1)$ is called the direct route, and $r_i(2)$, ..., $r_i(M_i)$ are called the "alternate routes" for node pair i. When a call for node pair i arrives, routes for it are attempted sequentially from $r_i(1)$, $r_i(2)$, ..., $r_i(M_i)$, until a route with a free wavelength is found.
- Wavelength assignment is performed by the algorithm presented in Section I-E.
- R is the wavelength reservation parameter, as defined in the wavelength-assignment algorithm in Section I-E. Unless otherwise stated, we assume that the value of R is zero.
- c is the wavelength conversion cost, as defined in the adaptive-shortest-cost routing algorithm in Section I-D. Unless otherwise stated, we assume that the value of c is zero.
- This work assumes that there is no access node blocking, i.e., calls cannot block because wavelengths or transceivers are not available on the fiber link that connects the access node to the network. This assumption allows us to focus on the properties of the network topology.

A. Additional Notation

We denote the path and the network-wide parameters by upper-case letters, and the link parameters by lower-case letters. Subscripts and superscripts refer to specific instances of links, node pairs, and routes.

- The term "traffic" means the rate of calls per unit time. The term "offered traffic" denotes the traffic that arrives (to the network, route, or link), and "carried traffic" denotes the traffic that is actually setup successfully (in the network, route, or link). The term "load" means the same as the term "traffic." We will employ the terms "call" and "connection" interchangeably.
- A route r denotes a sequence of adjacent links.
- P is the network-wide blocking probability.
- X_r is a random variable which denotes the number of idle wavelengths on route r. X_j is a random variable which denotes the number of idle wavelengths on link j.
- B_r is the blocking probability of a direct route r.
- Ba_r is the blocking probability of an alternate route r.
- $B_{r, X_j=m}$ is the blocking probability of a direct route r when link j has m idle wavelengths.
- $Ba_{r, X_j=m}$ is the blocking probability of an alternate route r when link j has m idle wavelengths.
- A^i is the offered traffic for node pair i.
- $\overline{A^i}$ is the carried traffic for node pair i.
- V_r^i is the traffic for node pair i that is offered to route r.
- $\overline{V_r^i}$ is the traffic for node pair i that is carried on route r.
- $\overline{V}_{r, X_j=m}^i$ is traffic for node pair i that is carried on route r when link j has m idle wavelengths.
- v_j is the carried traffic on link j.
- $v_{j, m}$ is the carried traffic on link j, when there are m idle wavelengths on link j.

- ρ is the network-wide average link utilization. The average link utilization for a single link is the average number of wavelengths used by lightpaths that traverse that link.

III. ANALYTICAL MODEL

Our analysis approach consists of two main components: 1) routing analysis and 2) path-blocking analysis. The routing analysis consists of a set of equations that determine the link-offered traffic from the path-blocking probabilities. The path-blocking analysis consists of a set of equations that determine the path-blocking probabilities from the link-offered traffic. An iterative method of repeated substitution [17], [22] is employed to solve the system of fixed-point nonlinear equations that result from the analysis. Our main contribution in the analytical model is to extend earlier analysis in [6], [19] to incorporate alternate routing and sparse full-wavelength conversion.

A. Overall Blocking Probability

The network-wide blocking probability is the ratio of lost traffic to the offered traffic, i.e.,

$$P = \frac{\sum_{i=1}^{N(N-1)} \left(A^i - \overline{A^i}\right)}{\sum_{i=1}^{N(N-1)} A^i}. \tag{1}$$

B. Carried Traffic for Node Pair i

The traffic for node pair i can be carried on any of the alternate routes. We express the total carried traffic for node pair i, $\overline{A^i}$, as the sum of the carried traffics on the alternate routes for node pair i, i.e.,

$$\overline{A^i} = \sum_{m=1}^{M_i} \overline{V}_{r_i(m)}^i. \tag{2}$$

C. Carried Traffic for Node Pair i on Route r

The carried traffic for node pair i on route r can be expressed in terms of the offered traffic and the blocking probability of the route as follows. If r is a direct route, we have

$$\overline{V_r^i} = V_r^i(1 - B_r). \tag{3}$$

If r is an alternate route, we have

$$\overline{V_r^i} = V_r^i(1 - Ba_r). \tag{4}$$

D. Offered Traffic for Node Pair i on Route r

Fig. 2 illustrates a system of alternate paths for node pair i. By the fixed-alternate routing algorithm, traffic is offered to alternate path $r_i(k)$ if all the routes $r_i(j)$, $1 \le j \le k - 1$, are blocked. Let P_j^i be the probability that the first j alternate routes for node pair i are blocked. Then, the traffic to node pair i that is offered to route $r_i(k)$, i.e., $V_{r_i(k)}^i$, is given by

$$V_{r_i(k)}^i = A^i P_{k-1}^i \tag{5}$$

where P_j^i is defined recursively as follows:

$$P_1^i = B_{r_i(1)} \tag{6}$$
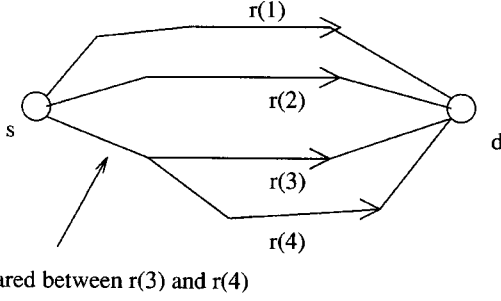
Link(s) shared between r(3) and r(4)

Fig. 2.   Illustration of alternate routes for a node pair.

and

$$P_j^i = P_{j-1}^i \times \text{Prob}\left(r_i(j) \text{ is blocked} \,|\, \text{all}\right.$$
$$\left. r_i(k) \text{ are blocked}, \ k = 1, 2, \ldots, j-1\right) \quad (7)$$

for $j \geq 2$. In this analysis, we will assume that blocking on any alternate route is *independent* of blocking on any other alternate route. From the assumption that alternate routes block independently, we have

$$\text{Prob}\left(r_i(j) \text{ is blocked} \,|\, \text{all}\right.$$
$$\left. r_i(k) \text{ are blocked}, \ k = 1, 2, \ldots, j-1\right) = Ba_{r_i(j)} \quad (8)$$

for $j \geq 2$. Therefore

$$P_j^i = B_{r_i(1)} \prod_{k=2}^{j} Ba_{r_i(k)}. \quad (9)$$

The assumption that alternate routes block independently is reasonable because alternate routes between any node pair are expected to contain link-disjoint routes, so that a link-disjoint route may be selected from the set of alternate routes to restore the connection upon a link failure. One event when this assumption (that routes block independently) is violated is when alternate routes share links. For example, routes $r(3)$ and $r(4)$ in Fig. 2 share a link and therefore blocking on $r(3)$ is related to blocking on $r(4)$. The results presented in this paper assume that alternate routes block independently. The work in this paper may be enhanced by taking into account the interdependencies between the blocking on alternate paths [26].

E. Carried Load on a Link

The carried load on link j, v_j, is the sum of the carried loads on all routes on which link j is a component link, i.e.,

$$v_j = \sum_{i=1}^{N(N-1)} \sum 1 \leq k \leq M_i j \in r_i(k) \overline{V}_{r_i(k)}^i. \quad (10)$$

F. Blocking Model of a Wavelength-Continuous Route

The blocking probability for a wavelength-continuous route is defined recursively in terms of the blocking characteristics of a basic element, which can be a single link or a two-link tandem. We utilize a single-link blocking model of a wavelength-continuous route proposed in [19]. Other blocking models of a wavelength continuous path, e.g., the two-link blocking model proposed in [13] can be utilized as well.

1) Single-Link Model: In the single-link model, each link j, $1 \leq j \leq E$, has associated with it a random variable, X_j, which indicates the number of idle wavelength on that link. We assume that the X_js are independent. Let $Y^{(2)}$ be a random variable indicating the number of idle wavelengths on a two-hop path, consisting of links i and j. The conditional probability that there are k idle wavelengths given that link i has n_a idle wavelengths and link j has n_b idle wavelengths, $P(Y^{(2)} = k | X_i = n_a, X_j = n_b)$, is determined combinatorially as follows.

