Decision Trees
An Early Classifier

Jason Corso

SUNY at Buffalo

Introduction to Non-Metric Methods

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  - For example (DHS), some teeth are small and fine (as in baleen whales) for straining tiny prey from the sea; others (as in sharks) come in multiple rows; other sea creatures have tusks (as in walruses), yet others lack teeth altogether (as in squid). There is no clear notion of similarity for this information about teeth.

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- Most of the other methods we study will involve real-valued feature vectors with clear metrics.
- We may also consider problems involving data tuples and data strings. And for recognition of these, decision trees and string grammars, respectively.

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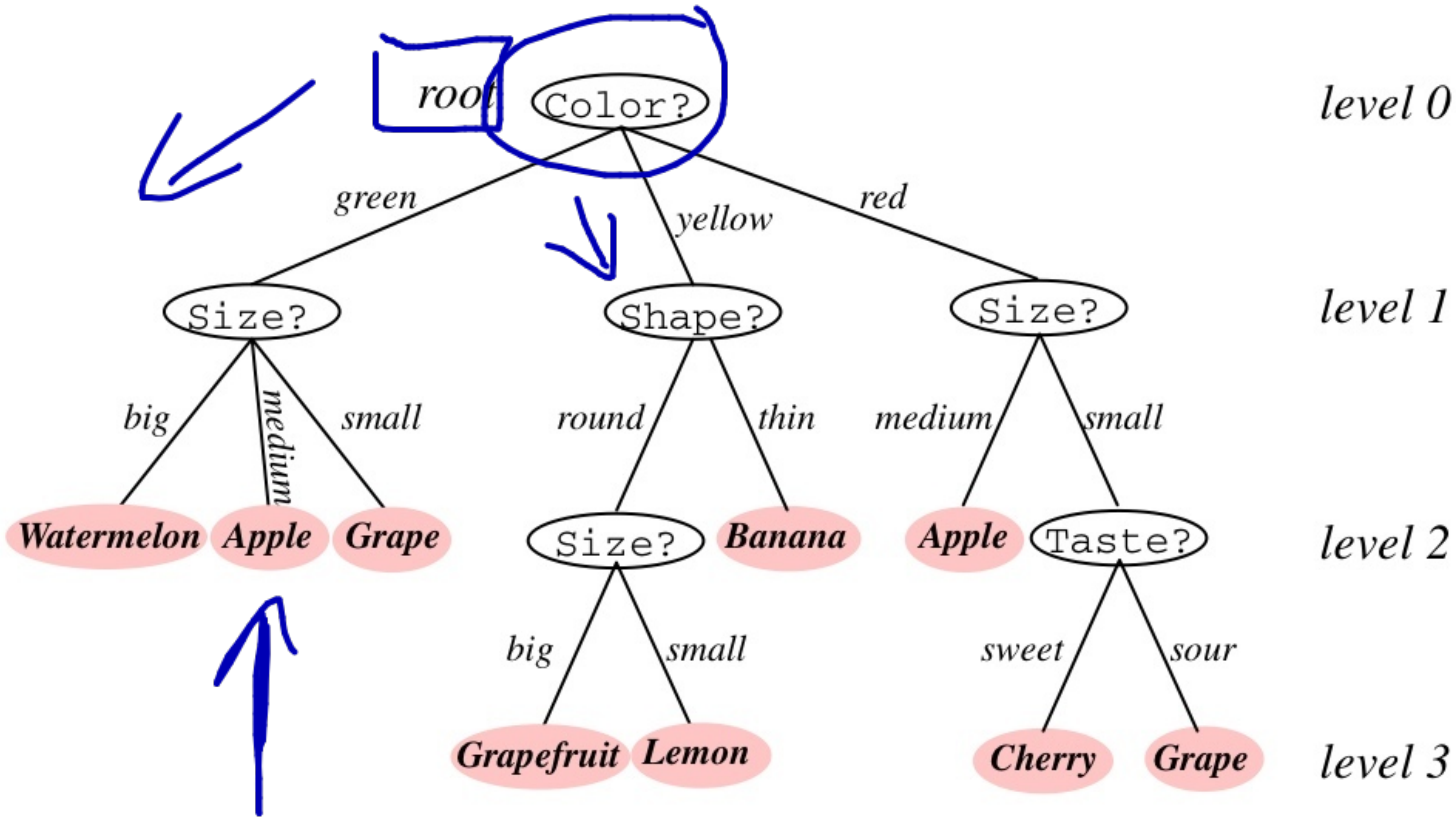
- What underlying measure led you the questions, if any?
- Most importantly, iterative yes/no questions of this sort require no metric and are well suited for nominal data.

