Shannon-type Inequalities, Submodular Width, and Disjunctive Datalog

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(Stealth Mode)

With Mahmoud Abo Khamis and Dan Suciu @ PODS 2017

HIGHLIGHTS
of Logic, Games and Automata

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A Query Evaluation Problem

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- Relation $R_F(\mathbf{A}_F)$ for each $F \in \mathcal{E}$, R_F for short
  - e.g. R_{123} for $R_{123}(A_1, A_2, A_3)$
  - e.g. T_{25} for $T_{25}(A_2, A_5)$

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How do we evaluate Q efficiently?

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$$Q : S() \leftarrow \bigwedge_{F \in \mathcal{E}} R_F ,$$ *// can all R_F be satisfied at once?*

Question

How do we evaluate Q efficiently?

Conjunctive, count, aggregate queries are fine too.

What do we mean by "efficiency"?

Database centric complexity framework

- Assumption 1: query size $\ll$ data size
  - Data complexity
  - Fixed parameter tractability (e.g. parameter = some function of query size)

$$\tilde{O}(\text{something}) = O\left(f(|\text{query}|) \cdot \text{polylog}(|\text{data}|) \cdot \text{something}\right)$$

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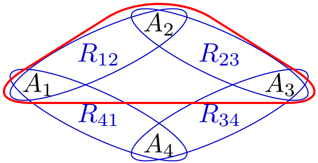
- Assumption 2: known constraints on input relations
  - Cardinalities of materialized relations, *or upper bounds*
    - cardinality constraints (CC)
  - Functional dependencies, *the more the merrier*
    - FD constraints (FDC)
  - Degree bounds, *the more the merrier*
    - Degree constraints (DC)

Example

Q : $S() \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$

$$\wedge A_1 + A_2 = A_3.$$

$$\mathcal{H} = ([4], \{12, 23, 34, 14, 123\})$$



A_1	A_2
2	1
4	1
4	2
5	2

A_2	A_3
1	c
1	d
2	c

A_3	A_4
c	3
d	4
d	5
e	6
e	7

A_4	A_1
3	2
4	4
4	5

- Cardinalities: $|R_{12}| = 4$, $|R_{23}| = 3$, ...

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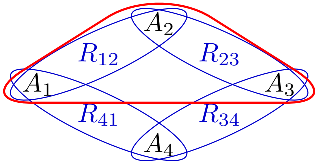

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- Cardinalities: $|R_{12}| = 4$, $|R_{23}| = 3, \ldots$
- FD: $\{A_1, A_2\} \to A_3$, $\{A_3, A_2\} \to A_1, \ldots$

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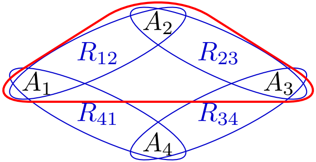

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- Cardinalities: $|R_{12}| = 4, |R_{23}| = 3, \ldots$
- FD: $\{A_1, A_2\} \to A_3, \{A_3, A_2\} \to A_1, \ldots$
- Degree Bounds: $\deg_{34}(A_4|A_3 = x) \stackrel{\text{def}}{=} |\sigma_{A_3=x}(R_{34})| \leq 2, \forall x, \ldots$

Degree- generalize cardinality- and FD-constraints

DC ⊇ CC ∪ FDC

- Degree Constraints (DC):

$$\deg_F(\mathbf{A}_Y | \mathbf{A}_X) \le N_{Y|X}, \qquad X \subset Y \subseteq F \in \mathcal{E}$$

Degree- generalize cardinality- and FD-constraints

$$\mathsf{DC} \supseteq \mathsf{CC} \cup \mathsf{FDC}$$

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$$|R_F| \leq N \Leftrightarrow \deg_F(\mathbf{A}_F | \mathbf{A}_\emptyset) \leq N_{F|\emptyset} \stackrel{\text{def}}{=} N.$$

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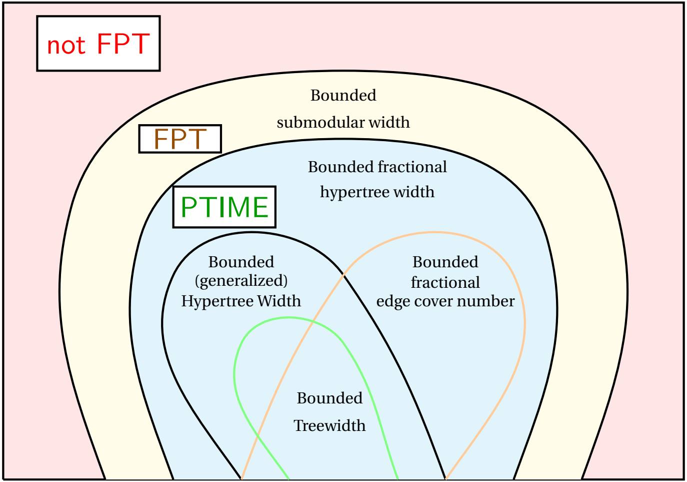
  - Functional Dependencies (FDC):

$$\mathbf{A}_X \to \mathbf{A}_Y \Leftrightarrow \deg_F(\mathbf{A}_Y | \mathbf{A}_X) \leq N_{Y|X} \stackrel{\text{def}}{=} 1.$$

Islands of tractability (redrawn from D. Marx's slides)

Prior results with cardinality constraints



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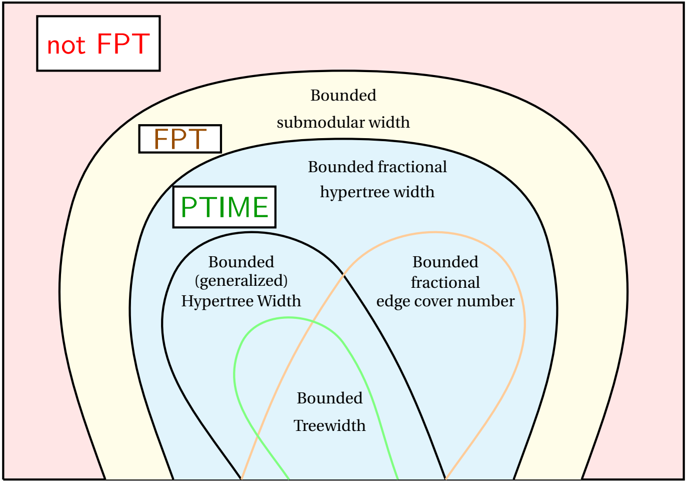
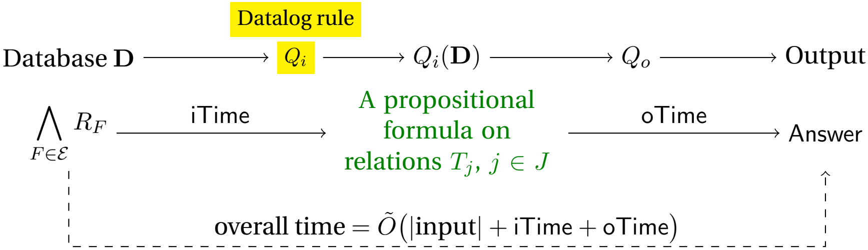


We want the same map with degree constraints.

A Meta Algorithm (i.e. Meta Query Plan)



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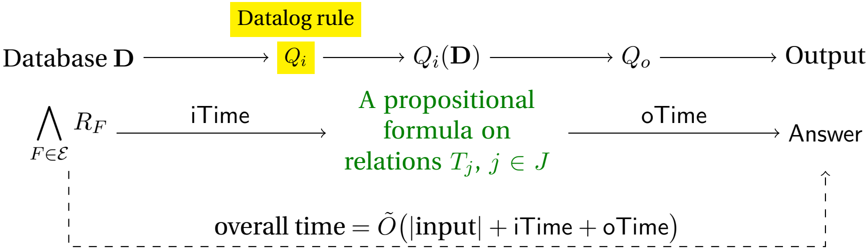

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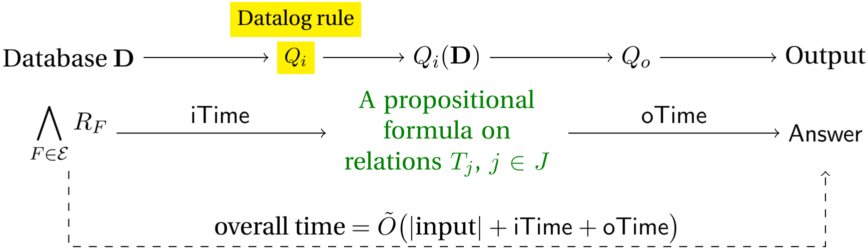

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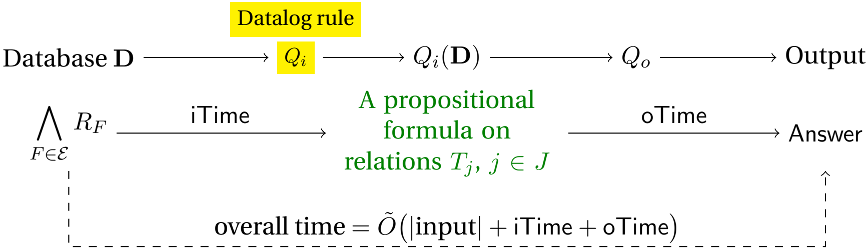