Consider throwing X_i blue balls at C different bins at random, and X_j red balls at random into the same C bins (independent of the blue balls). (Recall that C equals the number of wavelengths in a fiber link.) Then, $P(Y^{(2)} = k | X_i = n_a, X_j = n_b)$ is the probability that there are k bins with both blue and red balls, i.e.,

$$P(Y^{(2)} = k | X_i = n_a, X_j = n_b)$$
$$= \begin{cases} \dfrac{\dbinom{n_a}{k}\dbinom{C - n_a}{n_b - k}}{\dbinom{C}{n_b}}, & \text{if } \max(0, n_a + n_b - C) \\ & \quad \leq k \leq \min(n_a, n_b) \\ 0, & \text{otherwise.} \end{cases}$$
$$(11)$$

For an n-hop path r with links $l_1, l_2, \ldots, l_n$, the probability that there are k available wavelengths on the path, $P(Y^{(n)} = k)$, is defined recursively as follows:

$$P\left(Y^{(2)} = k\right) = \sum_{x=0}^{C} \sum_{y=0}^{C} P\left(Y^{(2)} = k\right) | X_{l1} = x, X_{l2} = y)$$
$$P(X_{l_1} = x) P(X_{l2} = y) \quad (12)$$

and

$$P\left(Y^{(n)} = k\right) = \sum_{x=0}^{C} \sum_{y=0}^{C} P\left(Y^{(2)} = k | Y^{(n-1)} = x, X_{l_n} = y\right)$$
$$P\left(Y^{(n-1)} = x\right) P(X_{l_n} = y). \quad (13)$$

The blocking probability of a wavelength-continuous direct route r, B_r, is therefore determined by

$$B_r = P\left(Y^{(n)} = 0\right) \quad (14)$$

and the blocking probability of a wavelength-continuous alternate route r, Ba_r, is given by

$$Ba_r = \sum_{i=0}^{i=R} P\left(Y^{(n)} = i\right). \quad (15)$$

2) Distribution of Idle Wavelengths on a Link: The idle wavelength distribution on a link j, $P(X_j = k)$, is determined as follows. The arrival process on a link j, when the link has m idle wavelengths, is Poisson with arrival rate $v_{j,m}$. The rate at which connections are terminated when there are m idle wavelengths (and hence $C - m$ active connections) on the link is given by $C - m$ since the average holding time for a connection is one. Therefore, the number of idle wavelengths
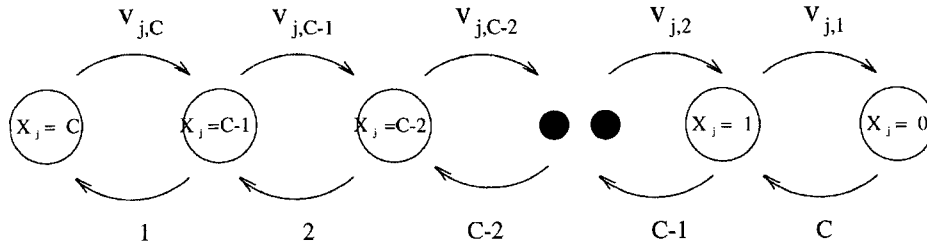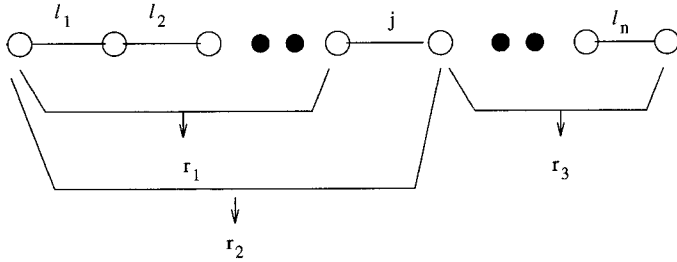
Fig. 3. Markov chain for idle wavelength distribution on link j.

Fig. 4. Decomposition of a path.

on the link, X_j, can be described by the Markov chain in Fig. 3. Solving the Markov chain, we obtain

$$P(X_j = m) = \frac{\prod_{i=1}^{m} (C - i + 1)}{\prod_{i=1}^{m} v_{j,i}} P(X_j = 0) \qquad (16)$$

$$P(X_j = 0) = \left[1 + \sum_{m=1}^{C} \frac{\prod_{i=1}^{m} (C - i + 1)}{\prod_{i=1}^{m} v_{j,i}} \right]^{-1}. \qquad (17)$$

3) State-Dependent Arrival Rate on a Link: We seek to determine $v_{j,m}$, which is the carried load on link j when $X_j = m$. From Section III-E, we have

$$v_{j,m} = \sum_{i=1}^{N(N-1)} \sum_{1 \leq k \leq M_i} \ j \in r_i(k) \overline{V}^{i}_{r(k), X_j = m} \qquad (18)$$

where $\overline{V}^{i}_{r(k), X_j = m}$ is traffic from node pair i that is carried on route $r_i(k)$ when the state of link j is $X_j = m$. From Section III-C, if r is a direct route, we have

$$\overline{V}^{i}_{r, X_j = m} = V_r^i (1 - B_{r, X_j = m}) \qquad (19)$$

and if r is an alternate route, we have

$$\overline{V}^{i}_{r, X_j = m} = V_r^i (1 - Ba_{r, X_j = m}). \qquad (20)$$

The offered traffic to route r from node pair i, V_r^i, can be calculated from the analysis in Section III-D.

4) State-Dependent Blocking Probability of a Wavelength-Continuous Path: $B_{r, X_j = m}$ and $Ba_{r, X_j = m}$ can be evaluated recursively as follows. Consider a n-hop path, r, with links $l_1, l_2, \ldots, j, \ldots, l_n$. We can express path r as $r = r_1 j r_3$, where $r_1 = l_1, l_2, \ldots$ is the initial part of path r that ends in the link before link j (see Fig. 4), and r_3 is the rest of the path r after link j. Let U be a random variable that indicates the

number of idle wavelengths in route r. Let U_1 be a random variable that indicates the number of idle wavelengths in route r_1, U_2 be a random variable that indicates the number of idle wavelengths in route $r_1 j$, and U_3 be a random variable that indicates the number of idle wavelengths in route r_3. Then

$$P(U_2 = k | X_j = m)$$
$$= \sum_{x=0}^{C} P(U_2 = k | U_1 = x, X_j = m) P(U_1 = x) \qquad (21)$$

$$P(U = k | X_j = m) = \sum_{x=0}^{C} \sum_{y=0}^{C}$$
$$P(U = k | U_2 = x, U_3 = y, X_j = m)$$
$$\times P(U_2 = x) P(U_3 = y). \qquad (22)$$

Therefore, we have

$$B_{r, X_j = m} = P(U = 0 | X_j = m) \qquad (23)$$

and

$$Ba_{r, X_j = m} = \sum_{i=0}^{i=R} P(U = i | X_j = m). \qquad (24)$$

G. Average Link Utilization

The network-wide average link utilization ρ is given by

$$\rho = \frac{\sum_{j=1}^{E} \sum_{m=1}^{C} mP(X_j = C - m)}{E}. \qquad (25)$$

H. Full Wavelength Conversion

Here, we assume that some nodes in the network have full wavelength-conversion capabilities. We divide each route r into segments, where each segment is a path with no wavelength-conversion nodes. So, a route r can be segmented as $r = r_1 r_2 \ldots r_k$, where each r_i, $1 \leq i \leq k$, is a wavelength-continuous path, and nodes shared by adjacent segments have full wavelength-conversion capability. We then compute the idle wavelength distributions X_i on each r_i by employing the analysis presented in Section III-F-2. Then, the probability of blocking on a (possibly wavelength-converted) direct route r is given by

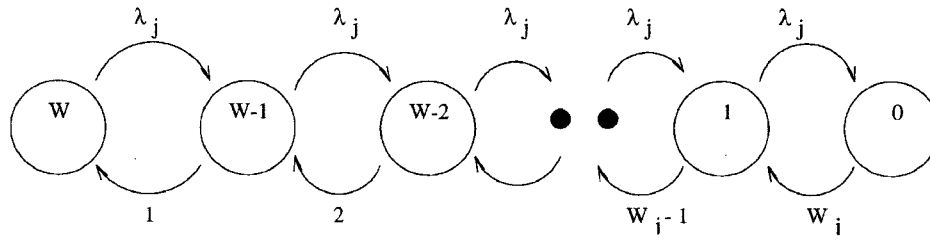$$B_r = 1 - \prod_{i=1}^{k} (1 - B_{r_i}). \qquad (26)$$

Fig. 5. Markov chain for the number of available wavelength converters at node j.