These sequence of questions are a decision tree...



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- We follow the link corresponding to the appropriate value of the pattern and continue to a new node, at which we check the next property. And so on.
- Decision trees have a particularly high degree of interpretability.

When to Consider Decision Trees

- Instances are wholly or partly described by attribute-value pairs.
- Target function is discrete valued.
- ~~Disjunctive hy~~pothesis may be required.
- Possibly noisy training data.
- Examples
  - Equipment or medical diagnosis.
  - Credit risk analysis.
  - Modeling calendar scheduling preferences.

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$\mathcal{D} = \{x, y\}$ Breiman

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- Any decision tree will progressively split the data into subsets.
- If at any point all of the elements of a particular subset are of the same category, then we say this node is **pure** and we can stop splitting.
- Unfortunately, this rarely happens and we have to decide between whether to stop splitting and accept an imperfect decision or instead to select another property and grow the tree further.

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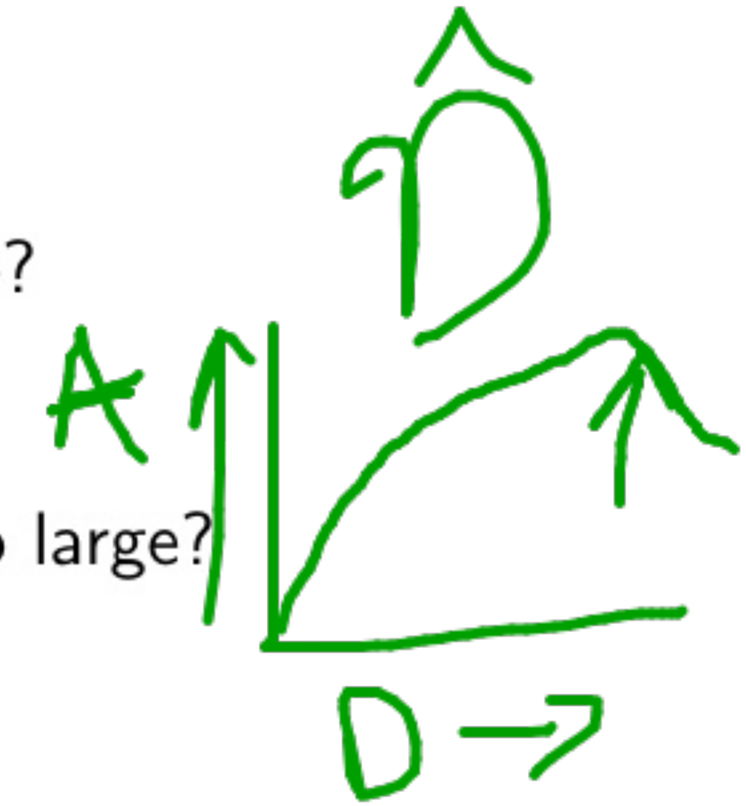

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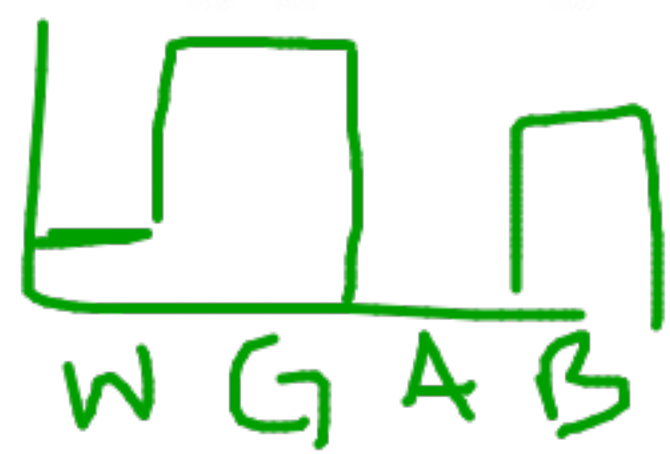

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  6. How should missing data be handled?