(a) $\mathsf{oTime} = |Q_i(\mathbf{D})| + |\mathsf{answer}|$

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(b) $\mathsf{iTime} = \max_{\mathbf{D} \models \mathsf{DC}} |Q_i(\mathbf{D})|$

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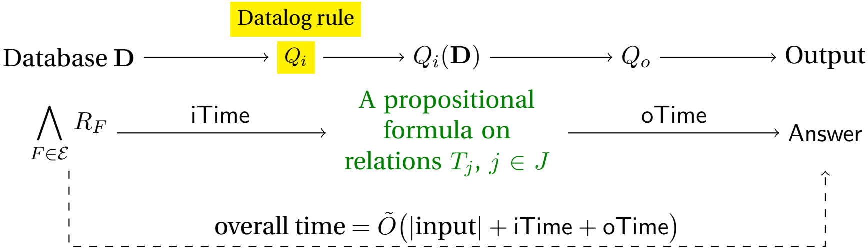

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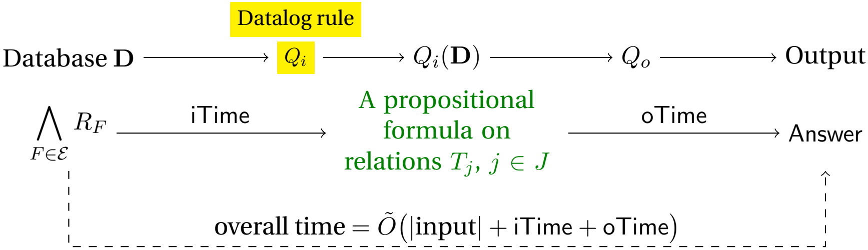

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- Design Q_i s.t. (a) holds and (b) as small as possible

Option 1: Full Conjunctive Query

Full conjunctive query

Database $\mathbf{D}$ $\longrightarrow$ Q_i $\longrightarrow$ $Q_i(\mathbf{D})$ $\longrightarrow$ Q_o $\longrightarrow$ Output

$\bigwedge_{F \in \mathcal{E}} R_F \xrightarrow{\text{iTime}} T_{[n]} \xrightarrow{\text{oTime}} \text{Answer}$

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- $\text{oTime} = |Q_i(\mathbf{D})| + |\text{answer}|$ trivially true
- $\text{iTime} = \max_{\mathbf{D} \models \text{DC}} |Q_i(\mathbf{D})|$ worst-case optimal algorithm
  - Known if all DC are cardinality constraints
    - NPRR [Ngo, Porat, Ré, Rudra PODS'12]
    - Leapfrog-Triejoin [Veldhuizen ICDT'14]
    - Generic Join [Ngo, Ré, Rudra SIGMOD Records 2013]
  - Unknown for general DC until our work

Detour: Tree Decompositions, Informally

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$$S() \leftarrow R(a,b,d) \wedge c < d \wedge T(c,b,d) \wedge U(b,e) \wedge V(c,e)$$
$$\wedge\, b + e = f \wedge W(b,e,g) \wedge \;\wedge X(i,j,h) \wedge e - b = k.$$

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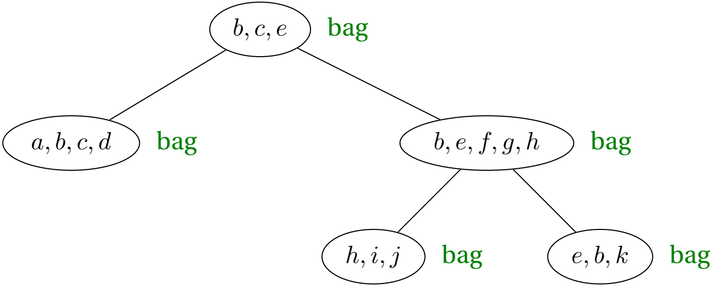

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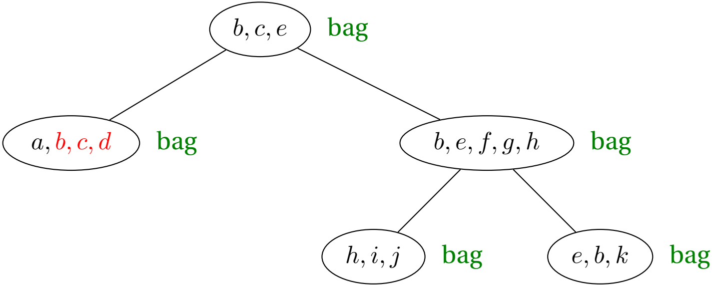

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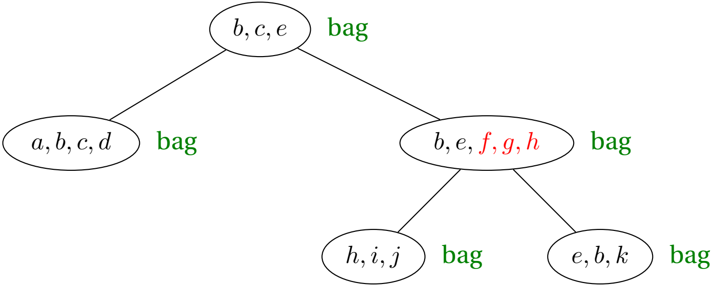

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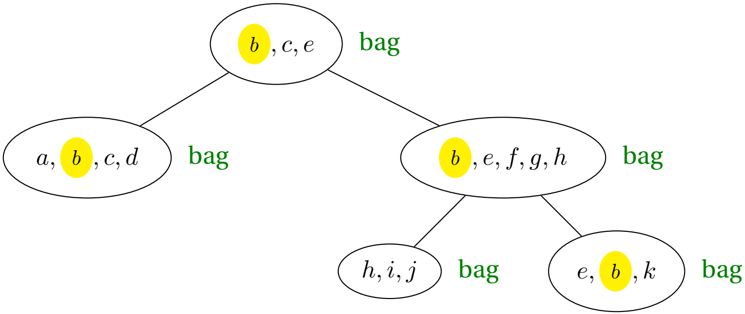

- Every relation is covered by some bag
- Bags conntaining a given variable are connected

Detour: Tree Decompositions, Formally

- Hypergraph $\mathcal{H} = ([n], \mathcal{E})$
- A Tree Decomposition of $\mathcal{H}$ is a pair $(\mathcal{T}, \chi)$ where
  - $\mathcal{T} = (V(\mathcal{T}), E(\mathcal{T}))$ is a tree

See [Gottlob et al 2016], Gems of PODS.

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- A Tree Decomposition of $\mathcal{H}$ is a pair $(\mathcal{T}, \chi)$ where
  - $\mathcal{T} = (V(\mathcal{T}), E(\mathcal{T}))$ is a tree
  - $\chi : V(\mathcal{T}) \to 2^{[n]}$ assigns a bag $\chi(v)$ to each tree-node v
    - Every hyperedge $F \in \mathcal{E}$ is covered by some bag $(F \subseteq \chi(v))$
    - Bags containing $\forall i \in [n]$ forms a subtree

See [Gottlob et al 2016], Gems of PODS.

Option 2: A Single Tree Decomposition

Fix $(\mathcal{T}, \chi)$

Multiple Conjunctive Rules

$$\text{Database } \mathbf{D} \longrightarrow Q_i \longrightarrow Q_i(\mathbf{D}) \longrightarrow Q_o \longrightarrow \text{Output}$$

$$\bigwedge_{F \in \mathcal{E}} R_F \xrightarrow{\text{iTime}} \bigwedge_{v \in V(\mathcal{T})} T_{\chi(v)} \xrightarrow{\text{oTime}} \text{Answer}$$

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  - $P_v : T_{\chi(v)} \leftarrow \bigwedge_{F \in \mathcal{E}} R_F \qquad v \in V(\mathcal{T})$

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- $Q_i(\mathbf{D})$ is Olteanu's factorized database (FDB)
- $\mathsf{oTime} = |Q_i(\mathbf{D})| + |\mathsf{answer}|$
  - Yannakakis for join, FDB/InsideOut for aggregates
- $\mathsf{iTime} = \max_{\mathbf{D} \models \mathsf{DC}} |Q_i(\mathbf{D})| \leq \max_{\mathbf{D} \models \mathsf{DC}} \max_{v \in V(\mathcal{T})} |P_v(\mathbf{D})|$
  - $P_v \ : \ T_{\chi(v)} \leftarrow \bigwedge_{F \in \mathcal{E}} R_F \qquad v \in V(\mathcal{T})$

$$\min_{(\mathcal{T},\chi)} \max_{\mathbf{D} \models \mathsf{CC}} \max_{v \in V(\mathcal{T})} |P_v(\mathbf{D})| \leq N^{\mathsf{fhtw}(\mathcal{H})} \leq N^{\mathsf{ghtw}(\mathcal{H})} \leq N^{\mathsf{tw}(\mathcal{H})+1}$$

Option 3: Multiple Tree Decompositions

$$\text{Database } \mathbf{D} \longrightarrow Q_i \longrightarrow Q_i(\mathbf{D}) \longrightarrow Q_o \longrightarrow \text{Output}$$

$$\bigwedge_{F \in \mathcal{E}} R_F \xrightarrow{\mathsf{iTime}} \bigvee_{(\mathcal{T},\chi)} \bigwedge_{v \in V(\mathcal{T})} T_{\chi(v)} \xrightarrow{\mathsf{oTime}} \mathsf{Answer}$$

- $\bigvee$ ranges over non-redundant TDs $(\mathcal{T}, \chi)$

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- $\bigvee$ ranges over non-redundant TDs $(\mathcal{T}, \chi)$
- oTime $= |Q_i(\mathbf{D})| + |\text{answer}|$
  - Union of Yannakakis on all TDs

Option 3: Multiple Tree Decompositions

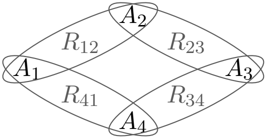
How to evaluate this?