Similarly, the probability of blocking on an (possibly wavelength-converted) alternate route r is given by

$$Ba_r = 1 - \prod_{i=1}^{k} (1 - Ba_{r_i}). \qquad (27)$$

Here, we are assuming that the routes r_i block independently, which is a reasonable assumption because wavelength conversion is available at the end-node of each segment r_i, and therefore there is no dependence due to wavelength continuity. We can then compute the state-dependent blocking probability of a wavelength-converted direct path, $B_{r, X_j = m}$, as follows. Let $j \in r_l$, i.e., link j is in the lth route segment. Then

$$B_{r, X_j = m} = 1 - (1 - B_{r_l, X_j = m}) \prod_{i=1, j \notin r_i}^{k} (1 - B_{r_i}). \qquad (28)$$

Similarly, we can compute the state-dependent blocking probability of a wavelength-converted alternate path as

$$Ba_{r, X_j = m} = 1 - (1 - Ba_{r_l, X_j = m}) \prod_{i=1, j \notin r_i}^{k} (1 - Ba_{r_i}). \qquad (29)$$

I. Sparse Wavelength Conversion

Here, we assume that some nodes in the network have limited wavelength-conversion capabilities. Let node j have W_j number of wavelength-converter units. Each converter unit can be utilized by one lightpath that traverses the node. We assume that the requests for wavelength-converter units at a node j is a Poisson process with rate λ_j. The number of available wavelength converters at node j, Z_j, can be represented as a Markov chain, illustrated in Fig. 5.

We can approximate the rate at which wavelength converters are requested for use at node j, λ_j as the rate at which routes that go through node j are blocked, i.e.,

$$\lambda_j = \sum_{i=1, j \in r_i(1)}^{N(N-1)} V^i_{r_i(1)} B_{r_i(1)}$$

$$+ \sum_{i=1}^{N(N-1)} \sum_{2 \leq k \leq M_i, j \in r_i(k)} V^i_{r_i(k)} Ba_{r_i(k)}. \qquad (30)$$

From the above Markov chain, we can determine the probability distribution of the number of available wavelength-con-

verter units at node j, Z_j, where $Z_j = k$ is the event that there are k available wavelength converters:

$$P(Z_j = k) = \frac{\prod_{i=1}^{k} (W_j - i + 1)}{(\lambda_j)^k} P(Z_j = 0) \qquad (31)$$

and

$$P(Z_j = 0) = \left[1 + \sum_{m=1}^{W_j} \frac{\prod_{i=1}^{m} (W_j - i + 1)}{(\lambda_j)^m} \right]^{-1}. \qquad (32)$$

Recall that each route r is divided into segments, where each segment is a path with no nodes with wavelength-converter units. So, a route r can be segmented as $r = r_1 r_2 \cdots r_k$, where each r_i, $1 \leq i \leq k$, is a wavelength-continuous route. We then compute the idle wavelength distributions X_i on each r_i (from Section III-F-2). Let Z_i, $2 \leq i \leq k - 1$, denote the number of wavelength converters available at the ith intermediate node that contains wavelength converters in the route r. We assume that the segments r_i block independently (this is reasonable since the r_i are link-disjoint), and the random variables Z_i are independent.

Let B_z be the probability that some intermediate node (with wavelength converters) in the route r does not have any free wavelength converters (so that all the available wavelength converters at that node are utilized by current connections). Then

$$B_z = 1 - \prod_{i=2}^{k-1} (1 - P(Z_i = 0)) \qquad (33)$$

and B_{rf}, the probability that some segment r_i has no idle wavelengths, equals

$$B_{rf} = 1 - \prod_{i=1}^{k} (1 - B_{r_i}). \qquad (34)$$

The probability that there are no continuous wavelengths available on route r and that each r_i has at least one free available wavelength is given by

$$B_{rc} = P(X_r = 0 \text{ and } X_{r_i} \geq 1, i = 1, \ldots, k) \qquad (35)$$

which can be evaluated recursively similar to the computation of blocking probability B_r in Section III-F-1. In the presence of sparse wavelength conversion, a route is blocked in the following two mutually exclusive cases: 1) some segment has no available wavelengths or 2) all segments have idle wavelengths, but there is no idle wavelength on the route r and some intermediate node (with wavelengths converters) does not have a free

wavelength converter. Therefore, the blocking probability of a direct path r is given by

$$B_r = B_{rf} + B_{rc} \times B_z. \tag{36}$$

In the above equation, we have assumed that the distribution of wavelength converters at intermediate nodes is independent of the idle wavelength distributions on the segments. We note that the blocking probability as computed above assumes that wavelength converters have to be available at each intermediate node. This assumption is "naive" in the sense that it may utilize more wavelength converters than may be necessary to establish a lightpath. It is possible that adjacent segments may have common free wavelengths and hence a wavelength converter may not be needed between the two segments.

We compute the blocking probability of a (possibly) wavelength-converted alternate route (similar to the above computation of the blocking probability of a possibly wavelength-converted direct route) as follows. Let Ba_{rf} be the probability that some segment r_i has at most R idle wavelengths. Let Ba_{rc} be the probability that there are no continuous wavelengths available on route r and that each r_i has more than R free available wavelengths. Then

$$Ba_{rf} = 1 - \prod_{i=1}^{k}(1 - Ba_{r_i}) \tag{37}$$

and

$$Ba_{rc} = P(X_r = 0 \text{ and } X_{r_i} > R, i = 1, \ldots, k). \tag{38}$$

Therefore, the blocking probability of a (possibly) wavelength-converted alternate route equals

$$Ba_r = Ba_{rf} + Ba_{rc} \times B_z. \tag{39}$$

For state-dependent blocking probabilities of a direct (or alternate) path, i.e., the blocking probability of a (possibly) wavelength-converted direct (or alternate) path r when link j has m idle wavelengths, $B_{r, X_j=m}$ (or $Ba_{r, X_j=m}$), we make the following modifications to (34)–(39). Let link j be in the lth segment r_l of the route r. Then

$$B_{rf, X_j=m} = 1 - (1 - B_{r_l, X_j=m}) \prod_{i=1, i\neq l}^{k}(1 - B_{r_i}) \tag{40}$$

$$B_{rc, X_j=m} = P(X_r = 0 \text{ and } X_{r_i} \geq 1, \\ i = 1, \ldots, k, \text{ and } X_j = m) \tag{41}$$

$$B_{r, X_j=m} = B_{rf, X_j=m} + B_{rc, X_j=m} \times B_z \tag{42}$$

$$Ba_{rf, X_j=m} = 1 - (1 - Ba_{r_l, X_j=m}) \prod_{i=1, i\neq l}^{k}(1 - Ba_{r_i}) \tag{43}$$

$$Ba_{rc, X_j=m} = P(X_r = 0 \text{ and } X_{r_i} > R, \\ i = 1, \ldots, k, \text{ and } X_j = m) \tag{44}$$

$$Ba_{r, X_j=m} = Ba_{rf, X_j=m} + Ba_{rc, X_j=m} \times B_z. \tag{45}$$

TABLE II
RUNNING TIME FOR EACH STEP IN THE ALGORITHM

Step	Description	Running time
1	Route traffic	$O(N^2M)$
2	Link loads	$O(EN^2MHC)$
3	Path distributions	$O(N^2MHC^3)$
4	Conditional path distributions	$O(N^2MH^2C^4)$
5	Wavelength-converter distributions	$O(N^3C)$
6	Wavelength-converted path distributions	$O(N^2MH^2C^3)$
7	Wavelength-converted conditional path distributions	$O(N^2MH^3C^4)$

IV. SOLUTION APPROACH

The evaluation of the blocking probability in (1) requires the solution of the system of equations (1)–(45). We utilize an iterative relaxation procedure to solve the system of nonlinear equations.

Initialization: Set path blocking probabilities $B_r = 0$ and $Ba_r = 0$ for all alternate routes r between all node pairs i, $1 \leq i \leq N(N-1)$.