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- So, DHS focuses on only binary tree learning.
- But, we note that in certain circumstances for learning and inference, the selection of a test at a node or its inference may be computationally expensive and a 3- or 4-way split may be more desirable for computational reasons.

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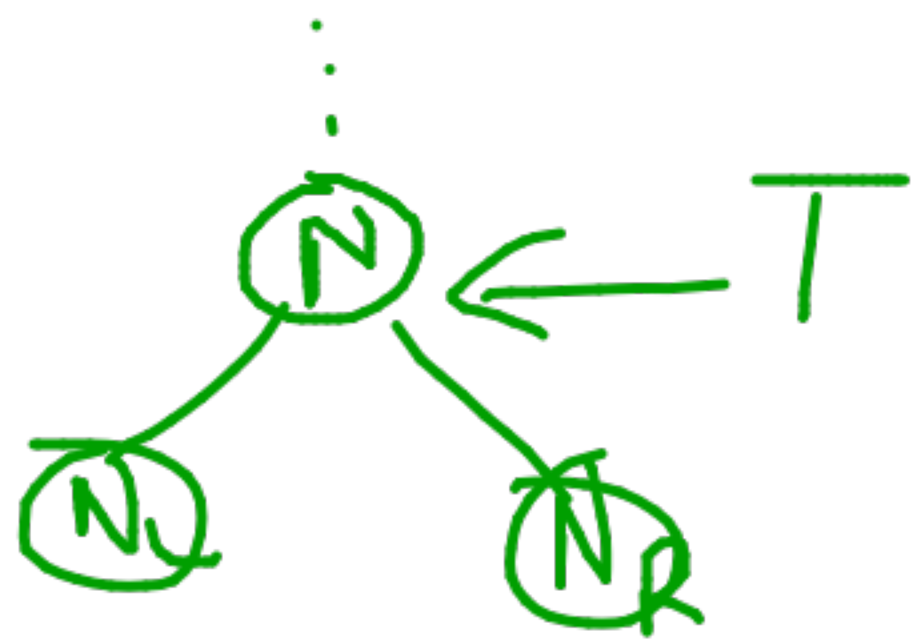

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- **Entropy impurity** is the most popular measure:

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$$i(N) = -\sum_j P(\omega_j) \log P(\omega_j) \quad . \tag{1}$$

It will be minimized for a node that has elements of only one class (pure).

