CSE 531 Homework Assignment 1

Due in class on Tuesday, Sep 11.

September 14, 2007

There are totally 5 problems, 10 points each. You should do them all. We will grade only 4 problems chosen at my discretion. If it so happens that you don't do one of the problems we don't grade, then no points will be deducted.

To "disprove" a statement, you must find a counter example to show that the statement is wrong. In general, your answers should be short but concise. (This will come with experience.)

Problem 1. Prove or disprove each of the following statements, given that $f, g : \mathbb{N}^+ \to \mathbb{R}^+$.

1. $n + 2 = o(2n)$

2. $2^n = o(2^{n+1})$

3. $f(n) = \omega(\sqrt{f(n)})$

4. $f(n) = \Theta(f(2n))$

5. $f(n) = o(2^{f(n)})$

6. $2^{7n} = \Theta(2^{6n})$

Problem 2. Suppose both $f(n)$ and $g(n)$ are functions from $\mathbb{N}^+$ to $\mathbb{R}^+$, and they tend to ∞ as $n \to \infty$. Prove or disprove each of the following statements.

1. $f(n) = \Omega(g(n))$ implies $\lg(f(n)) = \Omega(\lg(g(n)))$

2. $f(n) = \Omega(g(n))$ implies $2^{f(n)} = \Omega(2^{g(n)})$

3. $f(n) = \omega(g(n))$ implies $\lg(f(n)) = \omega(\lg(g(n)))$

4. $f(n) = \omega(g(n))$ implies $2^{f(n)} = \omega(2^{g(n)})$

5. $f(n) = \Theta(g(n))$ implies $\lg(f(n)) = \Theta(\lg(g(n)))$

6. $f(n) = \Theta(g(n))$ implies $2^{f(n)} = \Theta(2^{g(n)})$

Problem 3. Arrange the following functions in ascending order of their growth rates. That is, if $g(n)$ immediately follows $f(n)$ in the list, then $f(n) = O(g(n))$.

$$n! \quad n^{\lg n} \quad (\lg n)^n \quad 2^{n^{0.1}} \quad 2^{\sqrt{\lg n}}$$
$$n^3 \quad \sqrt{n}^{\lg \lg n} \quad (\lg^* n)^{2^n} \quad \sqrt{2}^{\lg n} \quad (\lg n)^{0.1}$$

Explain your ordering. (To avoid unnecessary explanation, order the functions first, like a, b, c, d, then explain why $a = O(b)$, $b = O(c)$, and $c = O(d)$, and so on.)

Problem 4. Find asymptotic upper and lower bounds for $T(n)$ in each of the following recurrences. Assume that $T(n)$ is constant and positive for $n \leq 2$. Make your bounds as tight as possible, and justify your answers.

1. $T(n) = 27T(n/9) + n\sqrt{n}$

2. $T(n) = 4T(n/2) + n^2/\lg n$

3. $T(n) = 2T(n/2) + n\lg n$

4. $T(n) = 8T(n/4) + n\lg n$

5. $T(n) = 9T(n/3) + n^2\sqrt{n}$

6. $T(n) = 2T(\sqrt{n}) + \lg n \cdot \lg\lg n$

7. $T(n) = T(n/2) + T(n/4) + n^2$

8. $T(n) = \frac{4\lg n}{\lg n - 1}T(n/2) + n^2\sqrt{n}\lg n$

Note: $g(n)$ is an asymptotic upperbound for $T(n)$ if $T(n) = O(g(n))$, and lowerbound if $T(n) = \Omega(g(n))$. If you can show $T(n) = \Theta(g(n))$, then that's enough for both upper and lower bounds.

Problem 5.   (i) Give an example of functions $f, g : \mathbb{N}^+ \to \mathbb{R}^+$ satisfying all of the following:

   * Both $f, g \to \infty$ as $n \to \infty$
   * $f = O(g)$, $f \neq \Theta(g)$, and $f \neq o(g)$

  (ii) Give an example of functions $f, g : \mathbb{N}^+ \to \mathbb{R}^+$ satisfying all of the following:

   * Both $f, g \to \infty$ as $n \to \infty$
   * $f = \Theta(g)$, $\lim_{n \to \infty} f(n)/g(n) \neq c$ for any constant c