CSE 455/555 Spring 2013 Quiz 7 of 14

Jason J. Corso, jcorso@buffalo.edu, SUNY Buffalo
Solutions by Yingbo Zhou

Name:

ID#:

Section: 455 or 555

2	8	10

Directions – *The quiz is closed book/notes. You have 10 minutes to complete it; use this paper only.*

Problem 1: Recall (2pts) (Answer in one sentence only.)

What is the maximum likelihood estimate of parameter θ given some data $\mathcal{D}$?
Solution:
The maximum likelihood estimate of the parameter θ for some data $\mathcal{D}$ is the parameter $\hat{\theta}$ that maximize the likelihood.

Problem 2: Work (8 pts) (Show all derivations/work and explain.)
Suppose that we have collected some data $\mathbf{X} = \{x_1, x_2, \ldots, x_N\}$. Assume that the data we collected follow the Poisson distribution:

$$p(x;\lambda) = \frac{\lambda^x \exp\{-\lambda\}}{x!}$$

Write the likelihood function and estimate the parameter λ_{ML} using maximum likelihood parameter estimation.
Hint: *log-likelihood function might be easier for derivation.*
Solution:

$$\mathcal{L}(\lambda;\mathbf{X}) = \prod_{n=1}^{N} p(x_n;\lambda) = \prod_{n=1}^{N} \frac{\lambda^{x_n} \exp\{-\lambda\}}{x_n!}$$

The log-likelihood:

$$l(\lambda;\mathbf{X}) = \sum_{n=1}^{N} \{x_n \log\lambda - \lambda - \log x_n!\}$$

Take the partial derivative w.r.t. λ and set to zero:

$$\frac{\partial l(\lambda;\mathbf{X})}{\partial\lambda} = 0$$

$$\frac{\sum_{n=1}^{N} x_n}{\lambda} - N = 0$$

$$\lambda_{ML} = \frac{\sum_{n=1}^{N} x_n}{N}$$