Iterate:

1) Route traffic: For all routes r, determine V_r and $\overline{V}_r$.
2) Link loads: For all links j, determine $v_{j, X_j=m}$.
   - Link idle wavelength distribution: For all links j, determine $P(X_j = m)$.
3) Wavelength-continuous path-blocking probabilities: For all r, determine B_r and Ba_r.
4) Wavelength-continuous conditional path-blocking probabilities: For all r, determine $B_{r, X_j=m}$ and $Ba_{r, X_j=m}$.
5) Wavelength-converter distributions: For each node i with wavelength converters, determine $P(Z_i = k)$.
6) Possibly wavelength-converted path-blocking probabilities: For all r, determine B_r and Ba_r.
7) Possibly wavelength-converted conditional path-blocking probabilities: For all r, determine $B_{r, X_j=m}$ and $Ba_{r, X_j=m}$.
8) Iterate k steps until $|P^k - P^{(k-1)}| < \epsilon$, where P^k and $P^{(k-1)}$ are the network-wide blocking probabilities in the kth and $(k-1)$th iterations, respectively.

It is not clear if the system of equations (1)–(45) has an unique solution, or if the algorithm presented above will converge to a solution point. However, in practice, we observe that the above algorithm converges to a solution point for all the representative networks that we considered, and that the solution is in reasonably good agreement with simulation results.

Complexity and Performance

The running times for each step in the above algorithm are shown in Table II. Here, H is the average hop distance of all the alternate routes between all node pairs, M is the number of alternate routes between node pairs, C is the number of wavelengths, and E is the number of links. We observe in practice that the algorithm converges to within an accuracy of $\epsilon = 10^{-6}$ in six to ten iterations, for the example network topologies examined in this work.
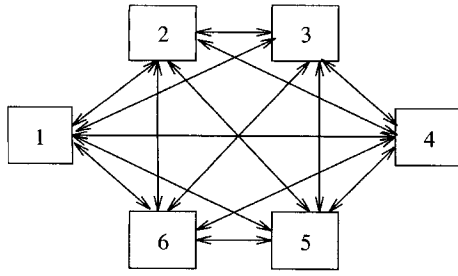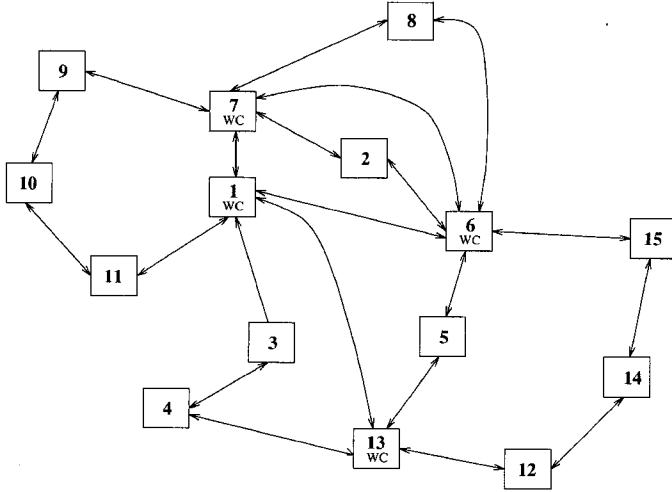
Fig. 6. A fully connected graph of six nodes.

Fig. 7. Network of interconnected rings, typical of a telecommunications network (wc = wavelength converter, if present).

Fig. 8. Twelve-node bidirectional ring.

V. ILLUSTRATIVE NUMERICAL EXAMPLES AND DISCUSSION

A. Network Topologies

We consider three network topologies for all our model and simulation studies: 1) a fully connected network with six nodes, 2) a network of interconnected rings with 15 nodes, and 3) a bidirectional ring network with 12 nodes. The three networks show different levels of connectivity, in terms of average hop distance, and in terms of the number of paths between node pairs. These networks were chosen as representing the diversity of topologies from a connectivity standpoint. The interconnected-rings network topology was provided to the authors by one of their project sponsors as being representative of a typical telecommunication network.

1) Fully Connected Network: Fig. 6 illustrates a network of six nodes where each node has a link to every other node. We assume that wavelength conversion, when present, is present at all nodes in the network. We study five configurations for alternate routing. The routing table at each node has one, two, three, four, or five alternate routes to each destination, in each configuration. We note that the 6-node fully connected network is 5-edge connected. We will employ the term "complete" network interchangeably with "fully connected" network in the rest of this work.

2) Interconnected Rings: Fig. 7 illustrates a 15-node network of interconnected rings. We assume that sparse wavelength conversion, when present, is at nodes 1, 6, 7, and 13, since these nodes have the maximum "traffic mixing" and can benefit most
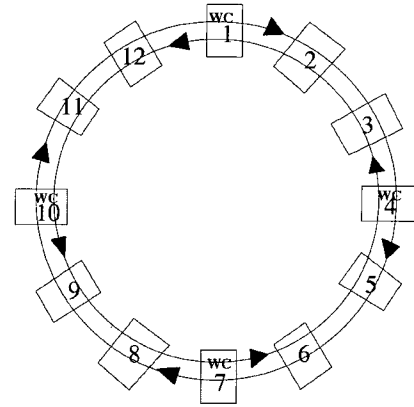
from wavelength conversion [16]. We study three configurations for alternate routing. The routing table at each node has one, two, or three alternate routes to each destination, in each configuration. We note that the interconnected rings network is 2-edge connected, i.e., there are at least two edge-disjoint paths between each node pair, and there is at least one node pair with exactly two edge-disjoint paths.

3) Bidirectional Ring: Fig. 8 illustrates a 12-node bidirectional ring. We assume that sparse wavelength conversion, when present, is at nodes 1, 4, 7, and 10. We study two configurations for alternate routing. In one configuration, the routing table at each node has at most one alternate route to each destination, and in the other configuration, the routing table at each node has two alternate routes to each destination ordered by increasing hop distance. We note that the bidirectional ring is 2-edge connected, i.e., there are two edge-disjoint paths between each node pair.

B. Simulation and Model Parameters

We have obtained results for each network with four and eight wavelengths. For each simulation configuration, five simulations runs were performed, each with a different seed for the random-number generator, resulting in a different call arrival sequence for each run. Each simulation run consisted of 200 000 calls. The reported simulation data are within the 95% confidence interval. We assumed that each node pair is equally loaded, i.e., the total offered load to the network is equally divided between all node pairs. Our simulation software was developed based on the discrete-event simulation method [29]. We utilized the Bellman–Ford algorithm [27], [28] for finding the shortest-cost path to set up the fixed-alternate routing tables. For adaptive routing, the simulation software performed a shortest-cost path computation for each connection setup. We considered two degrees of sparse wavelength conversion: one where selected nodes had one wavelength converter each, and another where the selected nodes had three wavelength converters each. Unless otherwise stated, the simulation and model studies assume that the wavelength-reservation parameter $R = 0$.

