Brief Announcement: On Regenerator Placement Problems in Optical Networks *

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ABSTRACT

Optical reach is defined as the distance optical signal can traverse before its quality degrades to a level that necessitates regeneration. It typically ranges from 500 to 2000 miles, and as a consequence, regeneration of optical signal becomes essential in order to establish a *lightpath* between a source-destination node pair whose distance exceeds the limit. In a *translucent optical network*, the optical signal is regenerated at selected nodes of the network before the signal quality degrades below the acceptable threshold. Given the optical reach of the signal, to minimize the overall network design cost, the goal of the *regenerator placement problem* is to find the minimum number of regenerators necessary in the network, so that every pair of nodes is able to establish a lightpath between them. In this paper, we study the regenerator placement problem and present complexity result for that.

ACM Categories: Computer Applications
General Terms: Algorithms, Theory
Keywords: optical networks, regenerator placement

1. INTRODUCTION

In a *translucent optical network*, the optical signal is regenerated at the regeneration points (typically a subset of the network nodes with the regeneration capability) to carry the signal over long distances. *Optical reach* (the distance an optical signal can travel before its quality degrades to a level that necessitates regeneration) usually ranges from 500 to 2000 miles [4]. To transmit an optical signal beyond this distance, it is essential to *re-amplify, reshape* and *re-time* (a process often called *3R regeneration*) it. The *Regenerator Placement Problem* (RPP) problem is to find i) the minimum number of regenerators and ii) their locations, so that a communication path can be established between every pair of source-destination nodes in the network.

*This research is supported in part by the U. S. Army Research Office, the Air Force Office of Scientific Research and Defense Threat Reduction Agency.

SPAA'10, June 13–15, 2010, Thira, Santorini, Greece.
ACM 978-1-4503-0079-7/10/06.

The RPP has been studied by a number of researchers [4, 5, 2]. However most of the published methods of locating regenerators operate by iteratively improving previously computed routes until they become feasible. These methods usually generate a path between each pair of nodes and then place regenerators, as needed, along those paths to make them feasible. However, this approach usually results in placing a significantly higher number of regenerators than are needed to ensure that a path can be established between every source-destination node pair.

Although most of the studies indicated earlier focused on the technological aspects of regenerator placement in optical networks, the theoretical computer science community also has investigated these problems [2]. In a recent paper in SPAA [2], the authors claim that their study is the first that presents a theoretical framework to study the RPP and related problems. They present polynomial time algorithms, NP-complete proofs, approximation algorithms and inapproximability results for four different versions of the RPP problem.

The contribution of this paper is the following:

- We show that the RPP can be effectively solved using an approximation algorithm for the minimum connected dominated set problem.
- We point out several serious flaws of the algorithm presented for the solution of RPP in [2].

2. REGENERATOR PLACEMENT PROBLEM

In this section, we first discuss the approach taken in [2] for the solution of the RPP. After pointing out a few limitations of their approach, we present our technique in subsection 2.2.

2.1 Flammini et al. approach to RPP

An optical network is modeled as an unweighted undirected graph $G = (V, E)$ in [2]. The length of a path is measured in terms of the number of edges that constitute the path and the notion of optical reach is incorporated by putting a bound (d) on the number of edges a lightpath can traverse before requiring regeneration. A connection between a source (s) and a destination (t) comprises of a sequence of lightpaths from s to a regenerator node, or from one regenerator node to another, or from one regenerator node to t (of length at most d) whose concatenation form a path from s to t. A solution to the RPP consists of identification of the smallest subset of nodes $U \subseteq V$ to place the regenerators, such that paths can be established between source-destination node pairs.

We make a few comments on the network model and solution technique proposed in [2].

Comment 1: Given that in reality the distance between nodes are not identical and deterioration of signal strength is proportional to the distance traversed by an optical signal, we feel that an edge-weighted graph would have been a more appropriate model for the RPP, instead of the un-weighted graph used in [2]. In our model described in section 2.2, we use an edge-weighted graph model, where the weights on the edges represent the distance between the nodes.