Database $\mathbf{D}$ $\longrightarrow$ Q_i $\longrightarrow$ $Q_i(\mathbf{D})$ $\longrightarrow$ Q_o $\longrightarrow$ Output

$$\bigwedge_{F \in \mathcal{E}} R_F \xrightarrow{\quad \mathsf{iTime} \quad} \bigvee_{(\mathcal{T},\chi)} \bigwedge_{v \in V(\mathcal{T})} T_{\chi(v)} \xrightarrow{\quad \mathsf{oTime} \quad} \mathsf{Answer}$$

- $\bigvee$ ranges over non-redundant TDs $(\mathcal{T}, \chi)$
- $\mathsf{oTime} = |Q_i(\mathbf{D})| + |\mathsf{answer}|$
  - Union of Yannakakis on all TDs
- $\mathsf{iTime} = \max_{\mathbf{D} \models \mathsf{DC}} |Q_i(\mathbf{D})| \leq$?

Example: $Q \; : \; S() \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$



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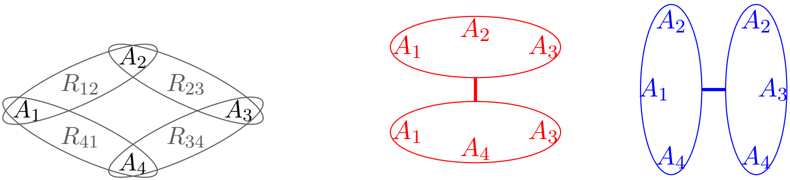

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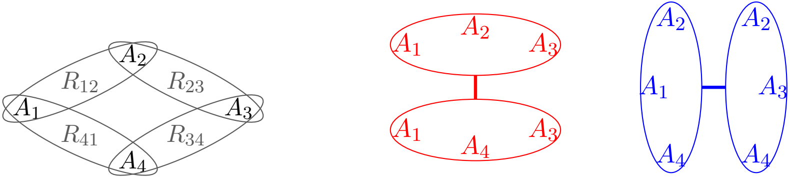
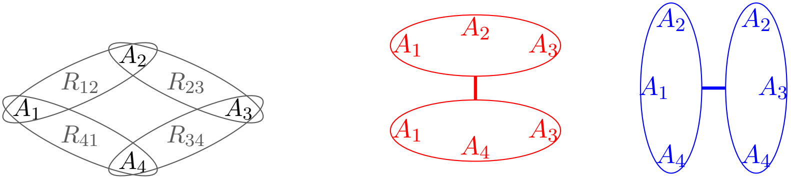


$$(T_{123} \wedge T_{134}) \vee (T_{124} \wedge T_{234}) \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$$

Example: $Q \ : \ S() \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$



$$(T_{123} \wedge T_{134}) \vee (T_{124} \wedge T_{234}) \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$$

By distributivity, rewrite an equivalent the head:

$$(T_{123} \vee T_{124}) \wedge (T_{123} \vee T_{234}) \wedge (T_{134} \vee T_{124}) \wedge (T_{134} \vee T_{234})$$
$$\leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$$

(Each “clause” has one bag per TD)

Option 3: Multiple Tree Decompositions

Multiple Disjunctive Datalog Rules!

Database $\mathbf{D} \longrightarrow Q_i \longrightarrow Q_i(\mathbf{D}) \longrightarrow Q_o \longrightarrow$ Output

$$\bigwedge_{F\in\mathcal{E}} R_F \xrightarrow{\text{iTime}} \bigwedge_{\mathcal{B}} \bigvee_{B\in\mathcal{B}} T_B \xrightarrow{\text{oTime}} \text{Answer}$$

- oTime $= |Q_i(\mathbf{D})| + |\text{answer}|$
  - Union of Yannakakis on all TDs
- iTime $= \max_{\mathbf{D}\models\text{DC}} |Q_i(\mathbf{D})| \leq \max_{\mathbf{D}\models\text{DC}} \max_{\mathcal{B}} |P_{\mathcal{B}}(\mathbf{D})|$
  - $P_{\mathcal{B}} : \bigvee_{B\in\mathcal{B}} T_B \leftarrow \bigwedge_{F\in\mathcal{E}} R_F$

disjunctive datalog rule

Option 3: Multiple Tree Decompositions

Multiple Disjunctive Datalog Rules!

$$\text{Database } \mathbf{D} \longrightarrow Q_i \longrightarrow Q_i(\mathbf{D}) \longrightarrow Q_o \longrightarrow \text{Output}$$

$$\bigwedge_{F \in \mathcal{E}} R_F \xrightarrow{\text{iTime}} \bigwedge_{\mathcal{B}} \bigvee_{B \in \mathcal{B}} T_B \xrightarrow{\text{oTime}} \text{Answer}$$

- oTime $= |Q_i(\mathbf{D})| + |\text{answer}|$
  - Union of Yannakakis on all TDs
- iTime $= \max_{\mathbf{D} \models \mathsf{DC}} |Q_i(\mathbf{D})| \leq \max_{\mathbf{D} \models \mathsf{DC}} \max_{\mathcal{B}} |P_{\mathcal{B}}(\mathbf{D})|$
  - $P_{\mathcal{B}} : \bigvee_{B \in \mathcal{B}} T_B \leftarrow \bigwedge_{F \in \mathcal{E}} R_F$

disjunctive datalog rule

$$\max_{\mathbf{D} \models \mathsf{CC}} \max_{\mathcal{B}} |P_{\mathcal{B}}(\mathbf{D})| \leq N^{\mathsf{subw}(\mathcal{H})} \leq N^{\mathsf{fhtw}(\mathcal{H})}$$

subw = submodular width (Daniel Marx, JACM'2013)

Example: $Q\ : S() \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$

Option 1:

$$Q_i : \qquad T_{1234} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$$

Example: $Q\ : S() \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$

Option 1:

$$Q_i : \qquad T_{1234} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$$

Option 2:

$$\text{either } Q_i : \qquad T_{123} \wedge T_{134} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$$

$$\text{or } Q_i : \qquad T_{124} \wedge T_{234} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$$

Example: $Q\ : S() \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$

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Option 3:

$$Q_i : \qquad (T_{123} \wedge T_{134}) \vee (T_{124} \wedge T_{234}) \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$$

Equivalent to:

$$P_{123,124} : \qquad T_{123} \vee T_{124} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$$

$$P_{123,234} : \qquad T_{123} \vee T_{234} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$$

$$P_{134,124} : \qquad T_{134} \vee T_{124} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$$

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Example: $Q\ : S() \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$

Option 1: $\mathsf{iTime} = \max_{\mathbf{D} \models \mathsf{DC}} |Q_i(\mathbf{D})| = N^2$

$$Q_i : \quad T_{1234} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$$

Option 2:

either $Q_i : \quad T_{123} \wedge T_{134} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$

or $Q_i : \quad T_{124} \wedge T_{234} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$

Option 3:

$$Q_i : \quad (T_{123} \wedge T_{134}) \vee (T_{124} \wedge T_{234}) \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$$

Equivalent to:

$$P_{123,124} : \quad T_{123} \vee T_{124} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$$
$$P_{123,234} : \quad T_{123} \vee T_{234} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$$
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Example: $Q \; : S() \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$

Option 1: $\textsf{iTime} = \max_{\mathbf{D} \models \textsf{DC}} |Q_i(\mathbf{D})| = N^2$

$$Q_i : \quad T_{1234} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$$

Option 2: $\textsf{iTime} = \max_{\mathbf{D} \models \textsf{DC}} |Q_i(\mathbf{D})| = N^2$

either $Q_i : \quad T_{123} \wedge T_{134} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$

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Option 3:

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Equivalent to:

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Example: $Q\ : S() \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$

Option 1: $\text{iTime} = \max_{\mathbf{D} \models \text{DC}} |Q_i(\mathbf{D})| = N^2$

$$Q_i : \quad T_{1234} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$$

Option 2: $\text{iTime} = \max_{\mathbf{D} \models \text{DC}} |Q_i(\mathbf{D})| = N^2$

either $Q_i : \quad T_{123} \wedge T_{134} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$

or $Q_i : \quad T_{124} \wedge T_{234} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$

Option 3: $\text{iTime} = \max_{\mathbf{D} \models \text{DC}} |Q_i(\mathbf{D})| = N^{3/2}$

$$Q_i : \quad (T_{123} \wedge T_{134}) \vee (T_{124} \wedge T_{234}) \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$$

Equivalent to:

$$P_{123,124} : \quad T_{123} \vee T_{124} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$$
$$P_{123,234} : \quad T_{123} \vee T_{234} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$$
$$P_{134,124} : \quad T_{134} \vee T_{124} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$$
$$P_{134,234} : \quad T_{134} \vee T_{234} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$$

Roadmap

Given degree constraints DC, and a disjunctive datalog rule

$$P \; : \; \bigvee_{B \in \mathcal{B}} T_B \leftarrow \bigwedge_{F \in \mathcal{E}} R_F$$

