Geometric Tomography: A Limited-View Approach for Computed Tomography[*]

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Categories and Subject Descriptors

I.4.5 [**Reconstruction**]: Summation methods

General Terms

Algorithms, Design

Keywords

Computed Tomography; Geometric Compressed Sensing; Topological Peeling.

ABSTRACT

Computed tomography(CT), especially since the introduction of helical CT, provides excellent visualization of the internal organs of the body. As a result, CT is used routinely in the clinical arena to obtain three- and four-dimensional data. Data is obtained by exposing patients to a beam of x-rays from a number (about 1000) of different angles (projections). Then, standard CT makes use of the Radon transform to generate 3D data, denoted $\vec{f}$, directly from projections, denoted $\vec{g}$. Thus, the projection relationship can be represented in matrix form by $\vec{g} = M\vec{f}$ where M represents the projection matrix. Note that techniques based on the Radon transform are in general limited by the Nyquist sampling criteria.

The increasing use of CT has resulted in a substantial rise in population-radiation-dose [1], which may lead to an increased incidence of cancer in the population. In addition to increased use, the number of projections in the CT acquisitions is increasing to improve image quality which further increases patient radiation exposure as well as the reconstruction time. This latter issue can be improved by using new technology, e.g., graphical processing units (GPUs) [2], but the problem of radiation dose does remains. Reduction of the number of projections can result in artifacts and reduced image quality. Thus, new approaches are being pursued.

Our novel algorithm is based on a key observation: Standard CT reconstruction techniques(such as filtered backprojection [3] or an algebraic reconstruction technique [4]) converges quickly when the intensity of all voxels are similar. This is because it "evenly" distributes the intensity of each pixel in a projection to all voxels along the corresponding projection ray. When all voxels have similar intensity, the value received by each voxel from one projection ray will be close to its actual intensity. As a results, a few projections will lead to a high quality reconstruction. Therefore, we have taken the approach to mathematically split up $\vec{g} = M\vec{f}$ into subproblems or Objects-of-Interest (OoI), effectively we are rewriting the general CT problem into:

$$
\begin{aligned}
\vec{g_{OoI_1}} &= M_{OoI_1}\vec{f_{OoI_1}} \\
\vec{g_{OoI_2}} &= M_{OoI_2}\vec{f_{OoI_2}} \\
&\cdots \\
g_{C-\vec{OoI}} &= M_{C-OoI}\vec{f_{C-OoI}}
\end{aligned}
\tag{1}
$$

Because of this unique characteristic, our approach can be understood as a geometric compressed-sensing approach [5]. By using this unique formulation of splitting up the problem into OoIs (region of similar voxel intensity), each subproblem includes a relatively small range of voxel values. By dividing the volume into subproblems, we establish an object-specific reconstruction.

Our approach is a two stage technique. In the first stage, we automatically isolate OoIs from a limited-view reconstruction, which are single or multiple anatomical structures. If the OoIs are detected, we proceed to stage two. In the second stage of processing, we use the extracted OoI volumes to drive a reconstruction process that independently reconstructs each OoI and its complement volume. To be able to independently reconstruct, we classify the projection rays that penetrate through the voxels within the C-OoI and those rays which penetrate through the OoIs. We split the problem into several 2D planes. In the 2D plane, we have a set of objects obtained from the rough segmentation, surrounded by lower density/contrast regions called complements. To separate the reconstruction of the objects from that of the complements, we first proximate the rough boundary of the objects by simple polygons, and then treat the polygons as obstacles. The complements are reconstructed by rays, which traverse only the complements, and the objects are reconstructed by rays penetrating the corresponding polygons. To efficiently classify all the rays, we use a point-line duality transformation on the set of bound-

ary segments of the polygons. Each vertex of the polygons transforms to a dual line, and each supporting line maps to a point in the dual space. The set of dual lines forms an arrangement which partitions the space into a set of convex cells. Each cell contains the set of rays intersecting the same subset of boundary segments of the polygons, that is, all rays traveling through the hourglass defined by the set of segments. All types of rays can be identified by sweeping the arrangement in an online fashion. To report all the cells and rays in a more efficient way, we use the Topological Peeling [6] [7]algorithm due to its optimal time and space complexities.

Finally the sparse sets of rays for each OoI is passed to a GPU-based reconstruction engine to generate the final 3D.