Comment 2: The authors in [2] make a distinction between *simple path case* and *non-simple path case*. They correctly note that if only simple paths are acceptable for lightpath establishment, then the number of regenerators needed to establish connection between the nodes can be significantly higher than the number of regenerators needed when non-simple paths are also acceptable. However, we would like to point out that the distinction should not be drawn between simple and non-simple path cases as non-simple path cases are acceptable under some conditions and unacceptable under some others. We elaborate our observation with an example shown in Figure 1. In the figure, the weights assigned on the edges represent the distance between the corresponding nodes in miles. If the optical reach is 3250 miles, we can establish a non-simple path $P_1 : A - B - C - D - F - G - B - C - H$ from node A to H, with signal regeneration at node D. The two path segments that make up the path from A to H are $(P_{1,1} : A - B - C - D)$ and $(P_{1,2} : D - F - G - B - C - H)$. However, this may not be an acceptable solution when we consider free wavelengths available on each of the fiber links. Let λ_1 be the only wavelength available on the fiber links AB and CH and let wavelengths $\{\lambda_1, \lambda_2, \ldots\}$ be available on the remaining links. In this scenario both the path segments $P_{1,1}$ and $P_{1,2}$ must use wavelength λ_1 to set up the lightpath. This will not be an acceptable solution since the link BC appears in both path segments. Thus in this case, the non-simple path P_1 from A to H will be unacceptable. However, the non-simple path P_2 from A to H given by $A - B - G - F - D - F - G - B - C - H$ will be perfectly acceptable, as this path will be composed of two path segments $P_{2,1} : A - B - G - F - D$ and $P_{2,2} : D - F - G - B - C - H$. The difference between the two cases is that in the first case the two path segments $P_{1,1}$ and $P_{1,2}$ share an edge, BC, that is traversed in the same direction and in the second case the two path segments $P_{2,1}$ and $P_{2,2}$ do not share any edge that is traversed in the same direction. Since in an optical network, traffic in opposite directions are normally carried by two different optical fibers, the non-simple path P_1 is unacceptable, whereas the non-simple path P_2 is acceptable. The authors in [2] do not make a distinction between these two types of non-simple paths. If X, Y, Z represent the number of regenerators that will be needed for a RPP problem instance for the cases where (i) only simple paths are acceptable, (ii) all simple and non-simple paths are acceptable and (ii) all simple and only non-simple paths that do not share any edge that is traversed in the same direction are acceptable, respectively, then $X \geq Z \geq Y$.

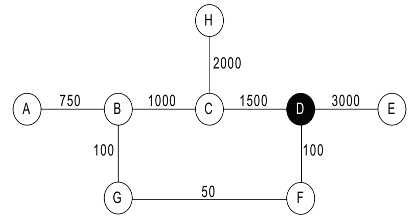

Figure 1: Example illustrates need for edge-disjointness among directed path segments

Comment 3: In proof of the Theorem 2.8 in [2], the authors describe an algorithm to find a solution to a version of RPP denoted by `RPP/∞/req`. `RPP/∞/req` problem states that given route requests between different source-destination pairs and an optical reach $d > 0$, find locations for the smallest number of regenerators. The algorithm starts by transforming the instance of `RPP/∞/req` problem to an instance of the set-cover problem. Unfortunately, the algorithm at times may fail to find a solution of the `RPP/∞/req` problem. The solution computed by this algorithm when executed on the graph $G = (V, E)$ is shown in Figure 2. For this graph the solution corresponding to the set-cover instance is the solution for the `RPP/∞/req` problem with $d = 2$. But it is not a solution for the `RPP/∞/req` problem for the graph G with $d = 2$ as the path length between the nodes v_2 and v_3 is greater than 2. Also the solution of the set-cover instance of `RPP/∞/req` problem does not always guarantee that a simple path can be found between every source-destination node pairs, once regenerators are placed at the locations identified by the algorithm. The Figure 3 shows another example where the solution returned by this algorithm for the `RPP/∞/req` problem with $d = 3$ is the solution corresponding to the set-cover instance of graph $G = (V, E)$. In Figure 3 the algorithm returns node r as the final solution for placement of a regenerator. However, if a regenerator is placed at only this node, the path between the leaf nodes v_1 and v_2 will be non-simple. In order to have at least one simple path between every pairs of nodes the correct solution has to place regenerators at nodes a and b.

2.2 Our approach to solution of the RPP

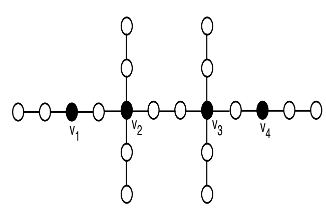

Figure 2: Solution returned by the algorithm for `RPP/∞/req` problem instance of graph G, where $d = 2$

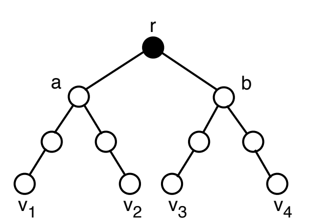

Figure 3: Solution returned by the algorithm for `RPP/∞/req` problem instance of graph G, where $d = 3$. There is no simple path between v_1 and v_2 in the solution

In our model $G = (V, E)$ is a connected *edge-weighted* graph with edge-weights representing distances between the corresponding nodes.

Path Segment w.r.t. node set V': Given a source node s, a destination node t and a subset $V' \subseteq V$, a subpath PS of a path from s to t (henceforth called a $s-t$ path) is referred to as a *path segment*, if the end-points of PS are in $V' \cup \{s, t\}$ and no intermediate node is in V'.

Optical reach constraint: Given a path in a network between a source and a destination, the optical reach constraint ensures that the distance of any path segment between two regenerator nodes, or the distance from the source to a regenerator or the distance from a regenerator to the destination on the $s-t$ path does not exceed fiber's optical reach.

Regenerator Placement Problem (RPP): Given $G = (V, E)$, the problem is to find the smallest $V' \subseteq V$ such that there exists a path between every pair of nodes $\{s, t\} \in V$ where (i) no path segment of the $s-t$ path has a length greater than R and (ii) no two path segments of the $s-t$ path share an edge that is traversed in the same direction.