Roadmap

Given degree constraints DC, and a disjunctive datalog rule

$$P \ : \ \bigvee_{B \in \mathcal{B}} T_B \leftarrow \bigwedge_{F \in \mathcal{E}} R_F$$

Question (Worst-case Output Size Bound)

Find a good upper-bound for $\max_{\mathbf{D} \models DC} |P(\mathbf{D})|$

Roadmap

Given degree constraints DC, and a disjunctive datalog rule

$$P \; : \; \bigvee_{B \in \mathcal{B}} T_B \leftarrow \bigwedge_{F \in \mathcal{E}} R_F$$

Question (Worst-case Output Size Bound)

Find a good upper-bound for $\max_{\mathbf{D} \models DC} |P(\mathbf{D})|$

Question (Algorithm)

Design an algorithm evaluating P *within the bound.*

Roadmap

Given degree constraints DC, and a disjunctive datalog rule

$$P \; : \; \bigvee_{B \in \mathcal{B}} T_B \leftarrow \bigwedge_{F \in \mathcal{E}} R_F$$

Question (Worst-case Output Size Bound)

Find a good upper-bound for $\max_{\mathbf{D} \models DC} |P(\mathbf{D})|$

Question (Algorithm)

Design an algorithm evaluating P *within the bound.*

Question (Gathering fruits)

Plug bound/algorithm into Meta Algorithm, what do we get?

Table of Contents

High-level View of the Bound

Given degree constraints DC, a disjunctive datalog rule

$$P \ : \ \bigvee_{B \in \mathcal{B}} T_B \leftarrow \bigwedge_{F \in \mathcal{E}} R_F$$

We shall prove bounds of the form

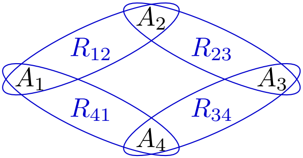
$$\max_{\mathbf{D} \models \mathsf{DC}} \log |P(\mathbf{D})| \leq \text{some function of } h$$

s.t. h is (approximately) entropic

and h satisfies degree constraints

An Idea From Gottlob-Lee-Valiant-Valiant, JACM'12

$$Q(A_1, A_2, A_3, A_4) \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}.$$



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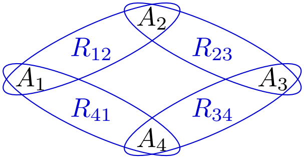


$$Q(A_1, A_2, A_3, A_4) \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}.$$

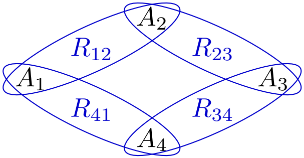
A_1	A_2
a	1
b	1
b	2

A_2	A_3
1	c
1	d
2	c

A_3	A_4
c	3
d	4
d	5

A_4	A_1
3	b
4	a
4	b

An Idea From Gottlob-Lee-Valiant-Valiant, JACM'12



$$Q(A_1, A_2, A_3, A_4) \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}.$$

A_1	A_2	A_3	A_4
a	1	d	4
b	1	c	3
b	1	d	4
b	2	c	3

A_1	A_2
a	1
b	1
b	2

A_2	A_3
1	c
1	d
2	c

A_3	A_4
c	3
d	4
d	5

A_4	A_1
3	b
4	a
4	b

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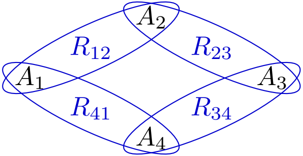

$$Q(A_1, A_2, A_3, A_4) \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}.$$

A_1	A_2	A_3	A_4	
a	1	d	4	1/4
b	1	c	3	1/4
b	1	d	4	1/4
b	2	c	3	1/4

A_1	A_2
a	1
b	1
b	2

A_2	A_3
1	c
1	d
2	c

A_3	A_4
c	3
d	4
d	5

A_4	A_1
3	b
4	a
4	b

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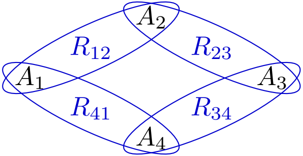

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b	1	c	3	1/4
b	1	d	4	1/4
b	2	c	3	1/4

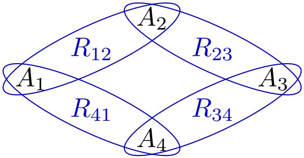
A_1	A_2	
a	1	1/4
b	1	2/4
b	2	1/4

A_2	A_3	
1	c	1/4
1	d	2/4
2	c	1/4

A_3	A_4	
c	3	2/4
d	4	2/4
d	5	0

A_4	A_1	
3	b	2/4
4	a	1/4
4	b	1/4

An Idea From Gottlob-Lee-Valiant-Valiant, JACM'12



$$Q(A_1, A_2, A_3, A_4) \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}.$$

A_1	A_2	A_3	A_4	
a	1	d	4	1/4
b	1	c	3	1/4
b	1	d	4	1/4
b	2	c	3	1/4

$H_{\mathsf{unif}}(A_1 A_2 A_3 A_4) = \log |Q|$

A_1	A_2	
a	1	1/4
b	1	2/4
b	2	1/4

A_2	A_3	
1	c	1/4
1	d	2/4
2	c	1/4

A_3	A_4	
c	3	2/4
d	4	2/4
d	5	0

A_4	A_1	
3	b	2/4
4	a	1/4
4	b	1/4

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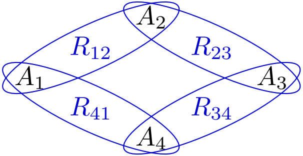

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A_1	A_2	A_3	A_4	
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b	1	d	4	1/4
b	2	c	3	1/4

$H_{\mathsf{unif}}(A_1 A_2 A_3 A_4) = \log |Q|$

A_1	A_2	
a	1	1/4
b	1	2/4
b	2	1/4

A_2	A_3	
1	c	1/4
1	d	2/4
2	c	1/4

A_3	A_4	
c	3	2/4
d	4	2/4
d	5	0

A_4	A_1	
3	b	2/4
4	a	1/4
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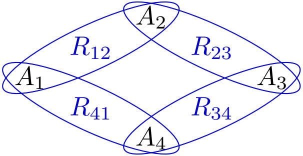
$H_{\mathsf{unif}}(A_1 A_2) \leq \log |R_{12}|$, $H_{\mathsf{unif}}(A_2 A_3) \leq \log |R_{23}|$, $H_{\mathsf{unif}}(A_3 A_4) \leq \log |R_{34}|$, $\ldots$

An Idea From Gottlob-Lee-Valiant-Valiant, JACM'12



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b	1	d	4	1/4
b	2	c	3	1/4

$H_{\mathsf{unif}}(A_1 A_2 A_3 A_4) = \log |Q|$

A_1	A_2	
a	1	1/4
b	1	2/4
b	2	1/4

A_2	A_3	
1	c	1/4
1	d	2/4
2	c	1/4

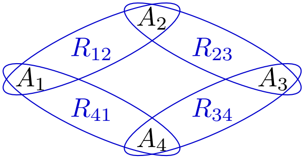
A_3	A_4	
c	3	2/4
d	4	2/4
d	5	0

A_4	A_1	
3	b	2/4
4	a	1/4
4	b	1/4

$H_{\mathsf{unif}}(A_1 A_2) \leq \log |R_{12}|, \quad H_{\mathsf{unif}}(A_2 A_3) \leq \log |R_{23}|, \quad H_{\mathsf{unif}}(A_3 A_4) \leq \log |R_{34}|, \quad \ldots$

$H_{\mathsf{unif}}(A_2 | A_1 = \text{‘a’}) \leq \log \left|\sigma_{A_1 = \text{‘a’}} R_{12}\right|, \quad H_{\mathsf{unif}}(A_2 | A_1 = \text{‘b’}) \leq \log \left|\sigma_{A_1 = \text{‘b’}} R_{12}\right|, \ldots$

An Idea From Gottlob-Lee-Valiant-Valiant, JACM'12



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b	1	d	4	1/4
b	2	c	3	1/4

$H_{\mathsf{unif}}(A_1 A_2 A_3 A_4) = \log |Q|$

A_1	A_2	
a	1	1/4
b	1	2/4
b	2	1/4

A_2	A_3	
1	c	1/4
1	d	2/4
2	c	1/4

A_3	A_4	
c	3	2/4
d	4	2/4
d	5	0

A_4	A_1	
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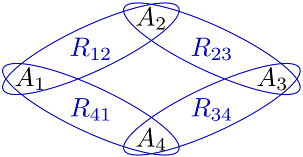
$H_{\mathsf{unif}}(A_1 A_2) \le \log |R_{12}|, \quad H_{\mathsf{unif}}(A_2 A_3) \le \log |R_{23}|, \quad H_{\mathsf{unif}}(A_3 A_4) \le \log |R_{34}|, \quad \ldots$

$H_{\mathsf{unif}}(A_2 | A_1 = \text{'a'}) \le \log \left|\sigma_{A_1=\text{'a'}} R_{12}\right|, \quad H_{\mathsf{unif}}(A_2 | A_1 = \text{'b'}) \le \log \left|\sigma_{A_1=\text{'b'}} R_{12}\right|, \ldots$

$H_{\mathsf{unif}}(A_2 | A_1) \le \log \max_x \left|\sigma_{A_1=x} R_{12}\right|$

