CSE 455/555 Spring 2013 Quiz 13 of 14

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Name:

ID#:

Section: 455 or 555

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Directions – *The quiz is closed book/notes. You have 10 minutes to complete it; use this paper only.*

Problem 1: Recall (2pts) (Answer in one sentence only.)

What is the Markov chain property?

Solution:
The Markov chain property states that the probability of each subsequent state depends only on the previous state, *i.e.* $P(S_{it}|S_{i1}, S_{i2}, \ldots, S_{it-1}) = p(S_{it}|S_{it-1})$

Problem 2: Work (8 pts) (Show all derivations/work and explain.)
Consider a primitive clinic in a village. People in the village have a very nice property that they are either healthy or have a fever. They can only tell if they have a fever by asking a doctor in the clinic. The wise doctor makes a diagnosis of fever by asking patients how they feel. Villagers only answer that they feel normal or cold.

Suppose the initial probability of a patient in fever state is 0.4. The transition probability from healthy to healthy state is 0.7, and from fever to fever state is 0.6. In addition, the probability of a patient in healthy state feels normal is 0.6, and the probability of a patient in fever state feels normal is 0.1.

Now consider a patient comes to the clinic for two days in a row and the doctor discovers that on the first day he feels normal, on the second day he feels cold. What is the probability of observing this sequence?

Solution:

Let H and F denote the healthy and fever state respectively, apply the forward recursion we get:

$$\begin{aligned}
\alpha_1(H) &= \pi_H b_H(\text{Normal}) = 0.6 \times 0.6 = 0.36 \\
\alpha_1(F) &= \pi_F b_F(\text{Normal}) = 0.4 \times 0.1 = 0.04 \\
\alpha_2(H) &= \left\{ \sum_{i=1}^{N} \alpha_1(i) a_{iH} \right\} b_H(\text{Cold}) = \{\alpha_1(H) \times a_{HH} + \alpha_1(F) \times a_{FH}\} \times b_H(\text{Cold}) \\
&= \{0.36 \times 0.7 + 0.04 \times 0.4\} \times 0.4 = 0.1072 \\
\alpha_2(F) &= \left\{ \sum_{i=1}^{N} \alpha_1(i) a_{iF} \right\} b_F(\text{Cold}) = \{\alpha_1(H) \times a_{HF} + \alpha_1(F) \times a_{FF}\} \times b_F(\text{Cold}) \\
&= \{0.36 \times 0.3 + 0.04 \times 0.6\} \times 0.9 = 0.1188
\end{aligned}$$

$$p(\text{Normal, Colde}) = \alpha_2(H) + \alpha_2(F) = 0.1072 + 0.1188 = 0.226$$