C. Results

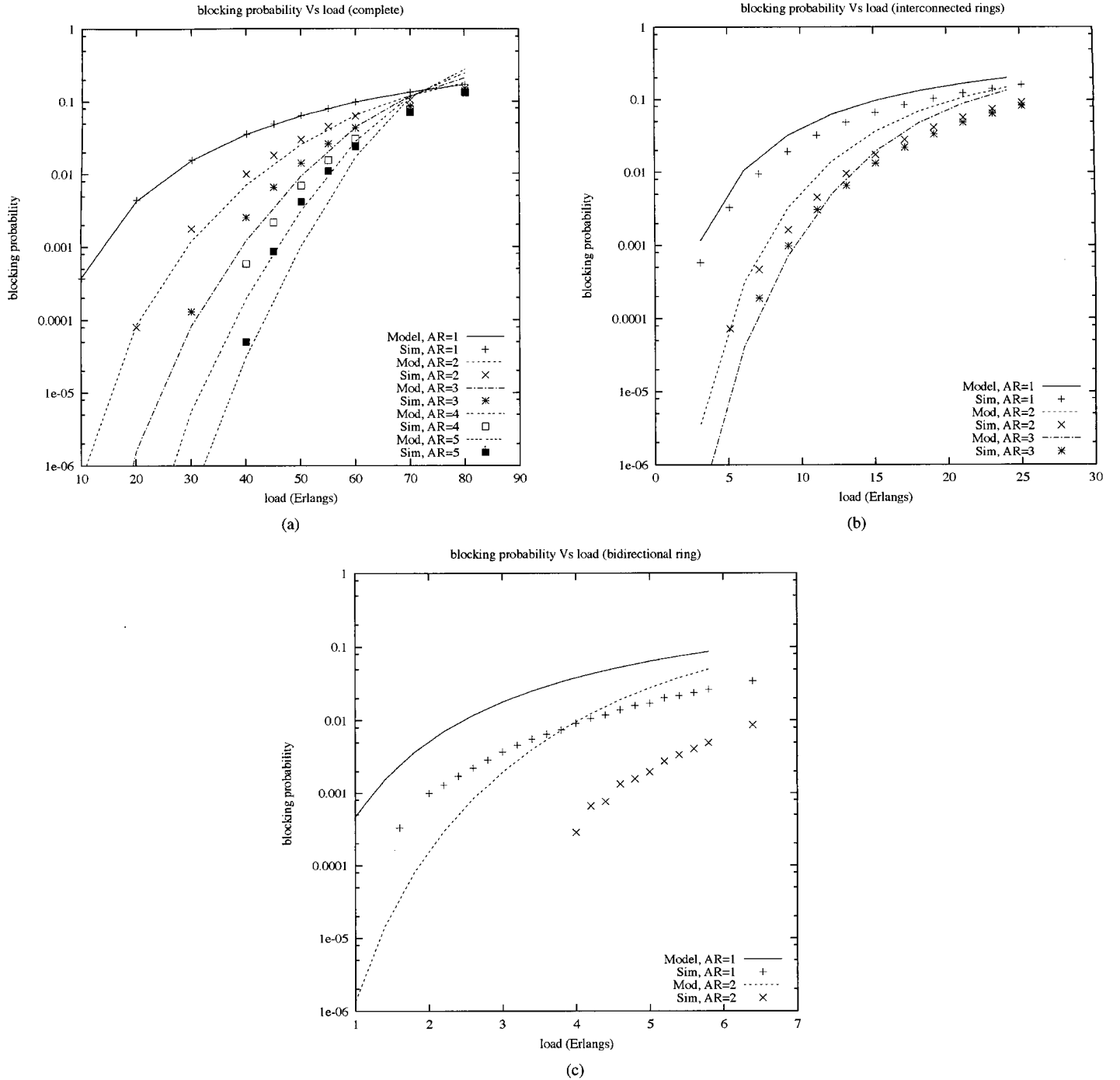We present the model and simulation results in two parts.

Fig. 9.   Accuracy of the alternate-routing model with no wavelength conversion. (a) 4-wavelength fully connected network. (b) 4-wavelength interconnected rings. (c) 4-wavelength bidirectional ring.

- In the first part, we study the accuracy of different aspects of the analytical model by comparing the model results with the corresponding simulation results. We also highlight some observations regarding alternate routing and wavelength conversion obtained from the model results.

- In the second part, we examine the simulation results and draw empirical generalizations and observations on the behavior of alternate routing and wavelength conversion.

In all the figures, "Model" refers to model results, "Sim" refers to simulation results, "AR" refers to the number of alternate routes, "Adaptive" refers to adaptive routing, "FW" refers to

full wavelength conversion, and "sparse" refers to sparse wavelength conversion.

D. Model Accuracy

1) Model Accuracy—Alternate Routing: In this subsection, we examine the accuracy of the alternate routing model when there is no wavelength conversion. Fig. 9 illustrates the accuracy of the model for the 4-wavelength fully connected network, interconnected-rings network, and the bidirectional-ring network with no wavelength conversion. Results for the 8-wavelength networks are similar and hence are not shown here. We ob-
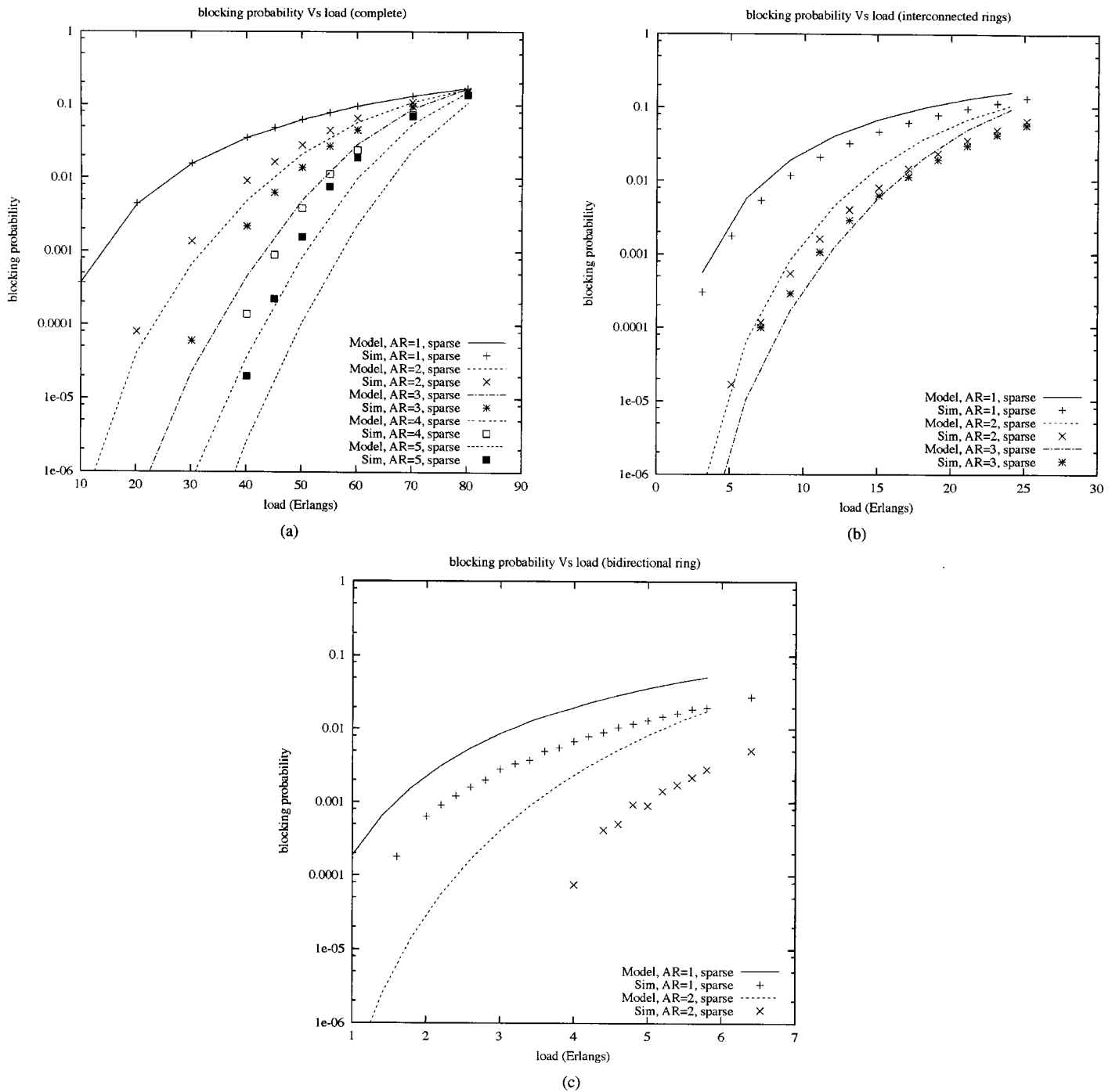
Fig. 10.   Accuracy of the alternate-routing model with sparse wavelength conversion. (a) 4-wavelength fully connected network. (b) 4-wavelength interconnected rings. (c) 4-wavelength bidirectional ring.

serve that the model is more accurate for the fully connected network than the other two networks. This is because 1) the average hop distance is one in the fully connected network (with any alternate route, it is at most two hops), whereas for the bidirectional ring, the average hop distance is more than three hops, and 2) the wavelength-continuous path blocking model is less accurate for longer paths [19]. We also observe that the model is more accurate at lower loads. In general, we expect the model to be more accurate for denser networks and at lower loads. Another interesting observation from the model for the fully connected network is that at high loads, the model results indicate

that a fewer number of alternate routes is better! This may be because, at high loads, alternate routes consume resources that would otherwise be used by direct routes. At high loads, the wavelength-reservation parameter R may need to be set appropriately to improve blocking performance.