An Idea From Gottlob-Lee-Valiant-Valiant, JACM'12

$$Q(A_1, A_2, A_3, A_4) \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}.$$



A_1	A_2	A_3	A_4	
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b	1	c	3	1/4
b	1	d	4	1/4
b	2	c	3	1/4

$H_{\mathsf{unif}}(A_1 A_2 A_3 A_4) = \log |Q|$

A_1	A_2	
a	1	1/4
b	1	2/4
b	2	1/4

A_2	A_3	
1	c	1/4
1	d	2/4
2	c	1/4

A_3	A_4	
c	3	2/4
d	4	2/4
d	5	0

A_4	A_1	
3	b	2/4
4	a	1/4
4	b	1/4

$H_{\mathsf{unif}}(A_1 A_2) \leq \log |R_{12}|, \quad H_{\mathsf{unif}}(A_2 A_3) \leq \log |R_{23}|, \quad H_{\mathsf{unif}}(A_3 A_4) \leq \log |R_{34}|, \quad \ldots$

$H_{\mathsf{unif}}(A_2 | A_1 = \text{'a'}) \leq \log |\sigma_{A_1=\text{'a'}} R_{12}|, \quad H_{\mathsf{unif}}(A_2 | A_1 = \text{'b'}) \leq \log |\sigma_{A_1=\text{'b'}} R_{12}|, \ldots$

$$H_{\mathsf{unif}}(A_2 | A_1) \leq \log \underbrace{\max_x |\sigma_{A_1=x} R_{12}|}_{\deg_{R_{12}}(A_2|A_1)}$$

Entropic Bound for Full Conjunctive Queries

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    - $h(Y|X) \stackrel{\text{def}}{=} h(Y) - h(X) \leq \log N_{Y|X}, \qquad X \subset Y \subseteq F \in \mathcal{E}$
- Good Bound, but not computable!

Hierarchy of Set Functions

$h : 2^{[n]} \to \mathbb{R}_+$, non-negative, monotone, $h(\emptyset) = 0$

$h(X) \leq h(Y)$ if $X \subseteq Y$

$\mathsf{SA}_n := \{h \mid h \text{ is sub-additive}\}$

$$h(X \cup Y) \leq h(X) + h(Y)$$

$\Gamma_n := \{h \mid h \text{ is submodular}\}$

$$h(X \cup Y) + h(X \cap Y) \leq h(X) + h(Y)$$

$\overline{\Gamma}^*_n$: topological closure of Γ^*_n

$\Gamma^*_n = \{h : h \text{ is entropic}\}$

M_n : Modular

$$h(X) = \sum_{x \in x} h(x)$$

Bounds for Full Conjunctive Query

- HDC $\stackrel{\text{def}}{=} \{h \quad | \quad h(Y|X) \leq \log N_{Y|X}, \quad \forall (X, Y, N_{Y|X})\}$

Bounds for Full Conjunctive Query

- $\mathsf{HDC} \stackrel{\text{def}}{=} \{h \quad | \quad h(Y|X) \leq \log N_{Y|X}, \quad \forall (X, Y, N_{Y|X})\}$
- Then,

$$\begin{aligned}
\max_{\mathbf{D} \models \mathsf{DC}} \log |Q(\mathbf{D})| &\leq \max_{h \in \overline{\Gamma}^*_n \cap \mathsf{HDC}} h([n]) && \text{entropic bound} \\
&\leq \max_{h \in \Gamma_n \cap \mathsf{HDC}} h([n]) && \text{polymatroid bound} \\
&\leq \max_{h \in \mathsf{SA}_n \cap \mathsf{HDC}} h([n]) && \text{sub-additive bound.}
\end{aligned}$$

Size Bounds for Full Conjunctive Queries

Bound	Entropic Bound	Polymatroid Bound
Definition	$\log \lvert Q \rvert \leq \max_{h \in \overline{\Gamma}^*_n \cap \mathsf{HDC}} h([n])$	$\log \lvert Q \rvert \leq \max_{h \in \Gamma_n \cap \mathsf{HDC}} h([n])$

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CC only	AGM bound (Tight) [Atserias et al. FOCS'08]	AGM bound (Tight) [Atserias et al. FOCS'08]

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DC	Entropic Bound for DC (Tight [our work])	Polymatroid Bound for DC (Not tight [our work])

Disjunctive Datalog: Size Bounds

$$P: \bigvee_{B\in\mathcal{B}} T_B(\mathbf{A}_B) \leftarrow \bigwedge_{F\in\mathcal{E}} R_F(\mathbf{A}_F) \qquad |P(\mathbf{D})| \stackrel{\text{def}}{=} \min_{\mathbf{T}:\mathbf{T}\models P} \max_{B\in\mathcal{B}} |T_B|$$

Theorem (our work)

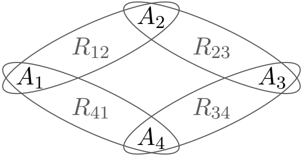
$$\max_{\mathbf{D}\models DC} \log|P(\mathbf{D})| \leq \underbrace{\max_{h\in\overline{\Gamma}^*_n\cap HDC} \min_{B\in\mathcal{B}} h(B)}_{\textit{Entropic bound}} \qquad \textit{Tight}$$

$$\leq \underbrace{\max_{h\in\Gamma_n\cap HDC} \min_{B\in\mathcal{B}} h(B)}_{\textit{Polymatroid bound}} \qquad \textit{Not Tight}$$

Imply all known bounds for (Full) Conjunctive Queries!

Earlier Example

$$P: \quad \bigvee_{B \in \mathcal{B}} T_B(\mathbf{A}_B) \leftarrow \bigwedge_{F \in \mathcal{E}} R_F(\mathbf{A}_F)$$



$$|P(\mathbf{D})| \stackrel{\text{def}}{=} \min_{\mathbf{T}:\mathbf{T} \models P} \max_{B \in \mathcal{B}} |T_B|$$

$$\mathsf{CC}: \quad |R_{12}| \leq N, \quad |R_{23}| \leq N, \quad |R_{34}| \leq N, \quad |R_{41}| \leq N.$$

$$P_{123,234} \quad : \quad T_{123} \vee T_{234} \quad \leftarrow \quad R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}.$$

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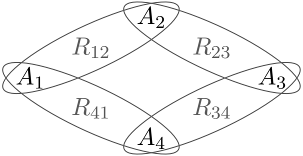

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$$P_{123,234} \quad : \quad T_{123} \vee T_{234} \quad \leftarrow \quad R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}.$$

$$\begin{aligned}
\max_{\mathbf{D} \models \mathsf{CC}} \log |P_{123,234}(\mathbf{D})| &\leq \max_{h \in \Gamma_n \cap \mathsf{CC}} \min\{h(A_1A_2A_3), h(A_2A_3A_4)\} \\
&\leq \max_{h \in \Gamma_n \cap \mathsf{CC}} \frac{1}{2}[h(A_1A_2A_3), h(A_2A_3A_4)] \\
&\leq \max_{h \in \Gamma_n \cap \mathsf{CC}} \frac{1}{2}[h(A_1A_2) + h(A_2A_3) + h(A_3A_4)] \\
&\leq \frac{3}{2} \log N.
\end{aligned}$$

Table of Contents

Roadmap

Given degree constraints DC and a disjunctive datalog rule

$$P : \bigvee_{B \in \mathcal{B}} T_B \leftarrow \bigwedge_{F \in \mathcal{E}} R_F$$

Answer (Worst-case Output Size Bound)

$$\max_{\mathbf{D} \models \mathsf{DC}} \log |P(\mathbf{D})| \leq \max_{h \in \Gamma_n \cap \mathsf{HDC}} \min_{B \in \mathcal{B}} h(B) = \text{polymatroid bound.}$$

Question (Algorithm)

Compute a model for P within $\tilde{O}(2^{\text{polymatroid bound}})$

Question (Gathering fruits)

Plug bound/algorithm into Meta Algorithm, what do we get?

Connection to Shannon-flow Inequalities

Lemma (Linearize it)

There exists non-negative $\boldsymbol{\lambda} = (\lambda_B)_{B \in \mathcal{B}}$*, with* $\|\boldsymbol{\lambda}\|_1 = 1$*, s.t.*

$$\max_{h \in \Gamma_n \cap HDC} \min_{B \in \mathcal{B}} h(B) = \max_{h \in \Gamma_n \cap HDC} \sum_{B \in \mathcal{B}} \lambda_B \, h(B) \tag{1}$$

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Lemma (Shannon-flow inequality)

There exists $\boldsymbol{\delta} \geq \mathbf{0}$ *s.t.* $2^{\textit{polymatroid bound}} = \prod_{(X,Y,N_{Y|X})} N_{Y|X}^{\delta_{Y|X}}$*, and*

$$\sum_{B \in \mathcal{B}} \lambda_B \cdot h(B) \leq \sum_{(X,Y,N_{Y|X})} \delta_{Y|X} \cdot h(Y|X), \quad \forall h \in \Gamma_n \tag{2}$$

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There exists non-negative $\boldsymbol{\lambda} = (\lambda_B)_{B \in \mathcal{B}}$, *with* $\|\boldsymbol{\lambda}\|_1 = 1$, *s.t.*

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(2) is a (vast) generalization of Shearer's lemma

PANDA (**P**roof-**A**ssisted e**N**tropic **D**egree-**A**ware)

- What?
  - Compute a model for our disjunctive datalog rule
  - Run within $\tilde{O}\left(2^{\text{polymatroid bound}}\right) = \tilde{O}\left(\prod_{(X,Y,N_{Y|X})} N_{Y|X}^{\delta_{Y|X}}\right)$:

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- How? Proof as symbolic instructions
  - Construct a Proof Sequence for the corresponding Shannon-flow inequality
  - Proof steps $\rightarrow$ relational operators.

