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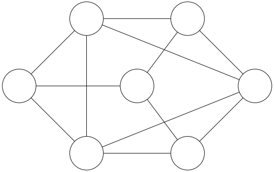
- Far away from polynomial time
- HC is NP-hard: it is unlikely that it can be solved in polynomial time.

Maximum Independent Set Problem

Def. An independent set of $G = (V, E)$ is a subset $I \subseteq V$ such that no two vertices in I are adjacent in G.



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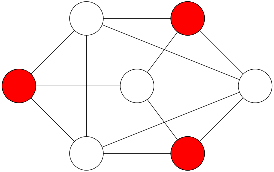

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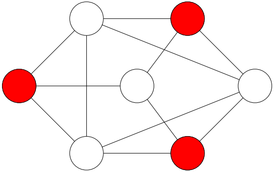

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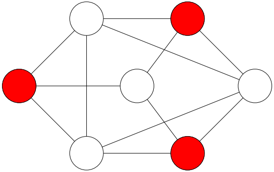

Maximum Independent Set Problem

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- Maximum Independent Set is NP-hard

Formula Satisfiability

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Input: boolean formula with n variables, with $\vee, \wedge, \neg$ operators.

Output: whether the boolean formula is satisfiable

- Example: $\neg((\neg x_1 \wedge x_2) \vee (\neg x_1 \wedge \neg x_3) \vee x_1 \vee (\neg x_2 \wedge x_3))$ is not satisfiable
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Outline

Decision Problem Vs Optimization Problem

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Fact For each optimization problem X, there is a decision version X' of the problem. If we have a polynomial time algorithm for the decision version X', we can solve the original problem X in polynomial time.

Optimization to Decision

Shortest Path

Input: graph $G = (V, E)$, weight w, s, t and a bound L

Output: whether there is a path from s to t of length at most L

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- Input: (3, 6, 100, 9, 60)
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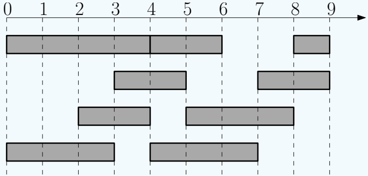

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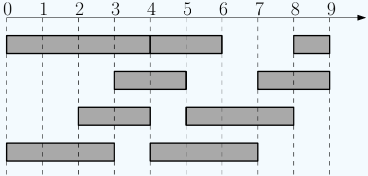

- $(0, 3, 0, 4, 2, 4, 3, 5, 4, 6, 4, 7, 5, 8, 7, 9, 8, 9)$

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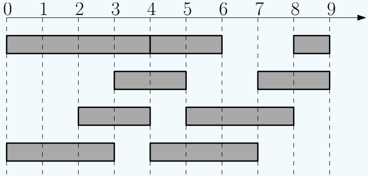

- $(0, 3, 0, 4, 2, 4, 3, 5, 4, 6, 4, 7, 5, 8, 7, 9, 8, 9)$
- Encode the sequence into a binary string as before

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Q: Does it matter how we encode the input instances?

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Q: Does it matter how we encode the input instances?

A: No! As long as we are using a "natural" encoding. We only care whether the running time is polynomial or not

Define Problem as a Function $X : \{0,1\}^* \to \{0,1\}$

Def. A decision problem X is a function mapping $\{0,1\}^*$ to $\{0,1\}$ such that for any $s \in \{0,1\}^*$, $X(s)$ is the correct output for input s.

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Def. A has a polynomial running time if there is a polynomial function $p(\cdot)$ so that for every string s, the algorithm A terminates on s in at most $p(|s|)$ steps.

Complexity Class P

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- The decision versions of interval scheduling, shortest path and minimum spanning tree all in P.

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Def. The message Alice sends to Bob is called a certificate, and the algorithm Bob runs is called a certifier.

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The Complexity Class NP

Def. B is an efficient certifier for a problem X if

- B is a polynomial-time algorithm that takes two input strings s and t, and outputs 0 or 1.
- there is a polynomial function p such that, $X(s) = 1$ if and only if there is string t such that $|t| \le p(|s|)$ and $B(s,t) = 1$.

The string t such that $B(s,t) = 1$ is called a certificate.

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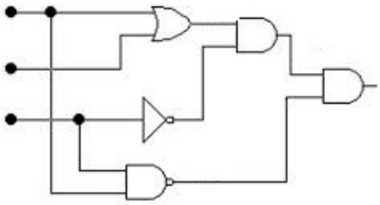
Circuit Satisfiablity (Circuit-Sat) Problem

Input: a circuit with and/or/not gates

Output: whether there is an assignment such that the output is 1?



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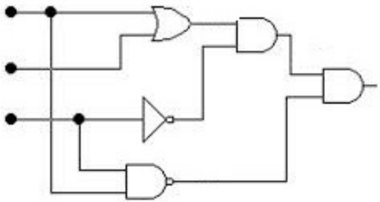

- Is Circuit-Sat $\in$ NP?