2) Model   Accuracy—Sparse   Wavelength   Conversion: Fig. 10 illustrates the accuracy of the model for the 4-wavelength fully connected network, interconnected-rings network, and bidirectional-ring network, with sparse wavelength conversion. In the sparse wavelength conversion configuration considered here, the selected nodes (refer to
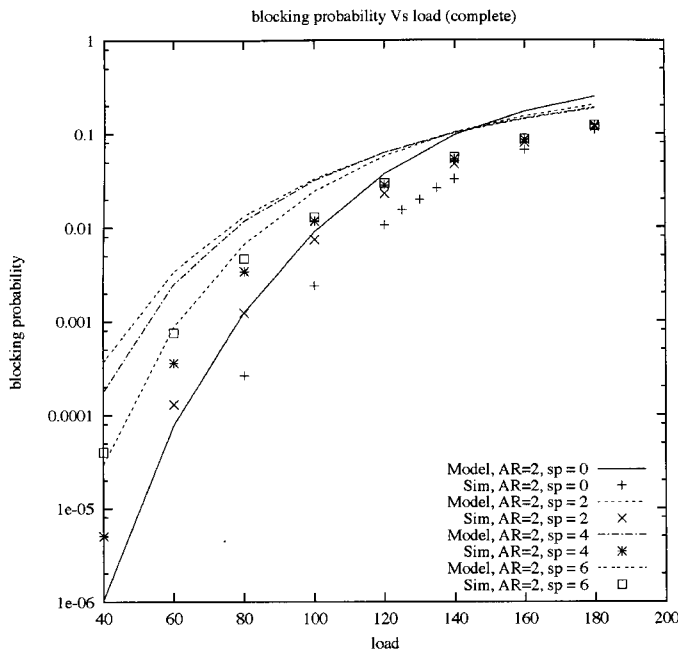
Fig. 11. Accuracy of the model for the 8-wavelength fully connected network with two alternate routes, when the wavelength-reservation parameter, R, takes on the values 0, 2, 4, and 6.

Section V-A for a specification of the nodes selected for sparse conversion) in each network were equipped with three wavelength-conversion units. Results for the 8-wavelength networks are similar, and hence are not shown here. We observe that the model is more accurate for the fully connected network in comparison to the other two networks. We also observe that the model is more accurate when there is sparse wavelength conversion than when there is no wavelength conversion. This is due to the fact the wavelength converters break up long wavelength-continuous paths, and contribute to ensuring the "independence" of idle wavelength distributions on adjacent links. In general, we expect the model to be more accurate for a network with wavelength conversion than for the same network without wavelength conversion.

3) Model Accuracy—Wavelength Reservation: Recall that the wavelength-reservation parameter R indicates the number of idle wavelengths that are reserved for the direct route, so that a lightpath on an alternate route can be established only when there are at least $R + 1$ available wavelengths on the alternate route (in the absence of wavelength conversion). Fig. 11 illustrates model accuracy for the 8-wavelength fully connected network, with two alternate routes, no wavelength conversion, and R taking on the values 0, 2, 4, and 6. The results for other networks and configurations are similar and hence are not shown here. We observe that the blocking probability increases with increasing values of R. This is due to the fact that, as we increase R, we prevent alternate routes from being established. We expect that, when the traffic pattern is skewed, or at heavy loads, it may be beneficial to set the wavelength-reservation parameter to nonzero values.

4) Model Accuracy—Link Utilization: In the model, the network-wide average link utilization is computed from (25). In the simulation, we compute the average link utilization as follows:

TABLE III
PERCENTAGE GAIN IN BLOCKING PROBABILITY FOR FULL AND SPARSE WAVELENGTH CONVERSION AVERAGED OVER A RANGE OF LOADS FOR DIFFERENT NUMBERS OF ALTERNATE ROUTES

Network	*Full* (4,8) wavelengths	*Sparse* (4,8) wavelengths
Fully connected		
1 route	-	-
2 routes	16,19	10,11
3 routes	32,29	24,21
4 routes	41,30	29,23
5 routes	60,33	36,26
Interconnected rings		
1 route	37,46	25,21
2 routes	60,74	34,24
3 routes	58,72	31,23
Bidirectional ring		
1 route	44,72	26,48
2 routes	66,91	41,50

TABLE IV
PERCENTAGE GAIN IN BLOCKING PROBABILITY OBTAINED BY ADDING AN ALTERNATE ROUTE, AVERAGED OVER A RANGE OF LOADS

Network	*No conversion* (4,8)	*Full conversion* (4,8)
Fully connected		
$1 \rightarrow 2$	74,80	79,86
$2 \rightarrow 3$	70,72	76,79
$3 \rightarrow 4$	64,66	68,73
$4 \rightarrow 5$	51,52	55,52
$5 \rightarrow$ adaptive	54,53	54,51
Interconnected rings		
$1 \rightarrow 2$	77,83	86,93
$2 \rightarrow 3$	40,36	28,35
$3 \rightarrow$ adaptive	60,65	50,41
Bidirectional ring		
$1 \rightarrow 2$	94,96	97,99

for each link, we compute its utilization as the time average of the number of wavelengths used on that link; the network-wide link utilization is the average value of link utilizations over all the links in the network.

Fig. 12 illustrates the model accuracy for the 8-wavelength fully connected network, interconnected-rings network, and the bidirectional-ring network, with no wavelength conversion. The results for 4-wavelengths are similar and hence are not shown here. We observe that the model is accurate at low loads and tends to diverge from the simulation at high loads, because the blocking probability model for a wavelength-continuous path is less accurate at higher loads.

5) Model Observations: In this subsection, we highlight some observations from the results of the model. Fig. 13 illustrates the model results for the 4-wavelength fully connected network, interconnected rings, and bidirectional ring. We observe the following interesting result for all networks: at low loads, the blocking probability with two alternate routes and no wavelength conversion is better than the blocking probability with one alternate route and full wavelength conversion. Furthermore, for the fully connected network, we observe that at low loads, and when the number of alternate routes is 1, 2, or 3, the benefits in blocking probability obtained by adding an alternate route is better than the benefit obtained by adding full wavelength conversion. In general, we expect that, *at low loads and when the number of alternate routes between node pairs*
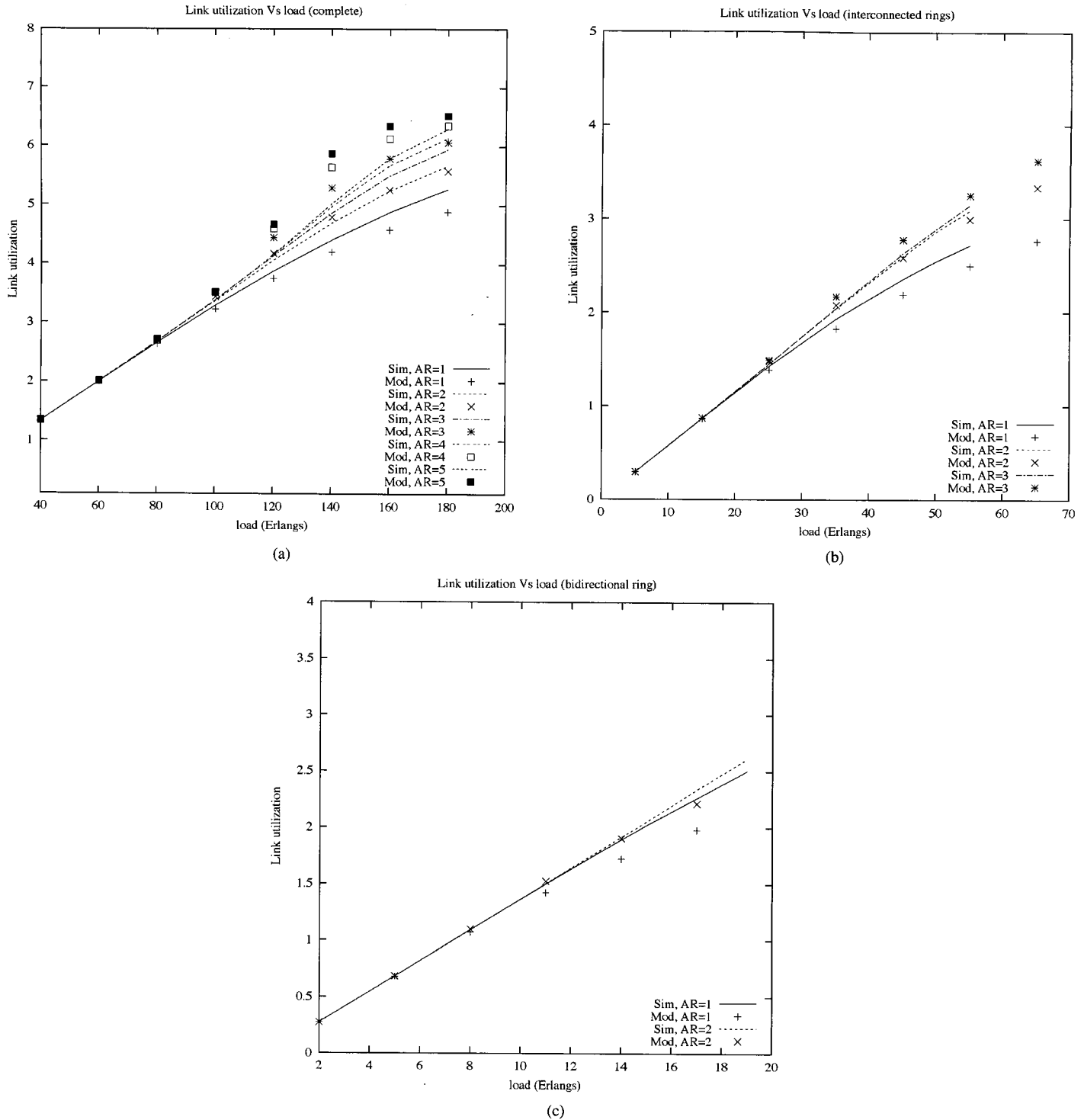
Fig. 12. Accuracy of the model's average link utilization. (a) 8-wavelength fully connected network. (b) 8-wavelength interconnected rings. (c) 8-wavelength bidirectional ring.