Proof sequence

Shannon-flow inequality: $h(Y|X) \stackrel{\text{def}}{=} h(Y) - h(X)$, $X \subseteq Y$

$$\sum_{B \in \mathcal{B}} \lambda_B \cdot h(B) \leq \sum_{(X,Y,N_{Y|X})} \delta_{Y|X} \cdot h(Y|X)$$

Proof sequence, convert RHS to LHS using following steps

(In)equality	Steps ($X \subseteq Y$)
$h(X) + h(Y\|X) = h(Y)$	$h(X) + h(Y\|X) \to h(Y)$
$h(Y) = h(X) + h(Y\|X)$	$h(Y) \to h(X) + h(Y\|X)$
$h(Y) \geq h(X)$	$h(Y) \to h(X)$
$h(Y\|X) \geq h(Y \cup Z\|X \cup Z)$	$h(Y\|X) \to h(Y \cup Z\|X \cup Z)$

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$h(Y|X) \geq h(Y \cup Z|X \cup Z)$	$h(Y|X) \to h(Y \cup Z|X \cup Z)$

Theorem

There is a proof sequence for every Shannon-flow inequality. The length is data-independent.

Proof Steps As Relational Operators

Shannon-flow inequality:

$$\sum_{B \in \mathcal{B}} \lambda_B \cdot h(B) \leq \sum_{(X,Y,N_{Y|X})} \delta_{Y|X} \cdot h(Y|X)$$

Proof sequence,

Steps ($X \subseteq Y$)	Relational Operator
$h(X) + h(Y\|X) \to h(Y)$	
$h(Y) \to h(X) + h(Y\|X)$	
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Proof sequence,

Steps ($X \subseteq Y$)	Relational Operator
$h(X) + h(Y\|X) \to h(Y)$	(join)
$h(Y) \to h(X) + h(Y\|X)$	
$h(Y) \to h(X)$	
$h(Y\|X) \to h(Y \cup Z\|X \cup Z)$	

Proof Steps As Relational Operators

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Steps ($X \subseteq Y$)	Relational Operator
$h(X) + h(Y\|X) \to h(Y)$	(join)
$h(Y) \to h(X) + h(Y\|X)$	(data partition)
$h(Y) \to h(X)$	
$h(Y\|X) \to h(Y \cup Z\|X \cup Z)$	

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$h(Y) \to h(X)$	(projection)
$h(Y\|X) \to h(Y \cup Z\|X \cup Z)$	

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Theorem

PANDA solves any disjunctive datalog rule P in time

$$\tilde{O}(N + \textbf{poly}(\log N) \cdot 2^{\textit{polymatroid bound for } P})$$

Example: $P: \; T_{123} \vee T_{234} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$.

$|R_{12}|, |R_{23}|, |R_{34}|, |R_{41}| \leq N$

Example: $P: \; T_{123} \vee T_{234} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$.

$$|R_{12}|, |R_{23}|, |R_{34}|, |R_{41}| \leq N \quad \Rightarrow \quad |P| \leq N^{3/2}$$

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$$\begin{aligned}
\log |P| &\leq \min(h(A_1A_2A_3), h(A_2A_3A_4)) && \text{(polymatroid bound)} \\
&\leq \frac{1}{2}\big(h(A_1A_2A_3) + h(A_2A_3A_4)\big) && \text{(linearize)} \\
&\leq \frac{1}{2}\big(h(A_1A_2) + h(A_2A_3) + h(A_3A_4)\big) && \text{(Shannon-flow)} \\
&\leq \frac{3}{2}\log N && \text{(Cardinality constraints)}
\end{aligned}$$

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Proof sequence	Proof Step
$h(A_1A_2) + h(A_2A_3) + h(A_3A_4)$	$\big(h(A_3A_4) \to h(A_4\vert A_3) + h(A_3)\big)$

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\end{aligned}
$$

Proof sequence	Proof Step
$h(A_1A_2) + h(A_2A_3) + h(A_3A_4)$	$\big(h(A_3A_4) \to h(A_4\|A_3) + h(A_3)\big)$
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$$|R_{12}|, |R_{23}|, |R_{34}|, |R_{41}| \leq N \quad \Rightarrow \quad |P| \leq N^{3/2}$$

$$\begin{aligned}
\log|P| &\leq \min(h(A_1A_2A_3), h(A_2A_3A_4)) && \text{(polymatroid bound)} \\
&\leq \frac{1}{2}\big(h(A_1A_2A_3) + h(A_2A_3A_4)\big) && \text{(linearize)} \\
&\leq \frac{1}{2}\big(h(A_1A_2) + h(A_2A_3) + h(A_3A_4)\big) && \text{(Shannon-flow)} \\
&\leq \frac{3}{2}\log N && \text{(Cardinality constraints)}
\end{aligned}$$

Proof sequence	Proof Step
$h(A_1A_2) + h(A_2A_3) + h(A_3A_4)$	$\big(h(A_3A_4) \to h(A_4\|A_3) + h(A_3)\big)$
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$h(A_1A_2) + h(A_2A_3) + h(A_3A_4)$	$\big(h(A_3A_4) \to h(A_4 \mid A_3) + h(A_3)\big)$
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Proof sequence	Proof Step
$h(A_1A_2) + h(A_2A_3) + h(A_3A_4)$	$\big(h(A_3A_4) \to h(A_4 \vert A_3) + h(A_3)\big)$
$h(A_1A_2) + h(A_2A_3) + h(A_4 \vert A_3) + h(A_3)$	$\big(h(A_4 \vert A_3) \to h(A_4 \vert A_2A_3)\big)$
$h(A_1A_2) + h(A_2A_3) + h(A_4 \vert A_2A_3) + h(A_3)$	$\big(h(A_2A_3) + h(A_4 \vert A_2A_3) \to h(A_2A_3A_4)\big)$
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Example: $P: \; T_{123} \vee T_{234} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}.$

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$h(A_1A_2) + h(A_2A_3) + h(A_4 \mid A_3) + h(A_3)$	$\big(h(A_4 \mid A_3) \to h(A_4 \mid A_2A_3)\big)$
$h(A_1A_2) + h(A_2A_3) + h(A_4 \mid A_2A_3) + h(A_3)$	$\big(h(A_2A_3) + h(A_4 \mid A_2A_3) \to h(A_2A_3A_4)\big)$
$h(A_1A_2) + h(A_2A_3A_4) + h(A_3)$	$\big(h(A_1A_2) \to h(A_1A_2 \mid A_3)\big)$
$h(A_1A_2 \mid A_3) + h(A_2A_3A_4) + h(A_3)$	

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Proof sequence	Proof Step
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$h(A_1A_2) + h(A_2A_3A_4) + h(A_3)$	$\big(h(A_1A_2) \to h(A_1A_2 \mid A_3)\big)$
$h(A_1A_2 \mid A_3) + h(A_2A_3A_4) + h(A_3)$	$\big(h(A_1A_2 \mid A_3) + h(A_3) \to h(A_1A_2A_3)\big)$

Example: $P : T_{123} \vee T_{234} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$.

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\end{aligned}$$

Proof sequence	Proof Step
$h(A_1A_2) + h(A_2A_3) + h(A_3A_4)$	$\big(h(A_3A_4) \to h(A_4\|A_3) + h(A_3)\big)$
$h(A_1A_2) + h(A_2A_3) + h(A_4\|A_3) + h(A_3)$	$\big(h(A_4\|A_3) \to h(A_4\|A_2A_3)\big)$
$h(A_1A_2) + h(A_2A_3) + h(A_4\|A_2A_3) + h(A_3)$	$\big(h(A_2A_3) + h(A_4\|A_2A_3) \to h(A_2A_3A_4)\big)$
$h(A_1A_2) + h(A_2A_3A_4) + h(A_3)$	$\big(h(A_1A_2) \to h(A_1A_2\|A_3)\big)$
$h(A_1A_2\|A_3) + h(A_2A_3A_4) + h(A_3)$	$\big(h(A_1A_2\|A_3) + h(A_3) \to h(A_1A_2A_3)\big)$
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$h(A_3A_4) \to h(A_4|A_3) + h(A_3)$

$h(A_4|A_3) \to h(A_4|A_2A_3)$

$h(A_2A_3) + h(A_4|A_2A_3) \to h(A_2A_3A_4)$

$h(A_1A_2) \to h(A_1A_2|A_3)$

$h(A_1A_2|A_3) + h(A_3) \to h(A_1A_2A_3)$

Example: $P: \; T_{123} \vee T_{234} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$.