1. THE VIDEO SEQUENCE

The video sequence begins with an animated explanation of the CT imaging setup (see Figure 1).
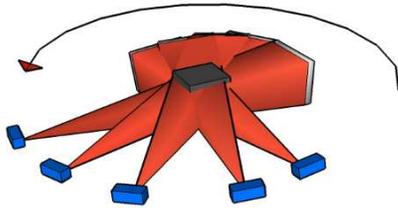
Figure 1: Illustration of the CT imaging setup. The image system acquires projection images from up to a 1000 different positions.

Next, we explain the inputs to our algorithm and how they are acquired, and we layout the main objectives which our algorithm should achieve. We then illustrate our main observations about the problem and how these observations allow us to formulate a geometric compressed sensing approach (see Figure 2).
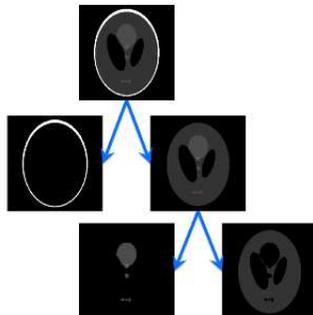

Figure 2: To make use of geometric compressed sensing, each voxel intensity region is handled as a sparse set of values.

Before reconstruction, we explain how to classify those projection rays which penetrate only through the voxels,

that lie within the C-OoIs, and which penetrate through the OoI. We show how a Topological Peeling is employed to traverse a 2D arrangement (see Figure 3). We next, show how 3D data are reconstructed using the classified rays (see Figure 4).
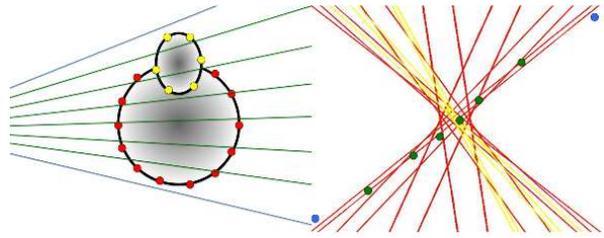
Figure 3: We illustrate the primal space on the left side and the dual space on the right side.
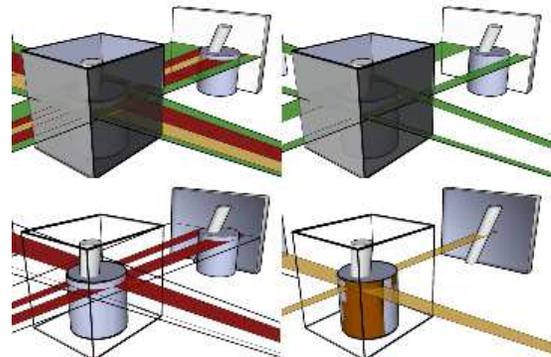
Figure 4: The reconstruction process after we classified the different rays. Each region is reconstructed from the outside in.

Finally, we show reconstruction results from standard techniques and from our approach by using rabbit data sets.

The video was produced by using Apple iMovie and Google SketchUp on an Apple iMac with Mac OS X 10.5.

2. REFERENCES

[1] Brenner, D.J., Hall, E.J.: *Computed Tomography – An Increasing Source of Radiation Exposure*, N Engl J Med 357(22) (2007) 2277-2284.

[2] Noël, P.B., Walczak, A.M., Hoffmann, K.R., Xu, J., Corso, J.J., Schafer, S.:*Clinical Evaluation of GPU-Based Cone Beam Computed Tomography*, MICCAI Workshop: High-Performance Medical Image Computing and Computer Aided Intervention (2008).

[3] Feldkamp, L.A., Davis, L.C., Kress, J.W.:*Practical cone-beam algorithm.*, J Opt Soc Am A 1(6) (1984) 612.

[4] Gordon, R., Bender, R., Herman, G.:*Algebraic reconstuction techniques (ART) for three-dimensional microsocpy and x-ray photography.*, J Theor Biol 29 (1970) 471-481.

[5] Candes, E., Romberg, J., Tao, T.:*Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information.*, Information Theory, IEEE Transaction on 52(2) (2006) 489-509.

[6] Chen, D.Z., Daescu, O., Hu, X., Wu, X., Xu, J.:*Determining an optimal penetration amog weighted regions in two and tree dimensions.*, Proceedings of the fifteenth annual symposium on computational geometry, New York, NY, USA, ASM (1999) 322-331.

[7] Chen, D.Z., Luan, Sl, Xu, J.:*Topoloical peeling and applications.*, International Journal of Computational Geometry and Applications 13(1) (2003) 135-172.