Reachability Graph: Given a network graph $G = (V, E)$ with edge weights representing the distances between the nodes, and an optical reach distance R, the *reachability graph* $G' = (V', E')$ corresponding to G is constructed as follows: $V' = V$ and two nodes v_i and v_j in V' will have an edge between them if the shortest path length between those two nodes in V is at most R.

Theorem 1: The Minimum Connected Dominating Set (MCDS) of the reachability graph $G' = (V', E')$ of the network graph $G = (V, E)$ represents the solution of Regenerator Placement Problem (RPP), in non-trivial cases in which G' is not a clique.

Proof: The proof follows from the following lemmas.

Lemma 1: If there's an α-approximation algorithm for the MCDS problem, then there's an α-approximation algorithm for the RPP.

Proof: Given an instance $[G = (V, E), w : E \to \mathbb{R}^+, R]$ of the RPP problem, construct an instance $G' = (V', E')$ of the MCDS problem in the following way: Set $V' = V$, and let $(u, v) \in E'$ iff $d_G(u, v) \leq R$. Here $d_G(u, v)$ denotes the shortest distance between the nodes u and v in G in terms of the distance function w. For convenience, define a distance function $w' : E' \to \mathbb{R}^+$ by assigning $w'(uv) = d_G(u, v)$.

Let $\emptyset \neq S \subseteq V = V'$ be a MCDS of the graph G'. Consider any two vertices s and t in V. Without loss of generality, assume $st \notin E'$. (Otherwise, there's a path from s to t of length $\leq R$ and we're done.) We need to show that there's a walk from s to t in G such that no two path segments in the walk share an edge that is traversed in the same direction twice. First, let s' be the vertex in S for which $w'(ss')$ is smallest, and t' be the vertex in S for which $w'(tt')$ is smallest. If $s \in S$ then $s = s'$. If $t \in S$, then $t = t'$.

Let $s' = s_1, s_2, \ldots, s_m = t'$ be the shortest path in $G'[S]$ between s' and t'. Since S is a connected dominating set, such a path exists. Now, for each edge $s_i s_{i+1} \in E'$ on this path, there's a corresponding (s_i, s_{i+1})-path in G with length $\leq R$. Hence, putting everything together, we can find a walk from s to t in G consisting of segments $s = s_0 \rightsquigarrow s_1$, $s_1 \rightsquigarrow s_2$, ..., $s_{m-1} \rightsquigarrow s_m$, $s_m \rightsquigarrow t = s_{m+1}$. Each segment has length $\leq R$.

Now, suppose there's some edge $uv \in E$ which is traversed twice in the same direction (from u to v). Suppose uv was used in the segment $s_i \rightsquigarrow s_{i+1}$ and then again in segment $s_j \rightsquigarrow s_{j+1}$. Without loss of generality, assume $i < j$. Now, on the $s_j \rightsquigarrow s_{j+1}$ segment, the length of the $s_j \rightsquigarrow u$ part has to be at least the length of the $u \rightsquigarrow s_{i+1}$ part. Otherwise, from s_i we could have gone to u, and then take the $u \rightsquigarrow s_j$ path to s_j; this "shortcut" would contradict that fact that the s', t'-path we chose was a shortest path.

Thus, the $v \rightsquigarrow s_{i+1}$ part is strictly shorter than the $s_j \rightsquigarrow u$ path. Consequently, from s_{i+1} we can go to v and then take the $v \rightsquigarrow s_{j+1}$ branch; this would be shorter than the current $s_j \rightsquigarrow s_{j+1}$ path, again contradicting the shortest path choice. This proves that a feasible solution for the MCDS instance is a feasible solution for the RPP instance.

Lemma 2: If there's an α-approximation algorithm for the RPP, then there's an α-approximation algorithm for the MCDS problem.

Proof: Consider an instance G of the MCDS problem. Construct an instance G' of the RPP by setting $G' = G$, and $R = 1$. Set the weight of each edge to be 1. It's easy to see that if a solution is feasible for RPP on G' then it is feasible for MCDS problem on G.

However, both lemma 1 and 2 holds except the trivial case when reachability graph G' is clique. If G' is a clique then solution of RPP will return 0 node while MCDS of G' will give exactly 1 node.

From recent [3], we know that MINIMUM CONNECTED DOMINATING SET (MCDS) can be approximated to within about $\ln n + O(1)$. We also know that MCDS cannot be approximated (unless $P = NP$) to within $\ln n - \Theta(\ln \ln n)$ [1].

Corollary 3: There's a $O(\ln n)$-approximation algorithm for the RPP.

Comment 4: In [2] the authors present an algorithm for the `RPP/∞/req` with approximation ratio of $\frac{3}{2} log\ m + 1$, where the demand matrix is all-to-all. If the demand matrix is all-to-all m is $O(n^2)$, where n is the number of nodes in the network. In this case the approximation ratio becomes $3 log\ n + 1$. In our MCDS based approach for the solution of RPP, we can provide better performance, as MCDS can be computed with approximation ratio $ln\ \delta + 2$ where δ is the maximum degree in the input graph [3].

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