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$h(A_3 A_4) \rightarrow h(A_4|A_3) + h(A_3)$

$R_{34}(A_3, A_4) \rightarrow R_{34}^{(\ell)}(A_3, A_4), R_3^{(h)}(A_3)$

$h(A_4|A_3) \rightarrow h(A_4|A_2 A_3)$

$h(A_2 A_3) + h(A_4|A_2 A_3) \rightarrow h(A_2 A_3 A_4)$

$h(A_1 A_2) \rightarrow h(A_1 A_2|A_3)$

$h(A_1 A_2|A_3) + h(A_3) \rightarrow h(A_1 A_2 A_3)$

Example: $P: \; T_{123} \vee T_{234} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$.

$$|R_{12}|, |R_{23}|, |R_{34}|, |R_{41}| \leq N \quad \Rightarrow \quad |P| \leq N^{3/2}$$

$h(A_3 A_4) \to h(A_4|A_3) + h(A_3)$

$h(A_4|A_3) \to h(A_4|A_2 A_3)$

$h(A_2 A_3) + h(A_4|A_2 A_3) \to h(A_2 A_3 A_4)$

$h(A_1 A_2) \to h(A_1 A_2|A_3)$

$h(A_1 A_2|A_3) + h(A_3) \to h(A_1 A_2 A_3)$

$R_{34}(A_3, A_4) \to R_{34}^{(\ell)}(A_3, A_4), R_3^{(h)}(A_3)$

$R_{34}^{(\ell)}(A_3, A_4) \to R_{34}^{(\ell)}(A_3, A_4)$

Example: $P : T_{123} \vee T_{234} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$.

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$h(A_1 A_2|A_3) + h(A_3) \rightarrow h(A_1 A_2 A_3)$

$R_{34}(A_3, A_4) \rightarrow R_{34}^{(\ell)}(A_3, A_4), R_3^{(h)}(A_3)$

$R_{34}^{(\ell)}(A_3, A_4) \rightarrow R_{34}^{(\ell)}(A_3, A_4)$

$R_{23}(A_2, A_3) \bowtie R_{34}^{(\ell)}(A_3, A_4) \rightarrow T_{234}(A_2, A_3, A_4)$

Example: $P : \; T_{123} \vee T_{234} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$.

$$|R_{12}|, |R_{23}|, |R_{34}|, |R_{41}| \leq N \quad \Rightarrow \quad |P| \leq N^{3/2}$$

$h(A_3 A_4) \to h(A_4|A_3) + h(A_3)$

$h(A_4|A_3) \to h(A_4|A_2 A_3)$

$h(A_2 A_3) + h(A_4|A_2 A_3) \to h(A_2 A_3 A_4)$

$h(A_1 A_2) \to h(A_1 A_2|A_3)$

$h(A_1 A_2|A_3) + h(A_3) \to h(A_1 A_2 A_3)$

$R_{34}(A_3, A_4) \to R_{34}^{(\ell)}(A_3, A_4), R_3^{(h)}(A_3)$

$R_{34}^{(\ell)}(A_3, A_4) \to R_{34}^{(\ell)}(A_3, A_4)$

$R_{23}(A_2, A_3) \bowtie R_{34}^{(\ell)}(A_3, A_4) \to T_{234}(A_2, A_3, A_4)$

$R_{12}(A_1, A_2) \to R_{12}(A_1, A_2)$

Example: $P: \; T_{123} \vee T_{234} \leftarrow R_{12} \wedge R_{23} \wedge R_{34} \wedge R_{41}$.

$$|R_{12}|, |R_{23}|, |R_{34}|, |R_{41}| \leq N \quad \Rightarrow \quad |P| \leq N^{3/2}$$

$h(A_3A_4) \rightarrow h(A_4\mid A_3) + h(A_3)$	$R_{34}(A_3, A_4) \rightarrow R^{(\ell)}_{34}(A_3, A_4), R^{(h)}_3(A_3)$
$h(A_4\mid A_3) \rightarrow h(A_4\mid A_2A_3)$	$R^{(\ell)}_{34}(A_3, A_4) \rightarrow R^{(\ell)}_{34}(A_3, A_4)$
$h(A_2A_3) + h(A_4\mid A_2A_3) \rightarrow h(A_2A_3A_4)$	$R_{23}(A_2, A_3) \bowtie R^{(\ell)}_{34}(A_3, A_4) \rightarrow T_{234}(A_2, A_3, A_4)$
$h(A_1A_2) \rightarrow h(A_1A_2\mid A_3)$	$R_{12}(A_1, A_2) \rightarrow R_{12}(A_1, A_2)$
$h(A_1A_2\mid A_3) + h(A_3) \rightarrow h(A_1A_2A_3)$	$R_{12}(A_1, A_2) \bowtie R^{(h)}_3(A_3) \rightarrow T_{123}(A_1, A_2, A_3)$

Table of Contents

Roadmap

Given degree constraints DC and a disjunctive datalog rule

$$P : \bigvee_{B \in \mathcal{B}} T_B \leftarrow \bigwedge_{F \in \mathcal{E}} R_F$$

Answer (Worst-case Output Size Bound)

Polymatroid bound $\max_{\mathbf{D} \models \mathsf{DC}} \log |P(\mathbf{D})| \leq \max_{h \in \Gamma_n \cap \mathsf{HDC}} \min_{B \in \mathcal{B}} h(B)$

Answer (Algorithm)

PANDA computes a model for P within $\tilde{O}(2^{\text{polymatroid bound}})$

Question (Gathering fruits)

Plug bound/algorithm into Meta Algorithm, what do we get?

Option 1: Full Conjunctive Query

Full conjunctive query

$$\text{Database } \mathbf{D} \longrightarrow Q_i \longrightarrow Q_i(\mathbf{D}) \longrightarrow Q_o \longrightarrow \text{Output}$$

$$\bigwedge_{F \in \mathcal{E}} R_F \xrightarrow{\quad \textsf{iTime} \quad} T_{[n]} \xrightarrow{\quad \textsf{oTime} \quad} \textsf{Answer}$$

$$\textsf{iTime} = \max_{\mathbf{D} \models \textsf{DC}} |Q_i(\mathbf{D})| \leq \max_{h \in \Gamma_n \cap \textsf{HDC}} 2^{h([n])} = \textsf{PANDA's} \text{ runtime}$$

PANDA is worst-case optimal whenever the polymatroid bound is tight!

Option 2: A Single Tree Decomposition

Let $(\mathcal{T}, \chi)$ be a tree decomposition of $\mathcal{H}$ to be chosen later

Multiple Conjunctive Rules

$$\text{Database } \mathbf{D} \longrightarrow Q_i \longrightarrow Q_i(\mathbf{D}) \longrightarrow Q_o \longrightarrow \text{Output}$$

$$\bigwedge_{F \in \mathcal{E}} R_F \xrightarrow{\quad \mathsf{iTime} \quad} \bigwedge_{v \in V(\mathcal{T})} T_{\chi(v)} \xrightarrow{\quad \mathsf{oTime} \quad} \text{Answer}$$

$$\mathsf{iTime} = \max_{\mathbf{D} \models \mathsf{DC}} |Q_i(\mathbf{D})| \leq \max_{\mathbf{D} \models \mathsf{DC}} \max_{v \in V(\mathcal{T})} |P_v(\mathbf{D})|$$

$$\leq 2^{\max_{v \in V(\mathcal{T})} \max_{h \in \Gamma_n \cap \mathsf{DC}} h(\chi(v))} = \text{PANDA's runtime}$$

Pick the best $(\mathcal{T}, \chi)$ before running PANDA:

$$\min_{(\mathcal{T}, \chi)} \max_{v \in V(\mathcal{T})} \max_{h \in \Gamma_n \cap \mathsf{CC}} h(\chi(v)) \leq \log N \cdot \mathsf{fhtw}(Q)$$

PANDA evaluates Q within $\tilde{O}(N^{\mathsf{fhtw}(Q)})$-time.

Option 3: Multiple Tree Decompositions

Multiple Disjunctive Datalog Rules!

$$\text{Database } \mathbf{D} \longrightarrow Q_i \longrightarrow Q_i(\mathbf{D}) \longrightarrow Q_o \longrightarrow \text{Output}$$

$$\bigwedge_{F\in\mathcal{E}} R_F \xrightarrow{\quad \mathsf{iTime} \quad} \bigwedge_{\mathcal{B}} \bigvee_{B\in\mathcal{B}} T_B \xrightarrow{\quad \mathsf{oTime} \quad} \mathsf{Answer}$$

$$\log \mathsf{iTime} = \max_{\mathbf{D}\models\mathsf{DC}} \log |Q_i(\mathbf{D})| \leq \max_{\mathbf{D}\models\mathsf{DC}} \max_{\mathcal{B}} \log |P_{\mathcal{B}}(\mathbf{D})|$$

$$\leq \max_{\mathcal{B}} \max_{h\in\Gamma_n\cap\mathsf{HDC}} \min_{B\in\mathcal{B}} h(B) = \max_{h\in\Gamma_n\cap\mathsf{HDC}} \max_{\mathcal{B}} \min_{B\in\mathcal{B}} h(B)$$

$$= \max_{h\in\Gamma_n\cap\mathsf{HDC}} \max_{(\mathcal{T},\chi)} \min_{v\in V(\mathcal{T})} h(\chi(v)) = \log(\text{PANDA's runtime})$$

$$\max_{h\in\Gamma_n\cap\mathsf{HDC}} \max_{(\mathcal{T},\chi)} \min_{v\in V(\mathcal{T})} h(\chi(v)) \leq \log N \cdot \mathsf{subw}(Q)$$

PANDA evaluates Q within $\tilde{O}(N^{\mathsf{subw}(Q)})$-time.