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- $\overline{\text{HC}} \in \text{Co-NP}$

The Complexity Class Co-NP

Def. For a problem X, the problem $\overline{X}$ is the problem such that $\overline{X}(s) = 1$ if and only if $X(s) = 0$.

Def. Co-NP is the set of decision problems X such that $\overline{X} \in$ NP.

Def. A tautology is a boolean formula that always evaluates to 1.

Tautology Problem

Input: a boolean formula

Output: whether the formula is a tautology

- e.g. $(\neg x_1 \wedge x_2) \vee (\neg x_1 \wedge \neg x_3) \vee x_1 \vee (\neg x_2 \wedge x_3)$ is a tautology

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- Thus Tautology $\in$ Co-NP

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- Thus, $X \in$ NP and P $\subseteq$ NP
- Similarly, P $\subseteq$ Co-NP, thus P $\subseteq$ NP $\cap$ Co-NP

Is P = NP?

Is $\mathsf{P} = \mathsf{NP}$?

- A famous, big, and fundamental open problem in computer science
- Most researchers believe $\mathsf{P} \neq \mathsf{NP}$
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- It would be too amazing if $P = NP$: if one can check a solution efficiently, then one can find a solution efficiently
- We assume $P \neq NP$ and prove that problems do not have polynomial time algorithms.
- We said it is unlikely that Hamiltonian Cycle can be solved in polynomial time:
  - if $P \neq NP$, then $HC \notin P$
  - $HC \notin P$, unless $P = NP$

Is NP $=$ Co-NP?

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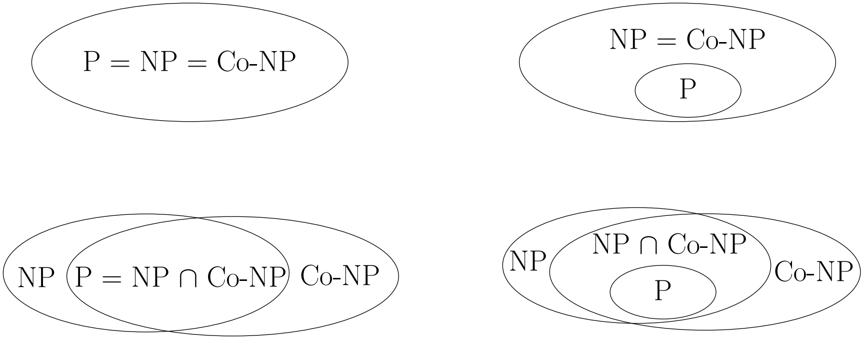
Is NP = Co-NP?

- Again, a big open problem
- Most researchers believe NP $\neq$ Co-NP.

4 Possibilities of Relationships

Notice that $X \in$ NP $\iff \overline{X} \in$ Co-NP and P $\subseteq$ NP $\cap$ Co-NP



- People commonly believe we are in the 4th scenario

Outline

1. Some Hard Problems
2. P, NP and Co-NP
3. Polynomial Time Reductions and NP-Completeness
4. NP-Complete Problems
5. Dealing with NP-Hard Problems
6. Summary

Polynomial-Time Reductions

Def. Given a black box algorithm A that solves a problem X, if any instance of a problem Y can be solved using a polynomial number of standard computational steps, plus a polynomial number of calls to A, then we say Y is polynomial-time reducible to X, denoted as $Y \leq_P X$.

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To prove positive results:

Suppose $Y \leq_P X$. If X can be solved in polynomial time, then Y can be solved in polynomial time.

To prove negative results:

Suppose $Y \leq_P X$. If Y cannot be solved in polynomial time, then X cannot be solved in polynomial time.

Polynomial-Time Reduction: Example

Hamiltonian-Path (HP) problem

Input: $G = (V, E)$ and $s, t \in V$

Output: whether there is a Hamiltonian path from s to t in G

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Lemma HP $\leq_{\mathsf{P}}$ HC.

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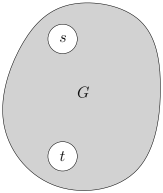
Lemma HP $\leq_\mathsf{P}$ HC.



Polynomial-Time Reduction: Example

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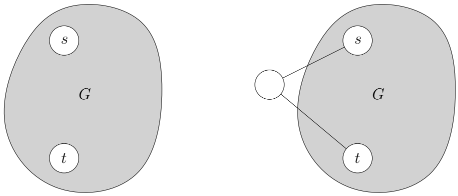
Input: $G = (V, E)$ and $s, t \in V$

Output: whether there is a Hamiltonian path from s to t in G

Lemma HP $\leq_{\mathsf{P}}$ HC.



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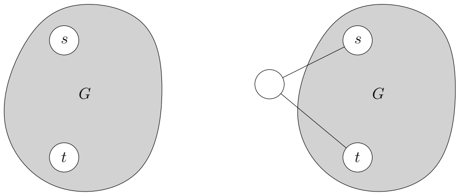

Obs. G has a HP from s to t if and only if graph on right side has a HC.