CSE 668 Spring 2023 Midterm Exam

Question 1

a) The model consists of the intersection of four half-spaces defined by their bounding planes. A plane is defined by the equation ax+by+cz=d. Three of the planes are x=0, y=0 and z=0. The fourth plane, the slanted one, contains the three points (1,0,0), (0,1,0), (0,0,1). Plugging in:

a(1)+b(0)+c(0)=d or d=a
a(0)+b(1)+c(1)=d or d=b

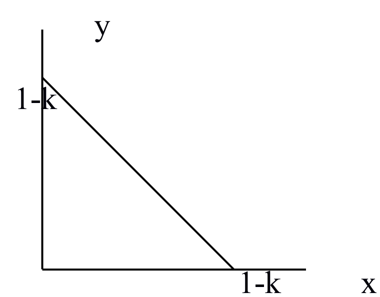
Combining these two equations, a=b=d. The third point gives us, after plugging in b=d=a,

a(0)+a(0)+c(1)=a so c=a. So the equation for the fourth plane is a(x+y+z)=a or x+y+z=1. So the four half-spaces whose intersection is the model are

1. { (x,y,z): $x\geq 0$ }
2. { (x,y,z): $y\geq 0$ }
3. { (x,y,z): $z\geq 0$ }
4. { (x,y,z): $x+y+z\leq 1$ }

(b) A sweep representation is defined by a spine and a crossection.

Spine: we can select any line or curve contained in the tetrahedron. Lets pick the line segment k(0,0,1) for $0\leq k\leq 1$. Then for any k, the crossection at the level z=k along the z axis is a 45 degree right triangle (see next page).



The hypotenuse formula is y= -x+(1-k). So the sweep model is defined as

Spine: k(0,0,1) for $0\leq k\leq 1$;
Crossection at point k along spine: { (x,y,z): $x\geq 0$, $y\geq 0$, $y\leq -x+(1-k)$ }

Question 2

(a) Explain how computation of an epipolar line facilitates stereopsis.

It reduces the correspondence search from 2-D to 1-D.

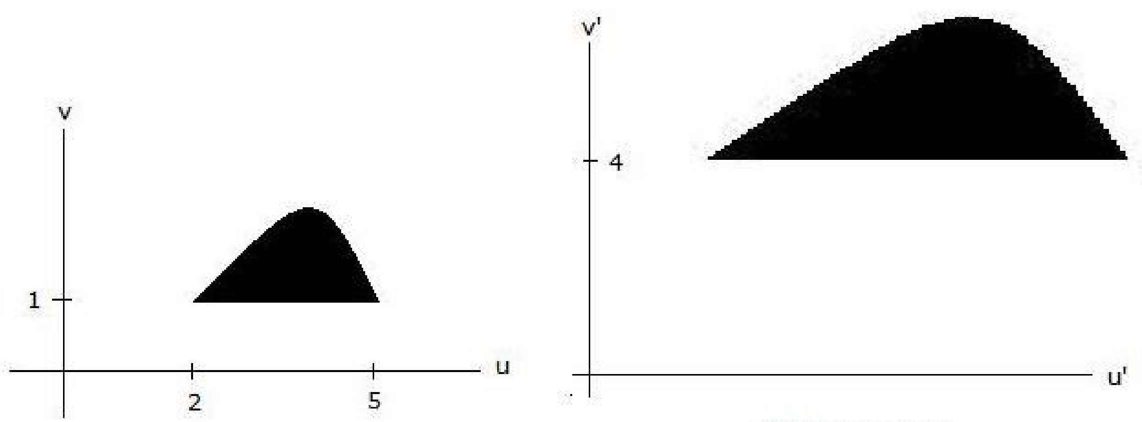
(b) The Longuet-Higgins Equation for a given camera model is $u^T F u' = 0$ where the fundamental matrix F is

$$F = \begin{bmatrix} 0 & 0 & 6 \\ 0 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

The left-eye image, corresponding to the unprimed vector u in the Longuet-Higgins Equation above, is shown below. The visible black object has a straight edge in the image at v=1 which is of length 3. The same straight edge is visible in the right-eye image at v'=4. What is its length?

$$\begin{bmatrix} u & v & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 6 \\ 0 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = 6\text{u} - v - u' + v' = 0$$

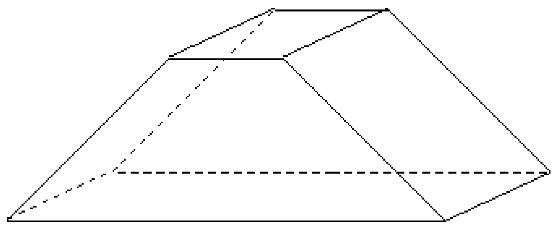
One end of the line is at u=5, v=1, v'=4 so $u' = 33$. The other is at u=2, v=1, v'=4 so u'=15. The length of the line in the right-eye image is 33-15=18.