Quantities of Interests

- $X \in \{\overline{\Gamma}^*_n, \Gamma_n, \mathsf{SA}_n\}$
- $Y \in \{\mathsf{HDC}, \mathsf{HFD}, \mathsf{HCC}, \log N \cdot \mathsf{ED}, \log N \cdot \mathsf{VD}\}$

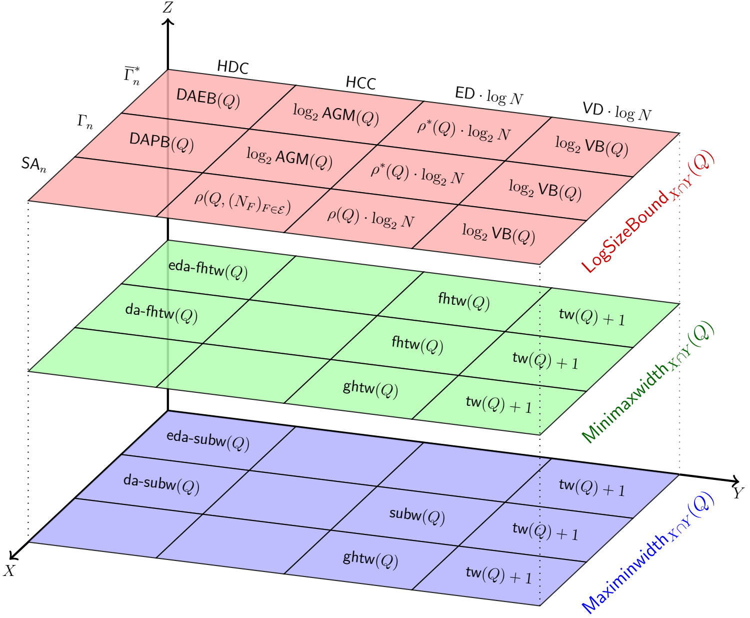
Define

$$\mathsf{LogSizeBound}_{X \cap Y}(P) \stackrel{\text{def}}{=} \max_{h \in X \cap Y} \min_{B \in \mathcal{B}} h(B)$$

$$\mathsf{Minimaxwidth}_{X \cap Y}(Q) \stackrel{\text{def}}{=} \min_{(\mathcal{T}, \chi)} \max_{v \in V(\mathcal{T})} \max_{h \in X \cap Y} h(\chi(v)),$$

$$\mathsf{Maximinwidth}_{X \cap Y}(Q) \stackrel{\text{def}}{=} \max_{h \in X \cap Y} \min_{(\mathcal{T}, \chi)} \max_{v \in V(\mathcal{T})} h(\chi(v)).$$

Summary of Bounds



Many Open Questions

- Is the entropic bound computable under $\mathsf{CC} \cup \mathsf{HDC}$ or DC?

Many Open Questions

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Many Thanks!
Questions?

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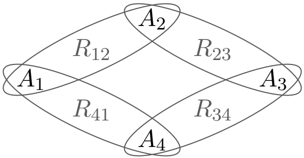

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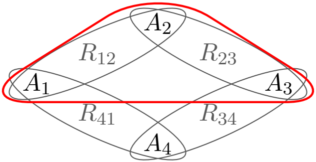

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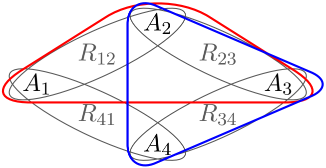

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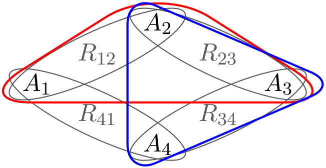

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A_1	A_2
a	1
b	1
b	2

A_2	A_3
1	c
1	d
2	c

A_3	A_4
c	3
d	4
d	5

A_4	A_1
3	b
4	a
4	b

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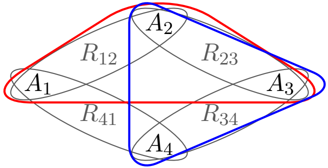

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A_4	A_1
3	b
4	a
4	b

A_1	A_2	A_3	A_4
a	1	d	4
b	1	c	3
b	1	d	4
b	2	c	3

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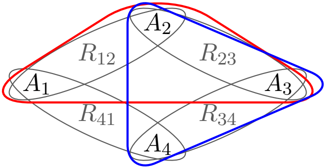

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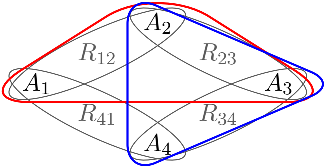

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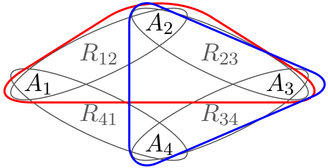

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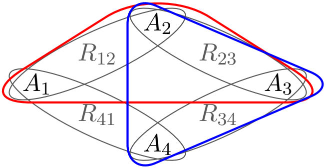
Inclusive Disjunction

Semantics of Our Disjunctive Datalog Rule



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No *minimal model* requirement

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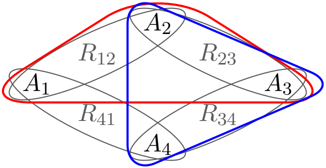

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A_1	A_2	A_3
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A_2	A_3	A_4
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Model size is $\max(|T_{123}|, |T_{234}|) = 3$

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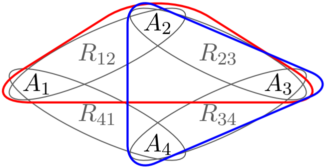

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Output size is the minimum over all models

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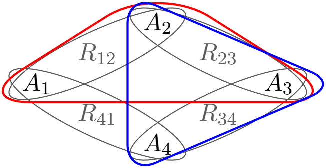

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A minimum-sized model of size 2

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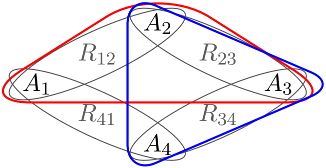

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A_1	A_2	A_3
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b	2	c

A_2	A_3	A_4
1	d	4

A minimum-sized model of size 2
Hence, the output size is 2

Output Size of a Disjunctive Datalog Rule

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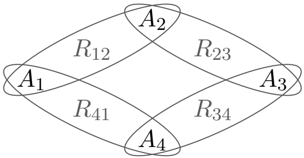

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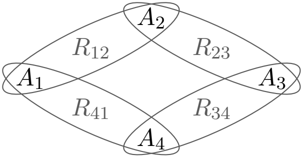

CC : $\quad |R_{12}| \leq N, \quad |R_{23}| \leq N, \quad |R_{34}| \leq N, \quad |R_{41}| \leq N.$

Output Size of a Disjunctive Datalog Rule

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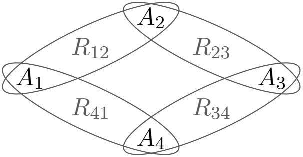

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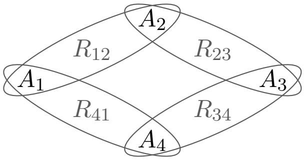

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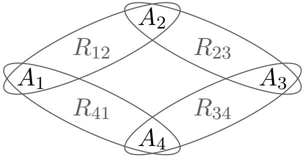

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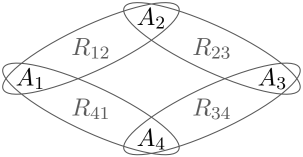

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$|P'(\mathbf{D})| = N^2$, for some $\mathbf{D}$

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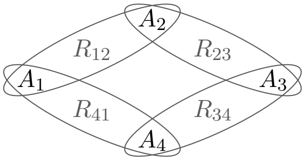

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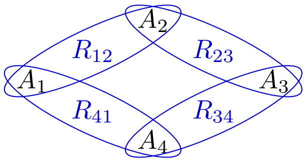
$|P'(\mathbf{D})| = N^2$, for some $\mathbf{D}$

Using Option 3, 4-cycle query answerable in $\tilde{O}(N^{3/2})$-time, matching [Alon, Yuster, Zwick'97]

Polymatroid Bound: Examples

$$Q(A_1, A_2, A_3, A_4) \leftarrow R_{12}(A_1, A_2), R_{23}(A_2, A_3),$$
$$R_{34}(A_3, A_4), R_{41}(A_4, A_1).$$



- $|R_{12}|, |R_{23}|, |R_{34}|, |R_{41}| \leq N$

$|Q| \leq N^2$

Polymatroid Bound: Examples

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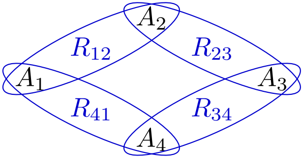

- $|R_{12}|, |R_{23}|, |R_{34}|, |R_{41}| \leq N$

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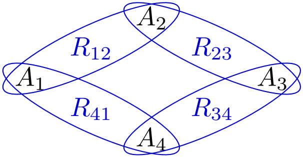

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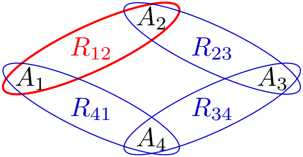

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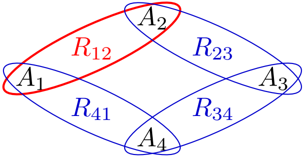

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- Construct a polymatroid h satisfying Q's constraints such that

$$11h(ABXYC) = 44 \log N$$

Beyond Worst-case Optimality

- Output-sensitive algorithms

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