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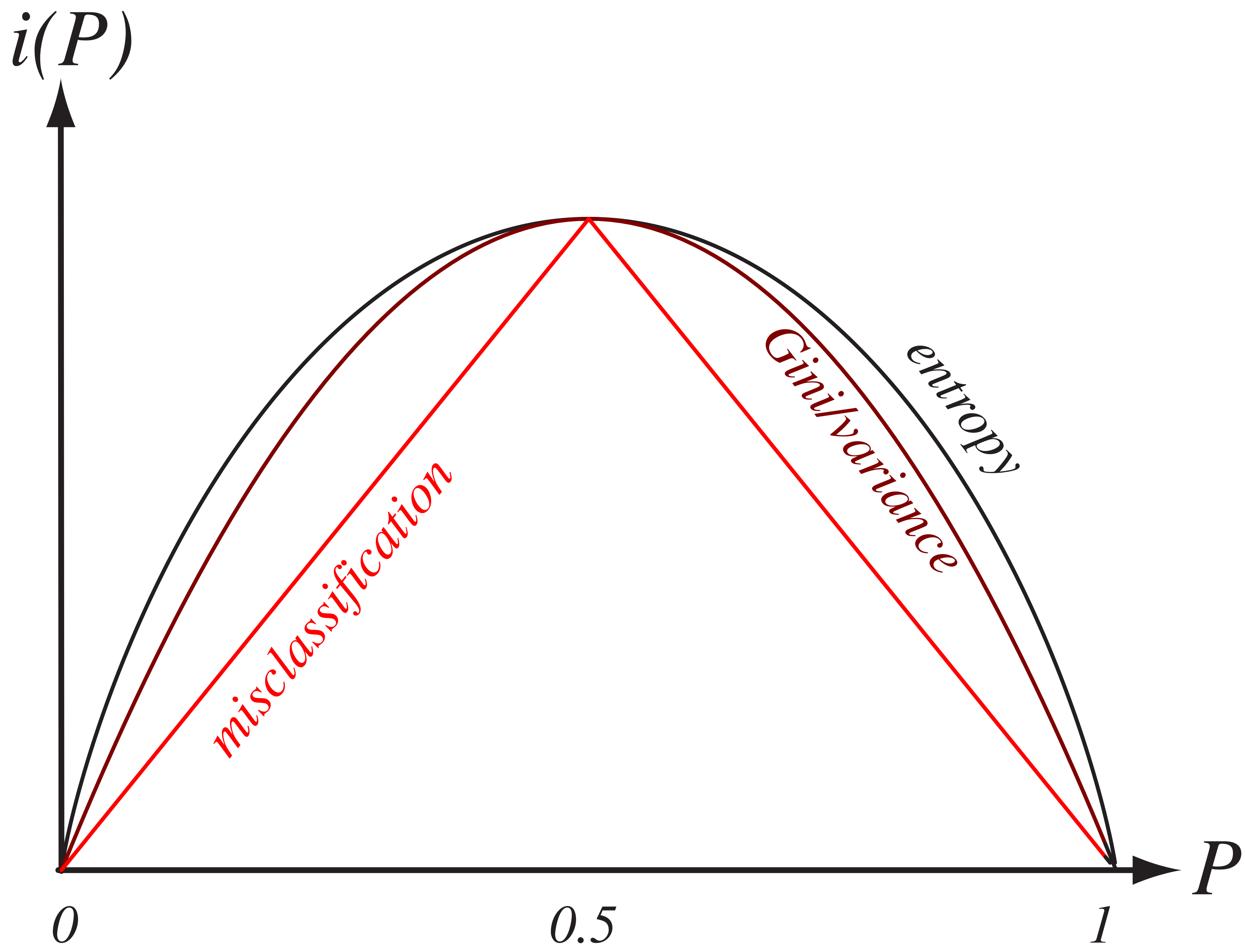
- The **misclassification impurity** measures the minimum probability that a training pattern would be misclassified at N:

$$i(N) = 1 - \max_j P(\omega_j) \tag{4}$$



For the two-category case, the impurity functions peak at equal class frequencies.

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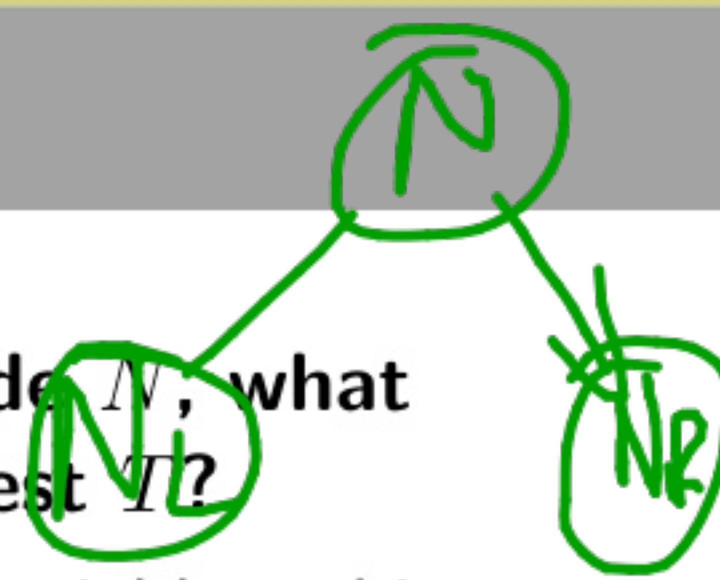

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COVER & Thomas

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- If the **entropy impurity** is used, this corresponds to choosing the feature that yields the highest **information gain**.

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- This is a local, greedy optimization strategy.
- Hence, there is no guarantee that we have either the global optimum (in classification accuracy) or the smallest tree.