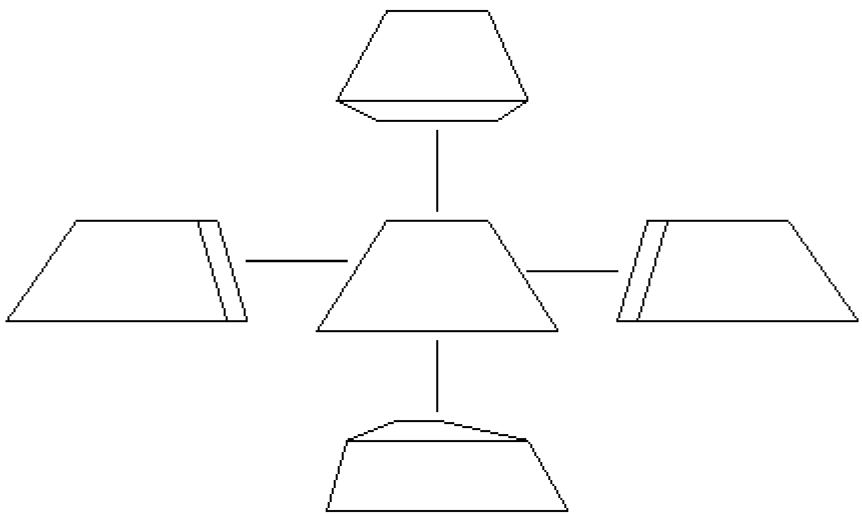
Left-eye image

Right-eye image

Question 3



(a) Adjacent aspects differ by just a single face. Starting from the aspect revealing only the front trapeziodal face, there are four aspects with two faces corresponding to the front plus any other face but the back. This part of the aspect graph looks like



(b) There is one set of topologically equivalent characteristic views with 1 face, one with 2 faces, and two with 3 faces. The 3 faces can be all adjacent, eg. top, front and one side are visible, or they not all adjacent, as when the left side, top and right side are visible. There is one with four faces (both sides, top and either front or back). So the answer is 5.

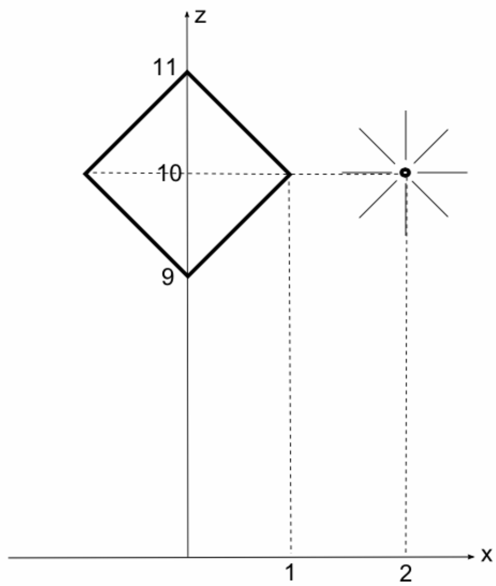
(c) Call the six faces f, b, l, r, t, o for front, back , left, right, top and bottom .

1. Aspects with one face: f, b, l, r, t, o, =6
2. Aspects with two faces: fl, fr, ft, fo; bl, br, bt, bo; lt, lo, rt, ro = 12
3. Aspects with tree faces: frt, flt, fro, flo; brt, blt, bro, blo; tlr = 9
4. Aspects with four faces: fltr, bltr =2

So total number of nodes in aspect graph = 6+12+9+2=29.

Question 4

The diagram below shows the crossection in the xz plane of a three dimensional scene. The Lambertian object in the center has a square crossection which is constant for all y. This object is illuminated by a line source of light (like a thin florescent tube) oriented parallel to the y axis, so it appears as a single point at x=2, z=10 in this crossection.



(a) What point or set of points in the image plane will be illuminated the brightest by the light reflected from the object? Explain your reasoning.

The reflected irradiance on the image plane will depend on two factors: the cosine of the angle between the source and the object surface normal, and the distance from the line source. On both counts, maxima occur at x=1, if dx/dz is defined as -1 at that point, else just to the left of that value of x, and for all y.

(b) Compute the ratio of irradiance at the point (x,y)=(1/2,1/2) in the image plane to the irradiance at the point (3/4,1/4).

The irradiance of the line source at the object falls of linearly with distance, and the reflected radiance from the object is proportional to the incident irradiance and the cosine of the angle between the surface normal, which is $[+1,-1]^T$ for both points, and the line connecting the point with the light source.

For (x,z)=(1/2,19/2) the distance to the source at (2,10) is sqrt(5/2) and the vector to the source is $[3/2\ 1/2]^T$. The cosine is the inner product of $[+1,-1]^T$ and $[3/2\ 1/2]^T$ divided by the product of their norms, which is 1/((sqrt(2)*sqrt(5/2))=1/sqrt(5).

For (x,z)=(1/4,37/4) the distance to the source at (2,10) is 3*sqrt(6)/4 and the vector to the source is $[7/4\ 3/4]^T$. The cosine is the inner product of $[+1,-1]^T$ and $[7/4\ 3/4]^T$ divided by the product of their norms, which is 1/((sqrt(2)*3*sqrt(6)/4)=2/(3sqrt(3)). So the desired ratio is

$$\frac{\text{irradiance at }(x,y)=(1/2,1/2)}{\text{irradiance at }(x,y)=(1/4,1/4)} = \frac{3*\text{sqrt}(5)}{2*\text{sqrt}(5)}\ \frac{3*\text{sqrt}(6)/4}{\text{sqrt}(5/2)} = \frac{9*\text{sqrt}(12/5)}{8} = 1.74$$