does not fully exploit the connectivity of the network topology (i.e., the number of alternate routes between node pairs is less than the edge connectivity of the network), the benefits in blocking probability obtained by adding an alternate route (and therefore exploiting more link-disjoint paths) may be significantly more than the benefits obtained by adding (any degree of) wavelength conversion. In the following subsection, we confirm these model observations by comparing them with the corresponding simulation results.

E. Observations From the Simulation

In this section, we examine the simulation results for the three representative networks. From the simulation results, we make general empirical observations and validate the observations highlighted in Section V-D-5.

Fig. 14(a) plots the simulation results for the fully connected network with four wavelengths. We observe the following: with any number of alternate routes, i.e., with 1, 2, 3, or 4 alternate

blocking probability Vs load (complete)

blocking probability Vs load (interconnected rings)
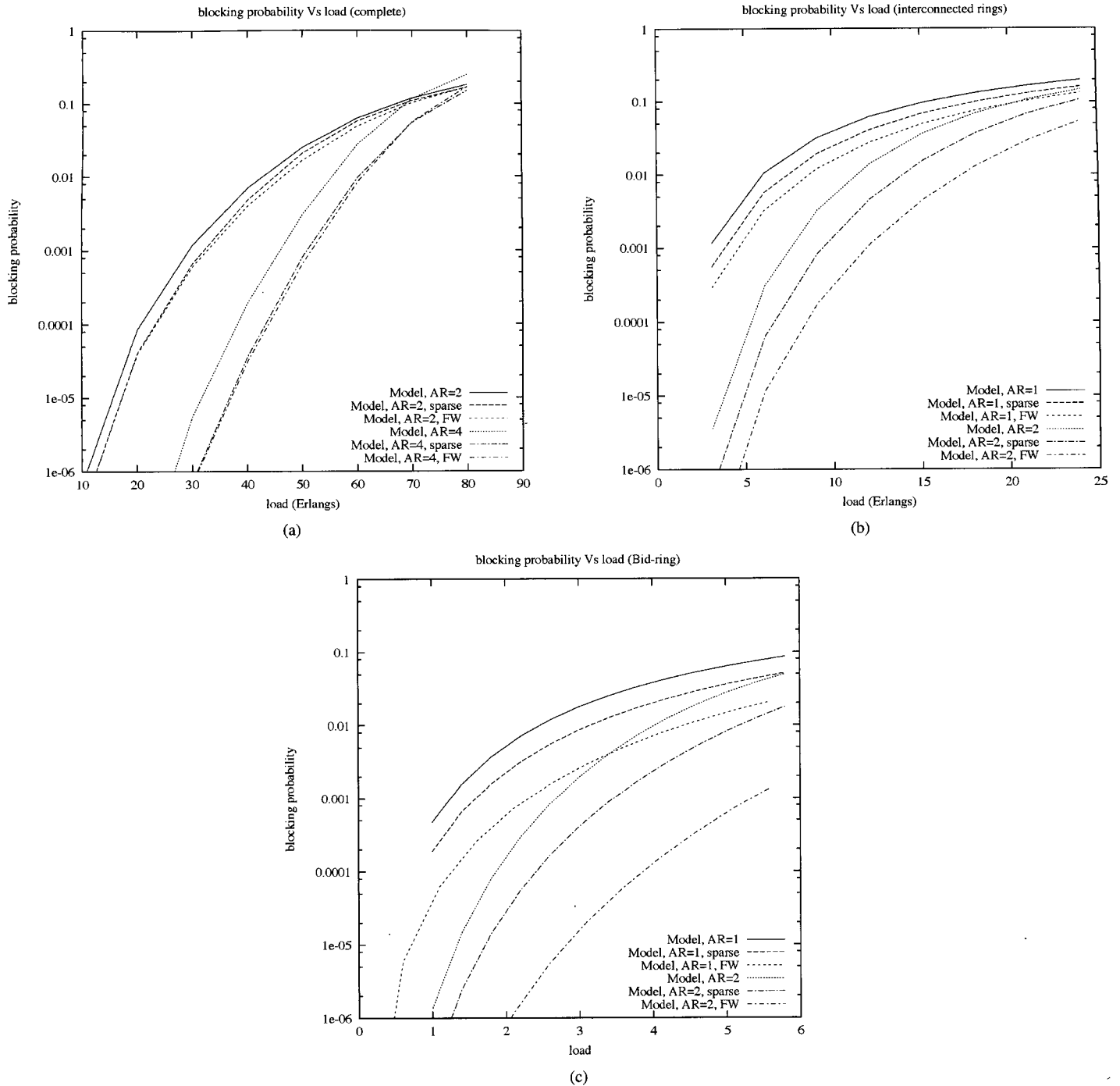
blocking probability Vs load (Bid-ring)

Fig. 13. Model results. (a) 4-wavelength fully connected network. (b) 4-wavelength interconnected rings. (c) 4-wavelength bidirectional ring.

routes, and at low loads, adding an alternate route improves the blocking probability more than adding full wavelength conversion. Further, we observe that, at low loads, the blocking performance of the network with fixed-alternate routing approaches that of adaptive routing as we increase the number of alternate routes.

Fig. 14(b) plots the simulation results for the interconnected-rings network with four wavelengths. We observe that the blocking probability of the network with two alternate routes and no wavelength conversion is better than that with one alternate route and full wavelength conversion at low loads. However, we also observe that, at low

loads, the blocking probability of the network with two alternate routes and sparse wavelength conversion is better than the blocking probability with three alternate routes and no wavelength conversion. This is because, with one alternate route, the network is underutilized since the network is 2-edge connected. When the number of alternate routes equal the edge connectivity of the network, i.e., equals two, adding another alternate route does not improve the blocking probability as much as adding wavelength conversion (since the added alternate routes share links with existing alternate routes for some node pairs). We observe that, at low loads, the blocking performance of the network with fixed-alternate
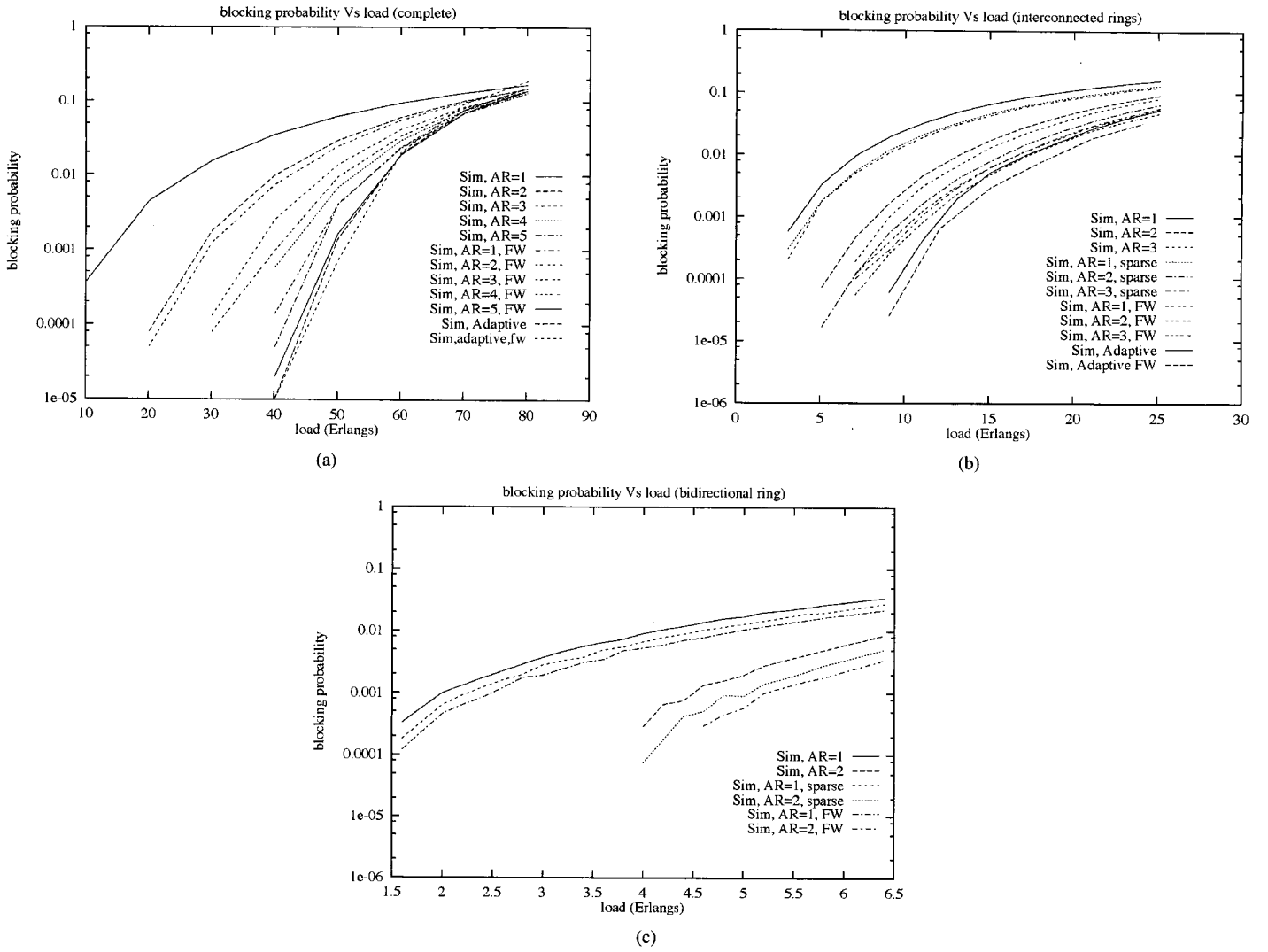
Fig. 14.   Simulation results. (a) 4-wavelength fully connected network. (b) 4-wavelength interconnected rings. (c) 4-wavelength bidirectional ring.

routing approaches that of adaptive routing as we increase the number of alternate routes.

Fig. 14(c) plots the simulation results for the bidirectional-ring network with four wavelengths. We observe that the blocking performance with two alternate routes is significantly better than that with one alternate route. In particular, the blocking probability of the network with two alternate routes and no wavelength conversion is better than that with one alternate route and full wavelength conversion at low loads.

The benefits of sparse and full wavelength conversion when the network has a certain number of alternate routes is illustrated in Table III. The percentage gain in blocking probability with wavelength conversion is the average value of the blocking probability gain over a range of loads. In sparse wavelength conversion, the selected nodes (refer to Section V-A for a specification of the nodes selected for sparse conversion) in each network were equipped with one wavelength-converter unit. We observe the following for all networks: 1) the benefits of wavelength conversion increases with number of alternate routes and 2) a large proportion of the gain in blocking probability with full wavelength conversion is obtained with sparse wavelength conversion.
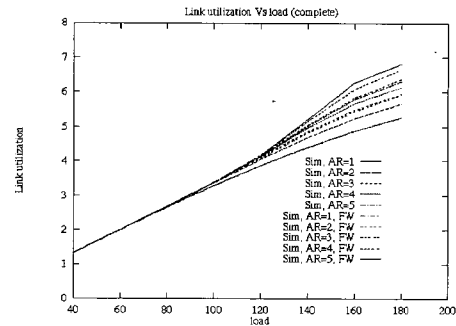

Fig. 15.   Average link utilization in the 8-wavelength fully connected network.

The benefits of adding an alternate route when the network has a certain number of alternate routes between node pairs is illustrated in Table IV, when the network has no wavelength conversion, and when the network has full wavelength conversion. We observe that the percentage gain in blocking probability by adding an alternate route decreases as we increase the number of alternate routes.

1) Link Utilization: Fig. 15 plots the average link utilization against the total offered load to the network, for the 8-wave-

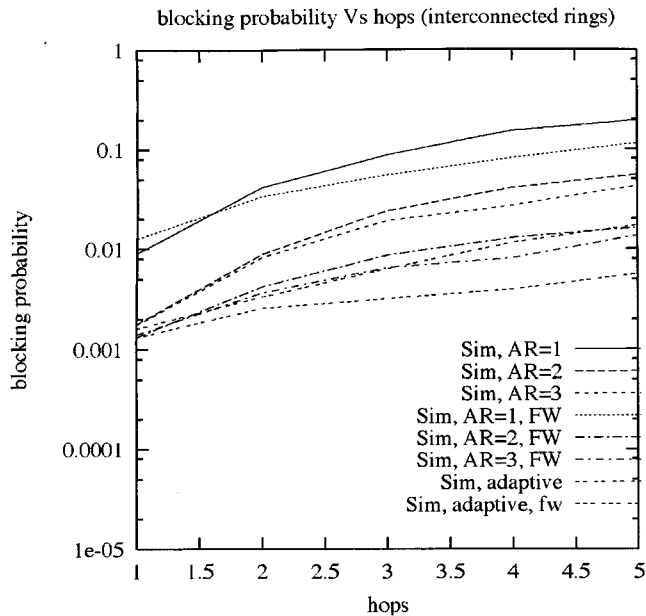blocking probability Vs hops (interconnected rings)

Fig. 16. Blocking probability versus minimum hop distance for the 4-wavelength interconnected-rings network when total network load is 15 Erlangs.

length fully connected network. We observe that the average link utilization increases with the number of alternate routes, and that the improvement in average link utilization is more at higher loads.

2) Fairness: Fig. 16 plots the average blocking probability against the number of hops (for the shortest-hop path) for the 4-wavelength, interconnected-rings network, when the total offered load to the network is 15 Erlangs. The blocking probability for a certain number of hops, h, $1 \leq h \leq 5$, is obtained by averaging the blocking probabilities of all node pairs whose shortest-hop-path distance is h. We observe that increasing the number of alternate routes improves fairness.

VI. CONCLUSION

This paper proposes an analytical model to analyze the performance parameters (such as blocking probability) of an optical network. This model incorporates a novel analysis of fixed-alternate routing in a wavelength-routed optical network that incorporates sparse wavelength conversion. Our results indicate that the model gives reasonably good estimates of network performance parameters including the blocking probability and the average link utilization. We found that the model is more accurate for denser network topologies and at lower loads. The model correctly (as corroborated by simulations) predicts that at high loads, alternate routing actually increases the blocking probability of the network. The model can be applied as a subroutine for use in iterative network design and optimization procedures, and to make empirical observations on the blocking performance of network topologies and configurations. Three representative network topologies were considered for the model and simulation studies. We found that, at low loads, and when the number of alternate routes between node pairs does not fully exploit the connectivity of the network topology (i.e., the number of alternate routes between

node pairs is less than the edge connectivity of the network), the benefits in blocking probability obtained by adding an alternate route (and therefore exploiting more link-disjoint paths) is more than the benefits obtained by adding wavelength conversion. For our example networks, we found that the blocking performance of fixed-alternate routing approaches that of adaptive-shortest-cost path routing with increasing number of alternate routes.

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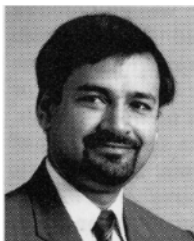
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